

Refraction of heat flow on subsurface contrast structures – the influence both on geothermal measurements and interpretation approaches

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Abstract: The paper deals with some problems of the heat flow refractions on the subsurface structures with contrasting thermal conductivities. The qualitative and quantitative analyses of these effects were made on the selected structure configurations. Analysed structures are important particularly for the study of temperature as well as heat flow density distributions influencing the Earth's heat flow measurements, the construction of terrestrial heat flow distribution maps and also for the interpretation of the heat flow density data. The related 2D and 3D mathematical problems were solved by means of the finite difference methods. The presented results have a great importance both for the solution of some problems of the applied geothermics (e.g. determination of the heat flow density values from measured data, their accuracy, eventual data corrections and relation of measured data to the surface heat flow density distributions) and also for the modelling of the thermal state of the lithosphere (e.g. determination of the boundary conditions and of the model check parameters, robustness of the modelling approaches, etc.).

Key words: geothermal models, refraction, interpretation, heat flow density, construction of maps, finite difference method

1. Introduction

Commonly, heat does not flow vertically from the Earth's interior. In fact, it is only in the case of horizontally layered media with horizontal surface and moreover, when the density of the heat flux from deeper Earth's parts is homogeneous. A lateral heat source and thermal conductivity heterogeneities, curved surface boundaries with varying temperature, non-vertical flows of various liquids, and many other factors may distort the heat flux away from

the vertical. The contacts of geological structures with contrasting thermal conductivity parameters play a very important role by the formation of the temperature field and by the distribution of heat characterized by the heat flow density absolute values and vector orientations. The importance of the thermal refraction increases mainly in subsurface Earth's parts. This fact is given both by higher lateral variability of structures and by greater local diversity of the thermal conductivity parameters.

The basic refraction effects were studied in models with the disturbing bodies bounded by the coordinate surfaces and buried in homogeneous half-space or in horizontally layered media (*Carslaw and Jaeger, 1959; Lee and Henyey, 1974; Hvoždara and Majcin, 1985; Hvoždara, 2000; 2009* and others). The thermal conductivity refraction on polyhedral as well as polygonal cross-section bodies was solved by various means, namely the finite difference method (*Ljubimova et al., 1976; 1983; Jones and Sydora, 1980; Šafanda, 1988; Majcin, 1992; Nagihara, 2003*), the finite element approaches (*Geertsma, 1971; Lee and Henyey, 1974; Jensen, 1983*) and the boundary integral equations method (*Hvoždara and Schlosser, 1985, Chen and Beck, 1991; Hvoždara and Valkovič, 1999; Hvoždara, 2008; Hvoždara and Majcin, 2009; 2011, etc.*).

The special attention was paid to salt bodies because the salt has the thermal conductivity coefficient two or more times greater than that one of other surrounding lithology commonly present in the basins. A similar thermal conductivity differences (contrasts) exist also on the contacts of not compacted sediments either with carbonates or with most of crystalline rock types. The conductivity contrasts characterized by the ratio of 1.5 are usual for real world situation in the upper parts of the Earth's crust. However some existing geothermal models have been made also for higher contrasts to show various special refraction effects. The refraction on anisotropic structures was studied only in very rare cases e.g. in the paper of *Šafanda (1995)*.

This contribution is devoted to the analysis of the influence of the heat flow refraction effects on the determinations of the heat flow density values, the visualisation of these data, and the geothermal interpretation approaches. For this purpose we enhanced the existing results of refraction modelling by both qualitative and quantitative analyses of the heat flow refractions on additional selected structure configurations.

2. Modelling method

The refraction effects were studied on model structures built up from polyhedral bodies in 3D refraction cases or by bodies of polygonal cross sections in 2D refraction models. The steady-state heat transfer conditions and no heat sources were considered in the heat conduction equation inside the regions under study (Fig. 1)

$$\operatorname{div} \left(k(x, y, z) \operatorname{grad} U(x, y, z) \right) = 0. \tag{1}$$

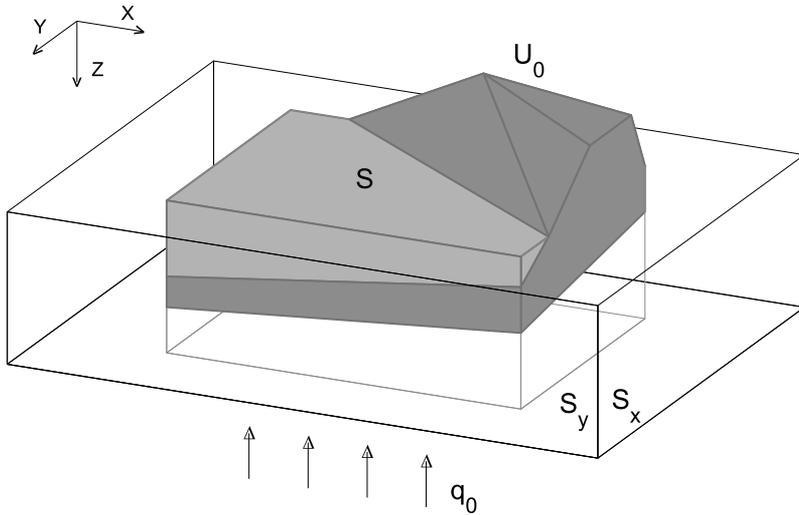


Fig. 1. Configuration of model problems.

Here U is the temperature distribution function and k is the thermal conductivity parameter which is constant or linearly dependent on depth within the separate structure blocks.

The constant temperature distribution U_0 is defined at the upper surface S of the model structures with or without the topography

$$U(x, y, z)|_s = U_0. \tag{2}$$

The heat flow density distribution $q_0(x, y)$ from bottom horizontal boundary at the depth h is defined by formula (3)

$$k(x, y, z) \left. \frac{\partial U(x, y, z)}{\partial z} \right|_{z=h} = q_0(x, y). \quad (3)$$

Commonly, a constant function $q_0(x, y) = q_c$ is considered. The model area is laterally bounded by coordinate surfaces S_x , S_y in x and y directions respectively, sufficiently distant from the main structure under study. The symmetry conditions (i.e. zero horizontal heat flow density values) are prescribed at these lateral model surfaces

$$k(x, y, z) \left. \frac{\partial U(x, y, z)}{\partial x} \right|_{S_x} = 0, \quad (4)$$

$$k(x, y, z) \left. \frac{\partial U(x, y, z)}{\partial y} \right|_{S_y} = 0.$$

At the interface planes $S_{1,2}$ between the two model media M_1 , M_2 the conditions of ideal contact should be fulfilled. These conditions are specified by the continuity of both the temperature distribution and the heat flow density component normal to $S_{1,2}$ in points $X_0 \equiv (x_0, y_0, z_0)$

$$\lim_{x_1 \rightarrow x_0} U(X_1) = \lim_{x_2 \rightarrow x_0} U(X_2). \quad (5)$$

$$\lim_{x_1 \rightarrow x_0} \left\{ k(X_1) \frac{\partial U(X_1)}{\partial n} \right\} = \lim_{x_2 \rightarrow x_0} \left\{ k(X_2) \frac{\partial U(X_2)}{\partial n} \right\}, \quad (6)$$

where $X_1 \in M_1$, $X_2 \in M_2$, $X_0 \in S_{1,2}$.

The problem (1–6) was solved by the finite difference numerical approach. The computational domain was covered by two- or three-dimensional heterogeneous grid. The grids are horizontally and vertically denser mainly near the model structure discontinuities and moreover, they have smaller grid spacing close to the upper surface of the region under study. We calculated the models alternatively for various grid point distributions to minimize the influence of the numerical approach on the solution accuracy. All cross points of both grid lines and the topography boundary of studied models are defined from the basic grid by spacing and consequently it is not necessary to make any regularization. The approximation of 2D mathematical problems (1–6) was made by the approaches proposed for five-point schemes

(*Samarski, 1971; Majcin, 1982*). The triangular iteration method of successive over-relaxation was applied to the solution of the related algebraic equations system (*Samarski and Andreev, 1976; Samarski and Nikolaev, 1978; Tsvyashchenko et al., 2000*).

The 3D problems were solved by enhancement of the above-mentioned methods to 7-point scheme with the same approximation accuracy of both material parameters and derivatives, and additionally by the generalization of a relaxation iteration scheme to three dimensions. The C language source codes were created for the structural models construction, the solution of mathematical problems and geothermal output data calculations too. Some supporting calculations and graphical outputs have been carried out within the MATLAB software environment.

3. Results of modelling

The presented models of heat flow refraction effects were calculated for the upper crust structures most common value conductivity contrasts of 1.5, though the thermal conductivity coefficient of these rocks varies within a range of about one order of magnitude.

We used the conductivity values typical for the Neogene sediments $\sim 2.0 \pm 0.5$ W/mK against the value of about 3.0 W/mK as representative one for most of the basement rocks. The distributions of calculated heat flow density components are plotted as a relative values (in percents) to the flow of heat amount q_c supplied from the bottom of the model. This allows to recalculate the refraction effects for various regional conditions, i.e. for various heat flows from deeper parts of the Earth, and finally to use the results also for quantitative analysis and interpretations of real data. The determination of the heat flow data and the construction of the terrestrial heat flow density maps require the vertical component of the heat flow density. This is the reason why we are analyzing mainly these data. In models with the horizontal upper surface of the model structure and with the constant temperature U_0 , the studied parameter is equal to the surface heat flow density distribution $q_s(x, y)$.

The geophysical branches, analysing the potential fields, work usually with the potential and/or with the gradient of the potential. Geothermics

has a specific problem caused by the fact that it determines, interprets and displays not only the temperature and temperature gradients but, in addition, also the heat flow density values. The core of the problem originates in the definition of this parameter by Fourier's law of the thermal conduction: $q = -k \text{grad}(U)$. Minimally, the step changes in thermal conductivity of rock complexes within the studied areas induce the jumps also in the heat flow density vector components distributions, with the exception of the special situations where a component of the heat flow density vector coincides with the boundary normal vector (i.e. the conditions of continuity (5), (6) are fulfilled for the component analysed).

The very simple 2D model of heat flow refraction on two semi-infinite layers (structure is shown in Fig. 2a) with different values of thermal conductivity – the vertical contact of contrasting rock complexes (Majcin, 1992;

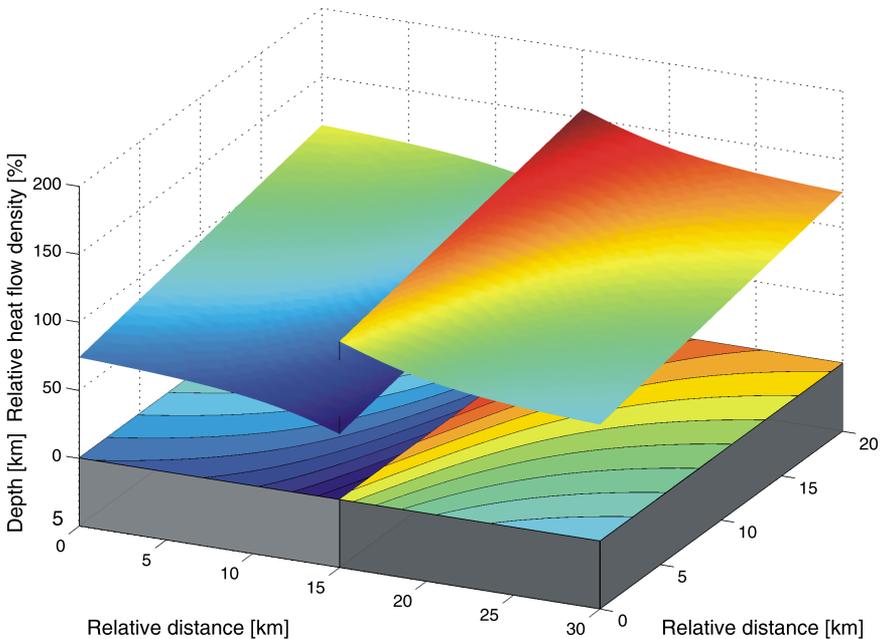


Fig. 2a. Relative surface heat flow density q_s/q_c distribution for the refraction model with the vertical boundary between two semi-infinite layers and with the additional heat flow increase along the vertical boundary. Model thermal conductivities: dark grey – 3.0 W/m K and medium grey – 1.5 W/m K.

Jaupard and Mareschal, 2011) – shows the jump in the distribution of the calculated surface heat flow density. This jump corresponds to the step change of the thermal conductivity parameter from k_2 to k_1 ($k_1 < k_2$) in horizontal direction (perpendicularly to contact of blocks). The size of the step change does not depend on the thickness T_L of these semi-infinite layers in contact. The depth range of the contrast boundary affects only the horizontal ambit of the refraction anomaly. The surface heat flow density distribution is perturbed on both sides of the surface discontinuity by more than 10% of the original heat flow density q_c over the distance $\sim T_L$ for common contrasts $k_2/k_1 = 2.0$ and up to half of this distance for slightly smaller contrasts about 1.5. The function describing the surface heat flow density distribution is strictly monotonic (decreasing in direction to more conductive rock types) equally on both sides of the contrasting rock blocks

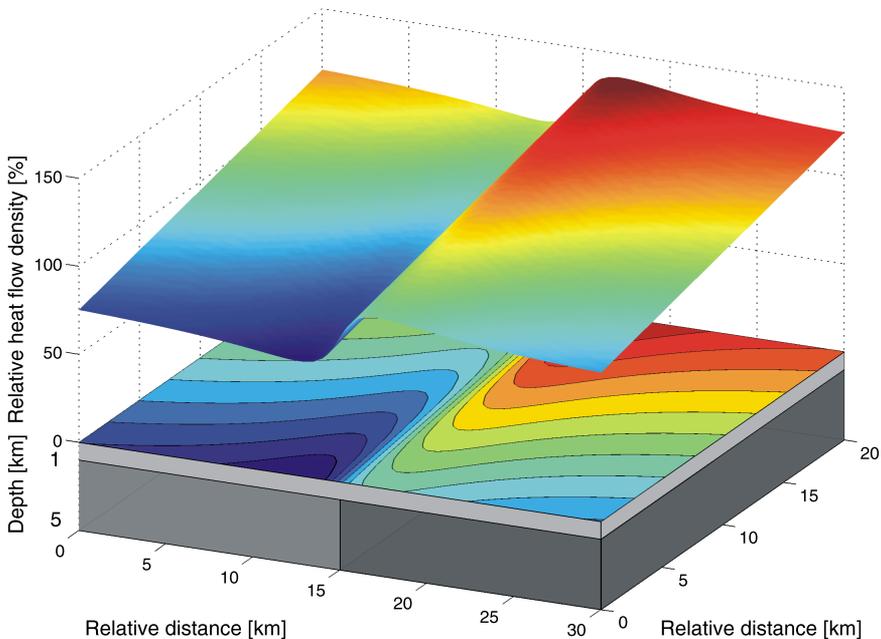


Fig. 2b. Relative surface heat flow density q_s/q_c distribution for the refraction model with neutral material over the vertical boundary between two semi-infinite layers and with the additional heat flow increase along the vertical boundary. Model thermal conductivities: dark grey – 3.0 W/m K, medium grey – 1.5 W/m K and light grey – 2.0 W/m K.

boundary (see Figs. 2a and 3a). However, it is not true over the whole region. The jump over the boundary increases the surface heat flow density value stepwise by the factor k_2/k_1 in the direction mentioned. The sets of such 2D refraction models (Majcin, 1992) allow us to determine also the horizontal changes of the surface heat flow density which are typical for various configurations defined by the thickness of semi-infinite layers, the thermal conductivity contrast and the value of flow supplied at the base of the model structure.

In real 3D case of the uniform heat flow refraction on vertical discontinuity of the thermal conductivity (structure is shown in Fig. 3a) the isolines are parallel to the discontinuity between two media (upper strip in Fig. 3a). The similar situation (parallel isolines and the step change of the heat flow density distribution) will be achieved also for the refraction of the heat flow density q_0 which is not constant in the direction perpendicular to this discontinuity i.e. $q_0(x) = q_c + f(x)$. Such model situation simulates the variability of the thermal activity across the contrast structure. On the other hand, a different surface heat flow density distribution and consequently the isolines shapes will be obtained for models with the heat flow density q_0 variable along the contrast structures. The basic analysis was made for the models of the refraction of the heat flow q_c with additional contribution in the form of linear increase along y axis as follows:

$$q_0(y) = q_c + \kappa y. \quad (7)$$

From the common properties of the problem (1–3,5–7) solution for an infinite layer and from the numerical modelling of the problem (1–7) in sufficiently great finite layer area, we find that the additional heat flow density increase (defined by the gradient κ) causes the bending of surface heat flow density isolines. However, the lines are broken over the discontinuity. These special heat flow refraction effects – discontinuities in the surface heat flow distribution – are related to the construction of the terrestrial heat flow maps over the tectonically complex structures with outcrops of contrasting rock complexes. Moreover, the horizontal distributions of the surface heat flow density are very important for the interpretation of data measured close to the vertical or nearly vertical boundaries between blocks with dissimilar thermal conductivities. The effects of this type were studied also on the structures with buried disturbing bodies like half-cylinders, rotational

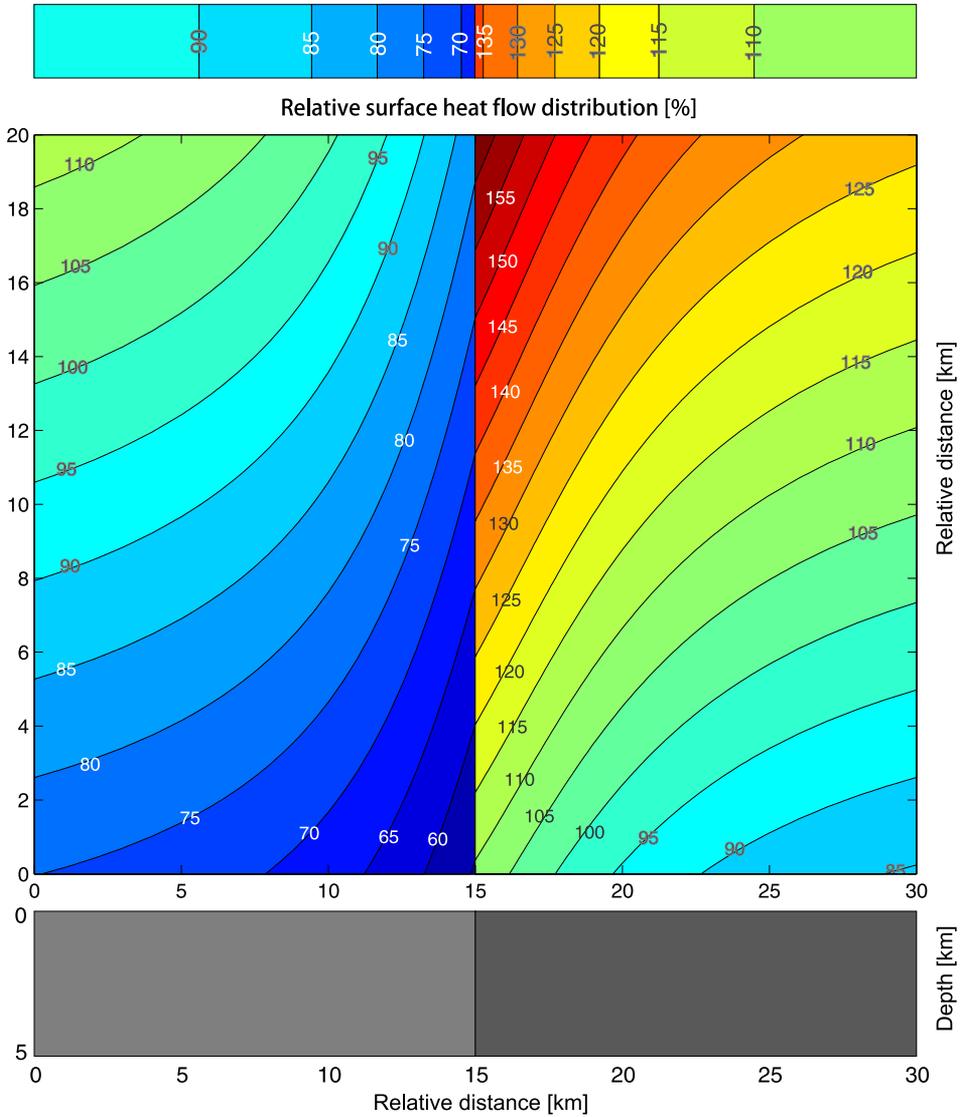


Fig. 3a. Isoline maps of the relative surface heat flow density q_s/q_c distribution for the refraction model described in Fig. 2a. Upper strip part – isolines for the refraction of the heat flow density q_c ; middle part – isolines for the refraction with the additional heat flow increase along the vertical boundary; lower part – model structure.

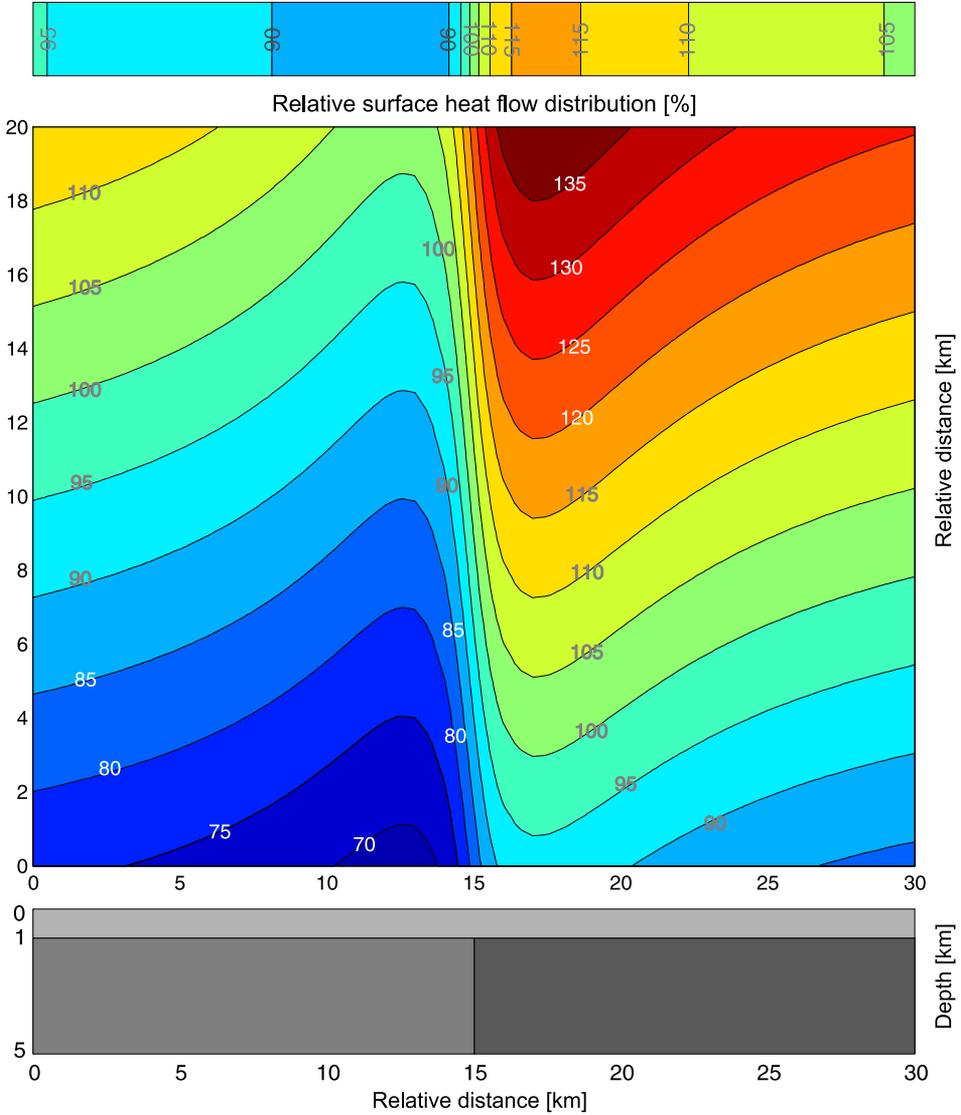


Fig. 3b. Isoline maps of the relative surface heat flow density distribution for the refraction model described in Fig. 2b. Upper strip part – isolines for refraction of constant heat flow density; middle part – isolines for the refraction with the additional heat flow increase along the vertical boundary; lower part – model structure.

half-ellipsoids, etc. (*Carslaw and Jaeger, 1959; Lee and Henyey, 1974; Hvoždara, 2009*), the boundaries of which are perpendicular to the upper boundary of half-space.

We present the models of surface heat flow distributions (Fig. 2a and Fig. 3a) for refraction on semi-infinite layers vertical contact with contrast $k_2/k_1 = 3.0/1.5$, moreover with or without the additional heat flow increase along the boundary equal to 2% of q_c per 1 km. The models with a covering neutral layer of thermal conductivity equal to 2.0 W/mK (Fig. 2b and Fig. 3b) show the diffusion effect removing the problem of the surface heat flow distribution discontinuity. The transitional zone width is strictly depending on the thickness of the neutral layer at the surface of model. Generally, for the real-world situations, it is not possible to assume that such very thick layers remove the step changes in the surface heat flow density distributions. The geological situations allow to apply dimensions in order of metres or tens of metres as a reasonable approximation for the transitional zone. Let us note that in some older publications (*Ljubimova et al., 1976; Czereminski, 1977* and others) the step changes in the surface heat flow density distribution over the surface conductivity discontinuities are omitted. Usually they are not considered owing to insufficient density of calculation points in critical zones and/or to usage of very simple “point to point” construction of continuous function in 2D or 3D region under study.

The models of heat flow refraction on aslant discontinuities of blocks enhance the basic knowledge by more realistic situations and they provide new results related to the surface heat flow density distribution near the contacts. Presented model of two dissimilar media with contact the inclination angle of which is 45 degrees (Fig. 4 and Fig. 5) supplements the results obtained by *Majcin (1992)*. The heat flow density increases (or vice-versa decreases) in the direction to the surface outcrop of the contrasting boundary within its close surroundings (upper strip part of Fig. 5). The course depends on configuration of conductivities in overlapping and underlying rock blocks. The models with relatively steep aslant contact boundaries of blocks (angle $\geq 45^\circ$) have a local surface heat flow density anomaly on the side of overlapping layer (Fig. 5, *Majcin, 1992*). The distribution of relative surface heat flow density q_s/q_c contains (Fig. 4) also extreme values which are non-representative in the sense both of practical geological situations and of interpretation of the measured data. The most frequent and also the most

interesting configuration of this model type – a sedimentary basin margin – has the positions of the thermal state characteristics measurements usually on the side of less conductive basin sediments. Here, the temperature field is influenced mainly by refraction of heat flow on the boundary with conductivity contrast and is partially affected by refractions on topography (Majcin and Polák, 1995 and Šafanda, 1988). Some problems related to topographic effects are discussed in the next parts of this contribution.

The previous models are applicable to the analysis of temperature field and surface heat flow density distributions at the boundaries of sedimentary basins with relatively great horizontal dimension, i.e. to the situation that is not influenced by other refraction effect. However, the surface structures of the West Carpathians and of other tectonically similar regions contain also frequently changed units with contrasting thermal conductivity parameters

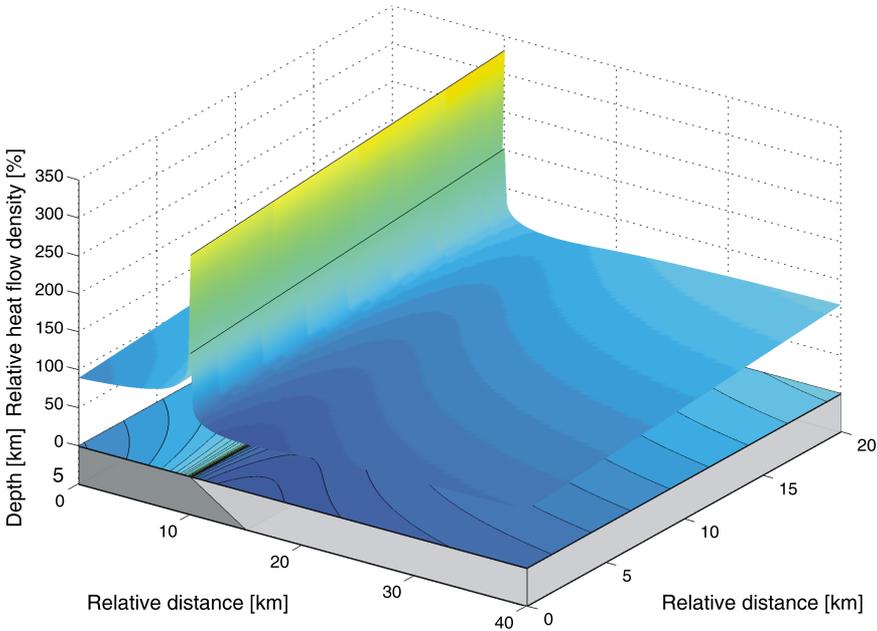


Fig. 4. Relative surface heat flow density q_s/q_c distribution for the refraction model with aslant boundary between two semi-infinite layers (basin margin) and with the additional heat flow density increase along the vertical boundary. Model thermal conductivities: dark grey – 3.0 W/m K and medium grey – 1.5 W/m K.

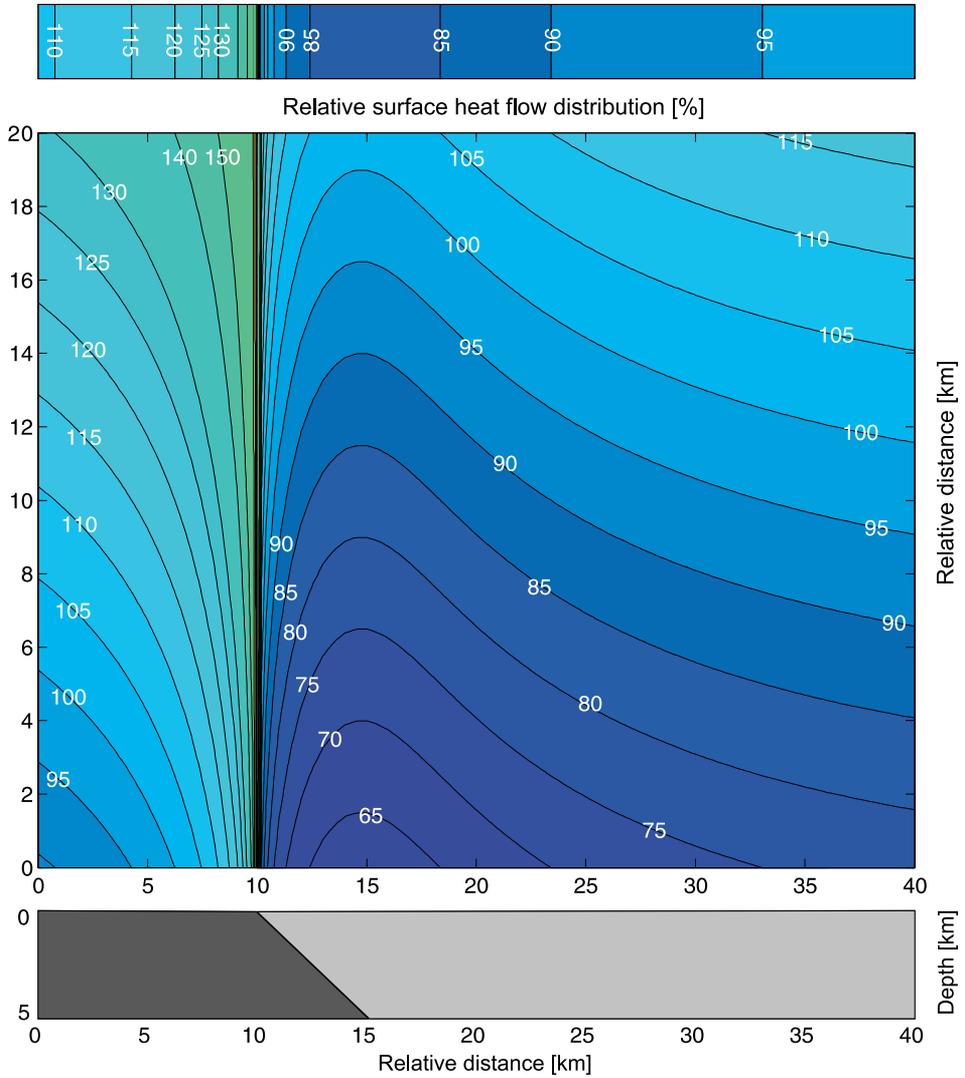


Fig. 5. Isoline maps of the relative surface heat flow density q_s/q_c distributions for the refraction model described in Fig. 4. Upper strip part – isolines for the refraction of the heat flow density q_c ; middle part – isolines for the refraction with the additional heat flow increase along the aslant boundary between rock blocks; lower part – basin margin model structure.

– less conductive filling of intra-mountain basins and outcrops of basement rocks usually those with relatively very high thermal conductivities. The refraction effects of such 2D closed structures were analysed on simplified (polygonal or curved) shapes (*Carslaw and Jaeger, 1959; Lee and Henyey, 1974; Majcin, 1992; Majcin and Polák, 1995; Hvoždara, 2000, 2009* and others).

These models are supplemented here by the surface heat flow density models for outcropping horst structure as well as for basin structure with both variable slope angles of contrast boundaries and additional heat flow density increase along the structure. The calculated surface heat flow density distributions (presented in Figs. 6a and 6b) reflect the dependency on slope angles and also on distances between outcropping contrast boundaries. Smaller distances induce higher mutual influence of refractions on slanted surfaces with the step change of the thermal conductivity coefficient. Hence the heat flow density distribution is symmetrical above the narrow outcrop structure (Fig. 6a) and almost symmetrical in the central part of the basin (Fig. 6b). Besides, the second model shows that the distance of surface basin borders is already sufficient for partial separation of the refraction influences of both lateral boundaries with thermal conductivity contrast.

The values of the heat flow density are usually not determined from the depth interval close to the surface (despite they are denoted as surface heat flow densities) and moreover, the depth range for the temperature gradient calculation is rather great. Both mentioned facts have a great importance for the calculation of the terrestrial heat flow density data as well as for the determination of the surface values of this thermal parameter. The influence of the refraction effects on horizontal as well as vertical distributions of both heat flow density and its vertical component is analysed on the sedimentary basin margin model (Fig. 7a) and on other additional related structural models as well. These models are selected as sufficiently representative for basic evaluation of the refraction effects in the real upper crust structures. We used very frequent thermal conductivity contrast 2.0/3.0 W/m K. The vertical distributions of the heat flow density vertical component were calculated with the vertical step of 100 metres for various selected positions over the sedimentary basin structure (Fig. 7a).

The calculated distributions (Fig. 7b) revealed that the depth variability of the heat flow density vertical component is noticeable and it reaches, in

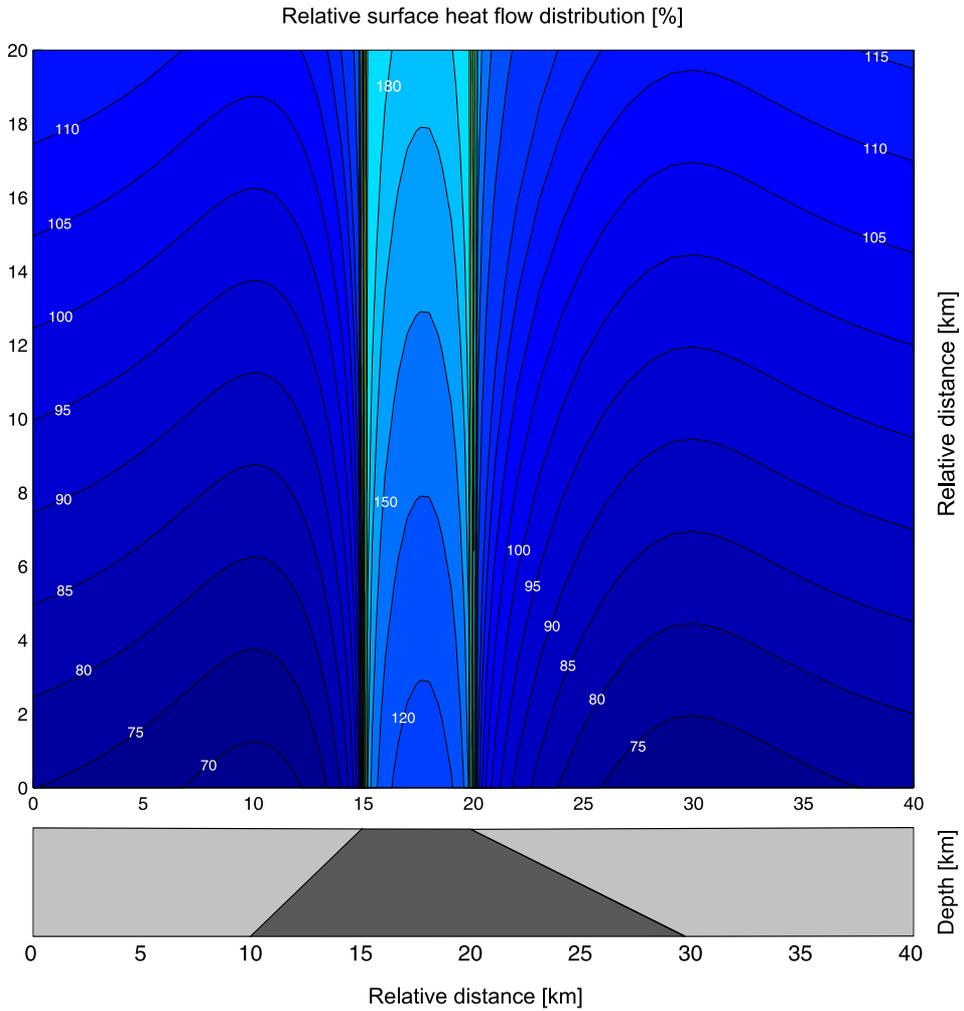


Fig. 6a. Isoline map of the surface heat flow density q_s/q_c distributions for the refraction on the outcropping horst structure with the additional heat flow density increase along the aslant boundaries between rock blocks. Model structure is shown at the bottom. Model thermal conductivities: dark grey – 3.0 W/m K and medium grey – 1.5 W/m K.

separate material blocks, the values about 20% of original heat flow and locally even more (upper part of profile for point C). The changes caused by the refraction diminish in relatively great distances – the profile at the check

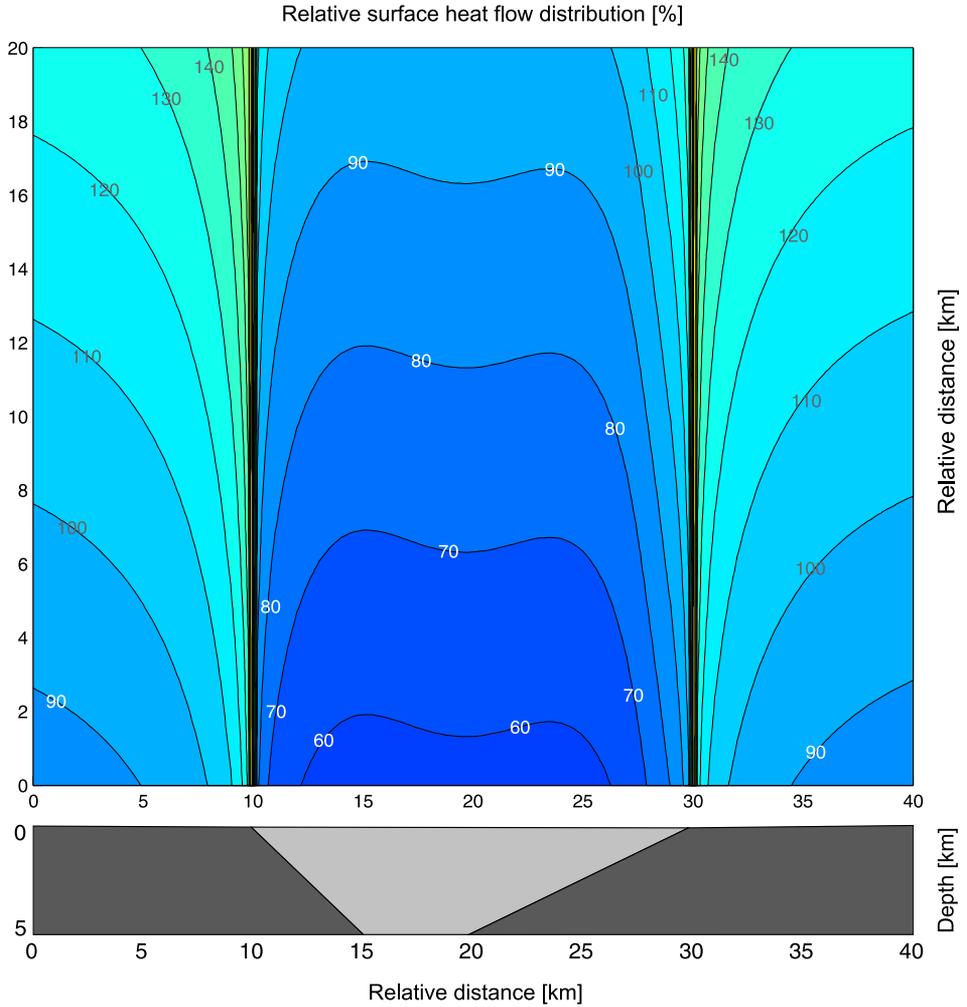


Fig. 6b. Isoline map of the surface heat flow density q_s/q_c distributions for the refraction on the narrow basin structure with the additional heat flow density increase along the aslant boundaries between rock blocks. Model structure is shown at the bottom. Model thermal conductivities: dark grey – 3.0 W/m K and medium grey – 1.5 W/m K.

point *J*. The step changes existing on the slanted lateral boundary between the sedimentary basin filling and the basement materials (distributions under the points D, E and F) are the most important from the interpretation

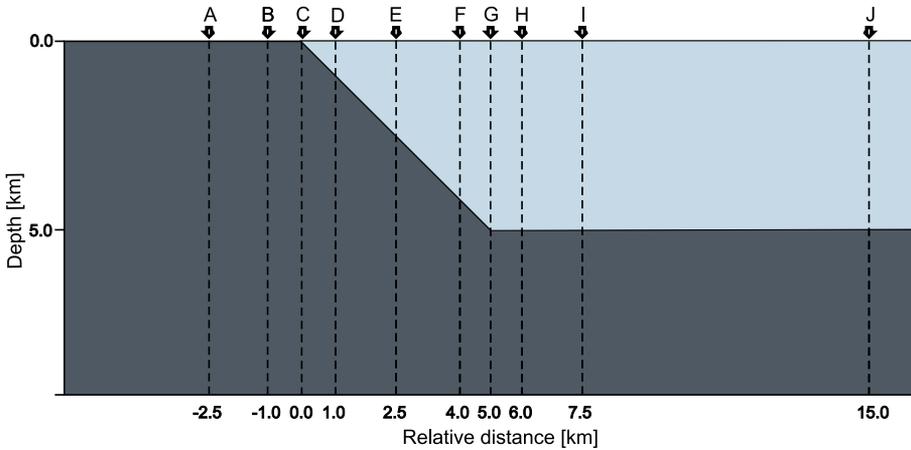


Fig. 7a. Structural model of the basin margin with checkpoints for calculation of vertical distributions of the relative vertical heat flow density component q_z/q_c . The positions of points are relative to the surface basin/basement contact (point C) and the shown distances correspond to basic refraction model with the depth 5 km and the inclination of the boundary equal to 45 degrees.

point of view. They are caused by the step change in direction of the heat flow stream lines (this effect results in cusped poly-curves) and by sudden change of absolute value of total heat flow density. The temperature isolines have such cusps at the aslant boundaries with the thermal conductivity contrasts too. The heat flow is refracted also on horizontal boundaries between basin filling and basement (the vertical distributions at the check points H, I and J). However, the vertical component of the heat flow density has no discontinuity because the conditions (5) and (6) are fulfilled. The special distribution of the heat flow at the toe of basin/basement lateral boundary (check point G) is not analysed because it is not supposed as representative for a real geological conditions.

The refraction of the heat flow is important for various cases of sloped layered media with different thermal conductivity parameters in geological conditions as well as in micro scale, for instance, in laboratory measurement of rock thermo-physical parameters.

The results achieved for the basic model are enhanced by additional combinations of structures. The refraction effects are represented by verti-

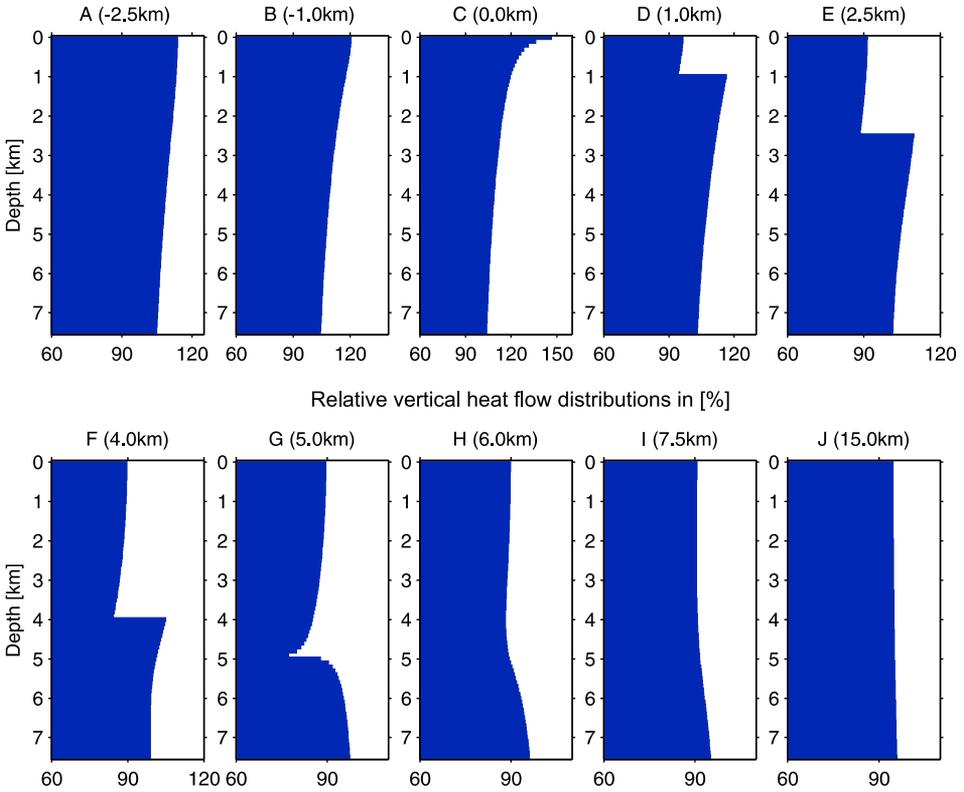


Fig. 7b. Depth distributions of the relative vertical heat flow density component q_z/q_c calculated in the check points *A – J* for the basic model of basin margin.

cal distributions of the vertical heat flow density component at the position which corresponds to the centre of sloped lateral boundary between the basin and the basement (point E defined for basic model). Figure 8 contains the plots of distributions for several models: the basic model with the constant thermal conductivity in basin, the model with the additional topography (sloped from the basin side and with the maximal height of 500 m) over the outcropping part of the basement rocks, and the model with basin filling having the thermal conductivity linearly depending on the depth (1.5 W/mK at the surface and 2.5 W/mK at the bottom of basin). The additional topography increases the vertical variability of plotted dis-

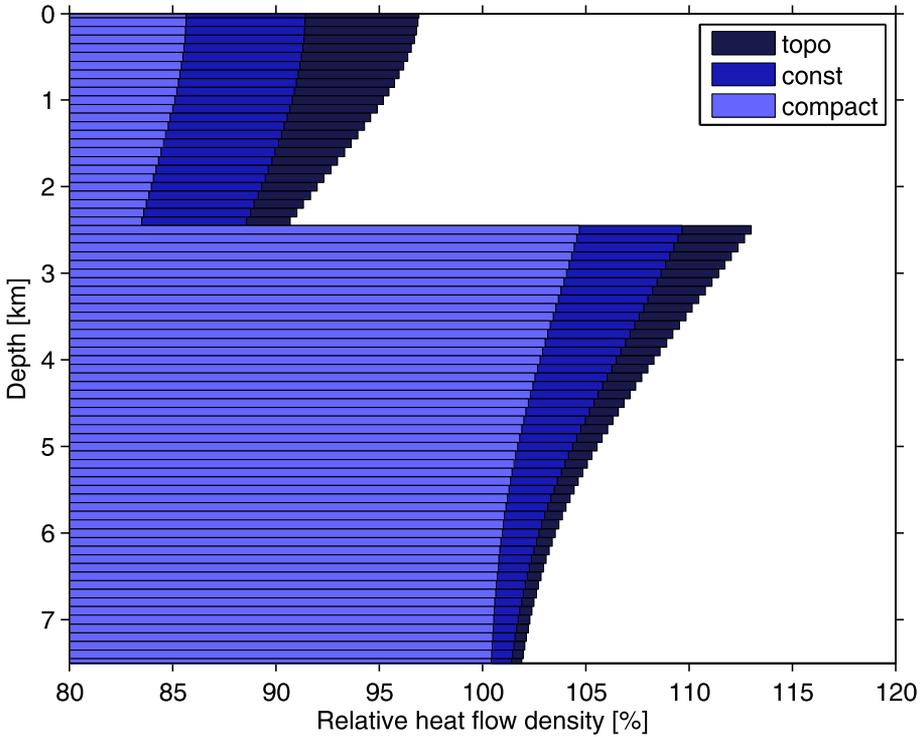


Fig. 8. Comparison of vertical distributions of the relative heat flow density vertical component q_z/q_c for various basin margin models. (const) – basin with constant thermal conductivity; (topo) – the same model with additional topography; (compact) – basin with linear dependency of the thermal conductivity coefficient on the depth. Plotted are the distributions related to the centre of the sloping boundary (check point E in Fig. 7a).

tributions. However, the model with non-constant thermal conductivities (approximating for example the conditions of compaction of the sediment filling) reduces the ranges of plotted parameter mainly in the basement. The step changes in the depth 2.5 km are nearly the same for all models. The results for models with various angles of the sloped contact between the contrasting structures of sedimentary basin and the basement are plotted in Fig. 9. The depth variability of the vertical heat flow density values as well as the range of step change strongly depends on the steepness of the contrast boundary. The mentioned values are in level of some percents of

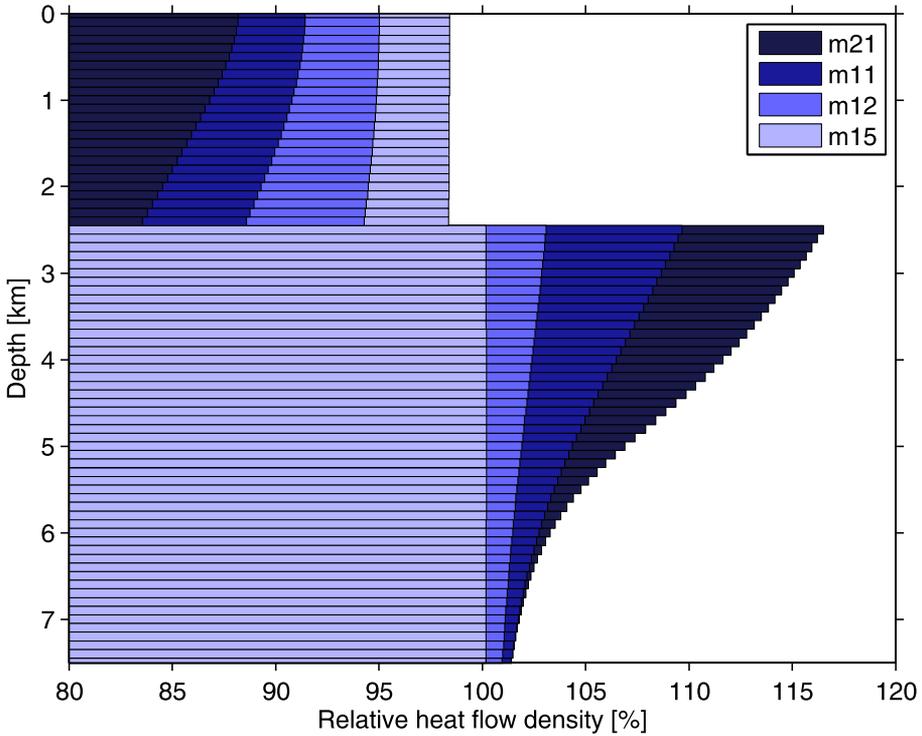


Fig. 9. Comparison of vertical distributions of the relative vertical heat flow density component q_z/q_c for various basin/basement boundary inclination models. The inclination is defined here by the ratio of the basin depth d and the horizontal extent h of sloping boundary between the basin and the basement. (m11) is the basic model with $d = h$; (m21) – $d/h = 2.0$; (m12) – $d/h = 0.5$; (m15) – $d/h = 0.2$. Plotted are the distributions related to the centre of the sloping boundary.

the original heat flow q_c for the case of relatively shallow basin/basement boundaries (angles about 10–20 degrees). The refraction effects on steeper slopes of contrast boundaries (~ 60 – 70 degrees) cause the step changes about 30–40% of the heat flow density while the variation is about 5% in the sedimentary filling and nearly 15% in upper part of the basement.

The last studied models provide the basic evaluation of the influence of the dimension of anomalous body on the distribution of the heat flow density at the half-space surface as well as in various depth levels below

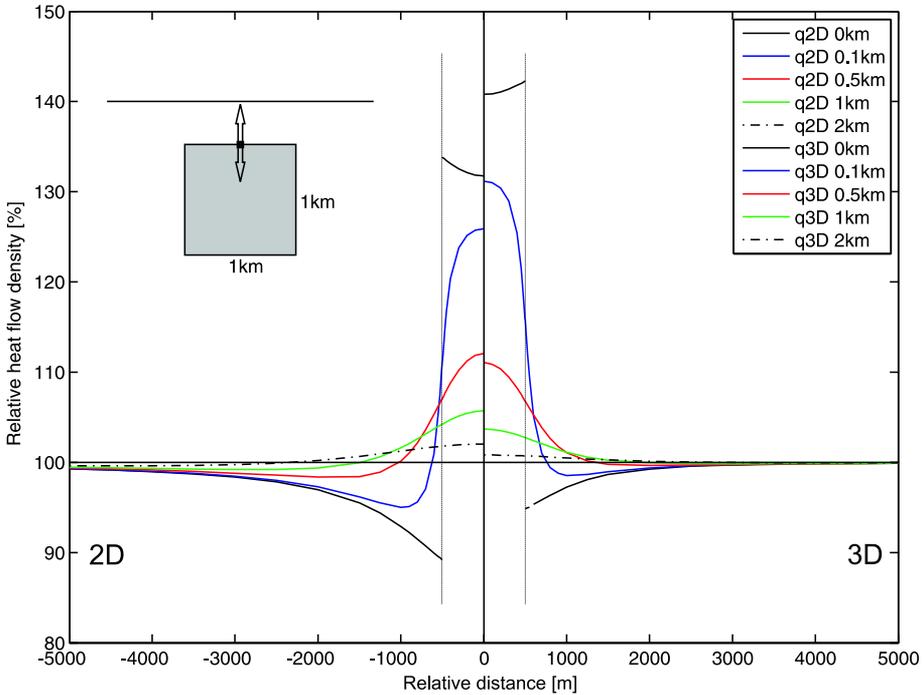


Fig. 10a. Comparison of the relative surface heat flow distributions q_s/q_c for the refraction on 2D and 3D anomalous body placed in the half-space at various depths (listed in figure legend) of upper face. Left side: 2D model with anomalous parallelepiped of rectangular cross section. Right side: 3D model with anomalous cube.

it. Figure 10a presents the models of the heat flow refraction on both the bounded anomalous bodies of cube shape and the horizontal infinite prism with square cross section. Both bodies have shape dimensions which allow to compare the refraction effects caused by them. The calculations were done for models with the thermal conductivity contrasts defined by value of 2.0 W/mK used for half-space and by value of 3.0 W/mK used for buried anomalous bodies. The surface heat flow density distributions in both 2D and 3D cases for the bodies placed directly to the surface of the half-space (created always from parts with discontinuity over the body edges) show that the refraction on 3D bounded body gives greater amplitudes than in 2D model case. The similar situation occurs also for instances where the

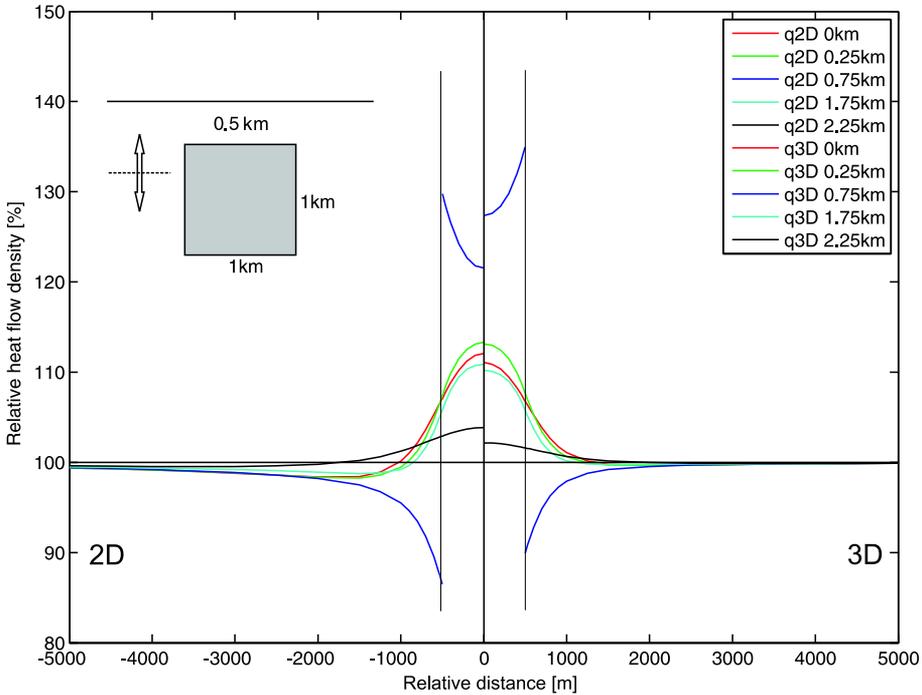


Fig. 10b. Comparison of horizontal distributions of the heat flow density vertical component q_z/q_c for refraction on 2D and 3D anomalous body placed in the half-space at various depth levels (listed in figure legend) of upper face. Left side: 2D model with anomalous parallelepiped of rectangular cross section. Right side: 3D model with anomalous cube body.

anomalous bodies are placed relatively close to the surface. If the bodies are buried under the depth level greater than half of their characteristic dimension, then the anomalies start to behave similarly like source types and the 2D model bodies cause slightly greater effects than 3D ones. We have received some compatible results also for other shapes of the anomalous bodies. Figure 10b contains the comparison of the distribution of the heat flow density vertical component with fixed space model configuration and for various depth levels starting from the half-space surface. In the depth level which crosses the centre of anomalous bodies the computed anomaly amplitude is greater for 3D case. However, the differences of refraction ef-

fects are very small in levels somewhat over and also under the 2D and 3D anomalous bodies. Admittedly, the step changes in the heat flow density vertical component distribution on lateral edges of bodies crossing them at all depth levels are very important from the interpretation point of view.

4. Conclusions

The presented computation results along with the qualitative and quantitative analysis of the refraction effects carried out on the selected structure configurations are important particularly for the study of both temperature and heat flow density distributions, for estimation of their influence on the Earth's heat flow measurements, for the construction of terrestrial heat flow distributions maps, for the interpretation of the heat flow density data, and for the geothermal modelling approaches.

Measured heat flow data and surface heat flow densities

The heat flow density data are commonly determined from the temperature measurements along with the thermal conductivity values from deeper parts of the upper crust and, moreover, from relatively great depth intervals. Usually it is due to the lack of reliable data (mainly temperatures) which are available for determination of this parameter. The geothermal gradients are determined very frequently only from one or two values measured at foot-positions by drilling of borehole and/or from surface mean temperature. This is typical for the hydrothermal regions. As a consequence the measured/calculated heat flow density data does not represent the value related to Earth's surface. The studied models of heat flow refraction on laterally inhomogeneous structures with common thermal conductivity contrasts show that the local variability and step changes in vertical distribution of heat flow density vertical component are very significant. The determined heat flow density values have to be corrected in the cases when they should be used as the surface heat flow data. This is important mainly for the surface heat flow distribution maps. The surface heat flow density data (distributions and separate values) are used also as the check parameters in geothermal modelling, but here it is possible to use the determined data directly and to compare them with adequate model data. Let

us note that the real models of the thermal fields of the lithosphere ought to contain enough computational points (i.e. in *Majcin, 1993*), that the local refraction anomalies will not be omitted and it will be possible to use the individual data obtained from boreholes.

Construction of terrestrial heat flow maps

The knowledge of the terrestrial heat flow density is of the fundamental importance in geophysical and tectonic analysis. The local, regional or global patterns of this parameter are usually displayed in the form of classical isoline maps (*Čermák and Haenel, 1988*). These maps of different scale and area coverage are constructed either by computer interpolation and contouring or by hand construction. The latter utilizes both the measured/determined data and the estimates based on such knowledge of the terrestrial heat flow density nature as heat transfer conditions, principal geological structure boundaries, tectonics development events of regions under study and so on.

Both classic approaches reveal smaller or greater problems in the construction of local maps of more precise scales owing to the existence of discontinuities of the surface heat flow density isolines over the boundaries of geological structures. The earlier analytical as well as numerical models, including our modelling results have shown that the greatest step changes occur mainly around the margins of both the big sedimentary basins and the intra-mountain basins.

The regional surface heat flow density maps are relatively smooth and do not exhibit discontinuities, as they represent the typical values for larger geological regions and describe the global changes between these units. The distributions are fitted to representative values and this requires also some reinterpretations of measured data – also the heat flow refraction effects on contrast structures of uppermost parts of crust.

The construction methods of the heat flow density distributions in various depth levels have the same drawbacks. The range of changes caused by the refraction is decreasing with the depth because of smaller mean variability of the thermal conductivity of rock complexes. The areal distributions of the heat flow density components may contain discontinuities given by definition of surfaces in which they are constructed, i.e. the Moho discontinuity.

The heat flow maps, mainly for regional distributions, are constructed

also using the division of surface by geographic coordinate networks. The sub-surfaces are characterised by certain mean value of plotted parameter (e.g. arithmetic mean from measured data, weighted mean and others). The fine divisions, sufficiently precise for the approximation of the surface geological structure boundaries should be used also for the solution of the problem caused by the discontinuities in local maps. Some other global networks with possibility to improve the presentation of data by refining of networks are usable for this purpose. One can consider as very suitable also the regularized triangular networks constructed for spherical or ellipsoidal Earth (*Pohánka, 2005; 2006; 2008*) providing the calculation of mean values by the interpolation of non-homogeneously distributed data.

Geothermal modelling approaches

The calculated model temperatures and the heat flow density distributions for various refraction problems allow to determine both the qualitative and quantitative properties of these types of anomalies. The examples of some representative structures were solved in this contribution as well. The knowledge of relations between structures with thermal properties and their refraction effects allows to decrease the ambiguity level of inverse geothermal problems solution. The mentioned relations together with utilization of other geological and geophysical data by starting model (i.e. *Majcin, 1993; Bielík et al., 2002; Dérerová et al., 2006*), along with application of integrated modelling approaches (i.e. *Bielík et al., 2002; Zeyen et al., 2002*), which provide further check parameters of models, can finally improve the robustness of geothermal modelling approaches.

We analysed both the sensitivity of the solutions on grid point distributions and practical representativeness of calculated surface heat flow density data for various configurations of the subsurface thermal conductivity contrast structures. The results from this contribution as well as from our previous papers will be used for computer-based generation of solution- and structure- adaptive grids for geothermal modelling in the real world conditions of the Western Carpathians and surrounding units.

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