REVIEW PAPER

Geoid versus quasigeoid: a case of physics versus geometry

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Abstract: For decades now the geodetic community has been split down the middle over the question as to whether geoid or quasigeoid should be used as a reference surface for heights. The choice of the geoid implies that orthometric heights must be considered, the choice of the quasigeoid implies the use of the so-called normal heights. The problem with the geoid, a physically meaningful surface, is that it is sensitive to the density variations within the Earth. The problem with the quasigeoid, which is not a physically meaningful surface, is that it requires integration over the Earth's surface.

Density variations that must be known for the geoid computation are those within topography and these are becoming known with an increasing accuracy. On the other hand, the surface of the Earth is not a surface over which we can integrate. Artificial "remedies" to this fatal problem exist but the effect of these remedies on the accuracy of quasigeoid are not known. We argue that using a specific technique, known as Stokes-Helmert's and using the increased knowledge of topographical density, the accuracy of the geoid can now be considered to be at least as good as the accuracy of the quasigeoid.

Key words: Gravity field, geoid, quasigeoid, contribution of topo-density variations, Lipschitz's conditions

1. Introduction

In this paper we wish to discuss the issues involved in the selection and production of a reference surface for heights. We wish to argue that the classical, physically meaningful surface, the **geoid**, as introduced in *(1873)* by *Listing*, is still not only the most natural surface to refer heights to but also that it can nowadays be determined to a sufficient accuracy from measurements on the surface of the Earth. We shall first explain what the geoid is and how the measurements of gravity, heights and mass density of the topography (that part of the Earth that extends between the Earth surface

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and the geoid) are obtained. These are the essential data needed for geoid determination.

Then we shall briefly explain why in the past 60 years or so, the ideas of using the geoid and of computing the geoid from various surface measurements kept falling into disrespect and how a different, artificial surface called the **quasigeoid**, could and should be used and computed from surface data. The production of this surface has its own problems, however, and these problems, unlike the problems with the geoid production, do not go away with the increased knowledge of the Earth composition.

We shall show how the main argument against choosing the geoid had lost its punch with advances in the theory of geoid computation and with an ever-increasing knowledge of topographical density anomalies.

2. The geoid

It is well known in surveying practice that heights of practical value have to be referred to mean sea level; the reasons were elucidated by many authors, among others by *Vaníček* in *1998.* Thus to obtain some heights of practical value the mean sea level underneath the continents has to be known. Such heights are intuitively attractive, and have been shown useful in most engineering applications. The mean sea level anywhere more or less follows a gravity equipotential surface of constant gravity potential W_0. Also surveying instruments in action are aligned with the local gravity vector, perpendicular to the gravity equipotential surfaces. Hence the gravity field clearly plays a very important role in practical height determination.

An equipotential surface of the Earth gravity field at a point is the horizontal (level) surface, passing through that point. As indicated in Fig. 1 there is only one such surface passing through any point and it is the surface that any homogeneous fluid will stabilize to if left alone. Sea water is not homogeneous because at different places it has different temperature, salinity, particle content, etc.; therefore, sea water in reality does not follow a horizontal surface. Ergo, horizontal currents at sea arise, some of them quite strong. Nevertheless, considering that the sea surface is very nearly an equipotential surface, within a range of plus or minus 2 metres, we can reasonably use an equipotential surface as the reference surface for heights.

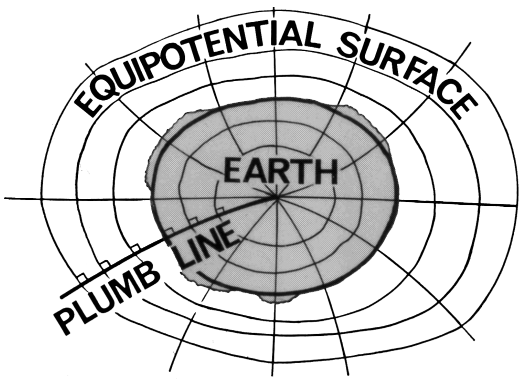

Fig. 1. Structure of Earth's gravity field.

Determination of such a horizontal surface, that best approximates the mean sea level and is called the geoid, is one of the themes of this contribution.

Two conceptually different kinds of height systems are commonly used:

1. **Orthometric heights** H are the quintessential "practical heights" above the sea level used in mapping and engineering practice. There are other less common heights that refer to the sea level and we shall see one of them, the normal height, in the next section. The orthometric height of a point of interest is measured along the plumbline, a line always tangent to the gravity vector, from the geoid to the point of interest.

   Dynamic heights, which are the most physically meaningful heights will not be treated here as they are not very popular in practice.

2. **Geodetic heights** h are heights above the bi-axial "geocentric reference ellipsoid," measured along the normal to the ellipsoid. They can be readily determined from observations from satellites but they are of very little practical use on their own. However, if the departure of the geoid from the geocentric reference ellipsoid N is subtracted from a geodetic height h, as we can see in Fig. 2, we get the orthometric height H, which can be then used in practice.

The departures of the geoid from the best fitting geocentric reference ellipsoid (presently estimated to have a semi-minor axis a equal to 6 378 137 m

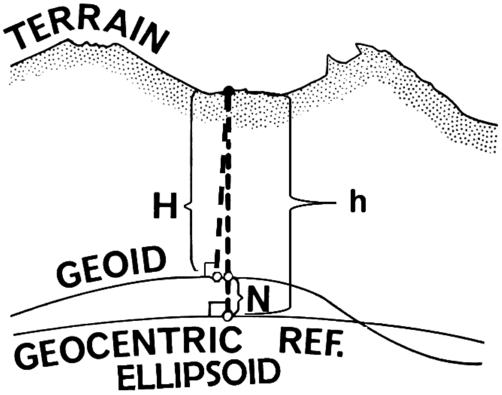

Fig. 2. Relation between geoidal, geodetic and orthometric heights.

and a flattening f equal to 1/298.25 *(Moritz, 1980a)*), called geoidal heights N, range approximately between –100 m and +100 m globally. Geoidal heights are useful as an intermediary between satellite-observed heights (geodetic) and practical heights (orthometric). These three heights are then related by:

$$H \approx h - N. \tag{1}$$

Figure 3 shows that geodetic height, h, can be computed from the satellite-determined position (given in the Cartesian coordinates x, y, z) exactly, if specific values for the size a and shape (flattening) f of the geocentric reference ellipsoid are adopted. The calculation is simply a matter of applying general geometrical principles *Vaníček and Krakiwsky, 1986.* In Figure 3, the center of the ellipsoid is coincident with the center-of-mass of the Earth by definition. We note that more often height differences, rather than heights themselves, are obtained from these calculations but to explain the reasons why this is done in practice is beyond the scope of this review paper.

Orthometric heights, or rather orthometric height differences, can be determined by a simple differential procedure which is quite accurate but it is also slow, expensive and prone to systematic errors. This classical process, the terrestrial levelling, has been used all around the world for well over a century. For economical reasons, the tendency now is to replace

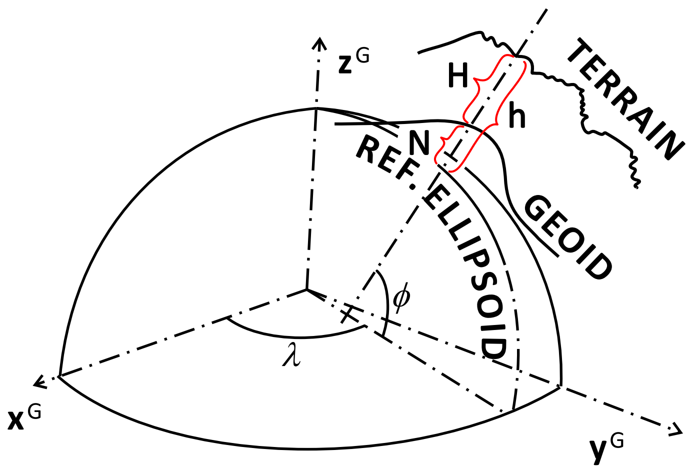

Fig. 3. Relation between Cartesian and curvilinear coordinates.

this process by satellite methods, which provide geodetic height differences. Satellite methods are almost as accurate as terrestrial levelling, particularly for larger distances, and much cheaper to use. If this approach is used, accurate knowledge of the geoidal heights on land becomes a prerequisite for converting geodetic heights to orthometric heights. Orthometric heights and geoidal heights are widely used around the world, particularly in the Americas and in portions of Africa and Asia. More recently, there has been the decision in Canada and in the US to adopt orthometric heights and a geoidal model as their national systems of heights *Véronneau and Huang (2011)*.

Figure 4 shows a map of the geoidal heights in Canada, superimposed by countour lines representing the relief, as an illustration. The geoidal map shown has been compiled by means of the Stokes-Helmert technique used at UNB for some 20 years. Note the high correlation of the geoid with topography and bathymetry.

3. The determination of the geoid

The determination of the geoid is a purely physical problem: if we knew

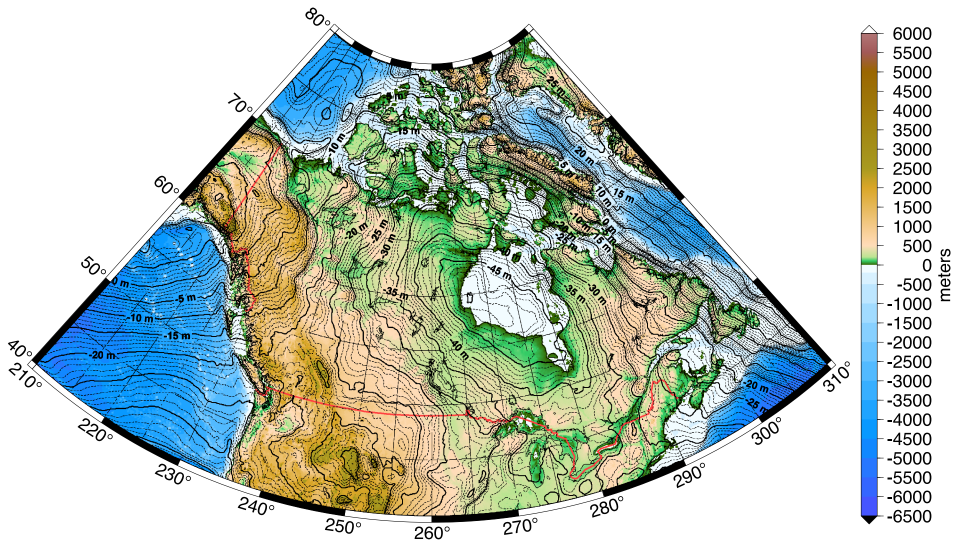

Fig. 4. Detailed geoid in Canada.

the mass density distribution within the Earth we could compute the gravity field, including gravity potential and thus the geoid, to any accuracy anywhere by using Newton's integration. We would then get the geoid by simply connecting all the points of the same required value W_0 of potential. Unfortunately, we do not know the density distribution within the Earth to sufficient accuracy to do this, so this approach cannot be used in practice.

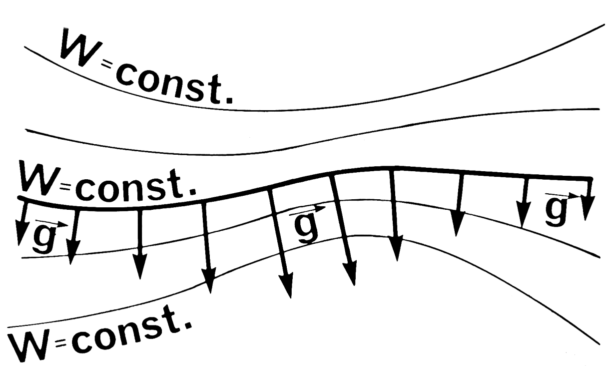

Fig. 5. The relation between gravity and its potential.

The only viable alternative is to use gravity values, which are cheap, plentiful and sufficiently accurate. If we have these, we can take advantage of the relation between gravity and gravity potential, as shown in Fig. 5. The gravity vector, $\vec{g}$, is related to the gravity potential, W, by the formula:

$$\vec{g} = \nabla W, \tag{2}$$

where ∇ is the gradient operator. The gravity, g, is the magnitude of the gravity vector, and is positive in the downward direction.

Gravity can be observed at the surface of the Earth and available in the form of gravity anomalies of different types. If we had the gravity anomalies Δg on the geoid (at the sea level), then we could use Stokes's formulation to compute the geoidal height $\mathbf{N}$ (already defined) for any desired position. Since on land we do not know Δg on the geoid, the observed values on the surface of the Earth, on the topography, have to be transferred onto the geoid first by a process known as the downward continuation through the topography. This strictly can be done only when the topographical mass density is known. However, the topo-density is not known to an accuracy that would allow us to do this and this approach thus cannot be contemplated in earnest. At least so it seems, but as we shall see below, a different course of action to the problem can be adopted to avoid the pitfalls of this approach.

Beyond the theoretical problem posed by the unknown topo-density, downward continuation is a numerically ill-conditioned problem for finely spaced gravity data, particularly in areas of large elevation (e.g. *Martinec, 1996*). However, this ill-conditioning is not fatal. The numerical error can be mitigated through regularization (e.g., *Novák et al., 2001; Kingdon and Vaníček, 2011*). Even without regularization, the error in geoid determination resulting from the ill conditioning of the downward continuation of gravity values only reaches a few centimeters in regions with elevations over 3 km (e.g., *Huang, 2002; Goli, 2011*), and less at lower elevations.

There are, of course, other data that can be used to compute the geoid, such as the deflections of the vertical, satellite altimetry, satellite dynamics, satellite gradiometry, etc. Satellite-derived global geopotential models are especially useful (e.g., models coming from the GRACE *Tapley et al., 2005* and GOCE *Rummel et al., 2009* missions), as they can provide the long-wavelength features of the geoid more accurately than terrestrial data.

However, for the sake of keeping the discussion simple and transparent, we shall not get into other techniques as gravity data is the mainstay of all geoid computations.

4. The quasigeoid

The fact that the topo-density was not known with an adequate accuracy back in the 1960's (and this problem lingers on still today) led Molodenskij to declare the geoid impossible to determine to a sufficient accuracy and to introduce an alternative quantity known as the quasigeoid (*Molodenskij et al., 1960*). Methods of determining the quasigeoid have since been somewhat refined, especially by the formulation in terms of analytical continuation as described by *Bjerhammer (1963)*, but also by numerous other mathematical and theoretical developments (e.g., *Krarup, 1973; Hörmander, 1976; Moritz, 1980b; Holota, 1997*). The interplay of the quasigeoid with the geoid and the reference ellipsoid, is shown in Fig. 6.

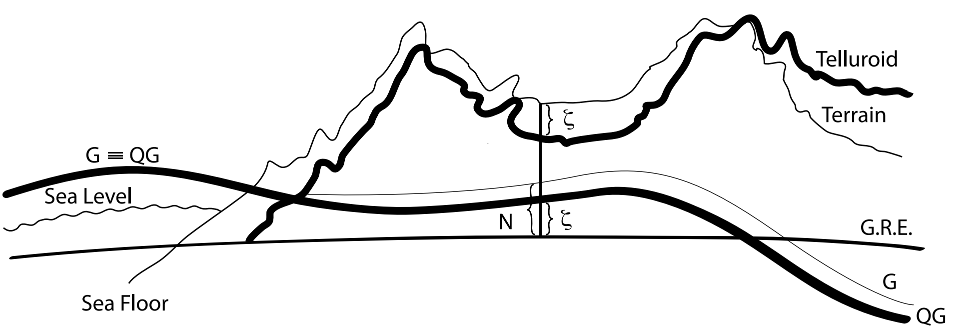

Fig. 6. The relation among the quasigeoid, geoid and reference ellipsoid.

The vertical distance between the quasigeoid and the reference ellipsoid is called the quasigeoidal height (a.k.a. height anomaly) ζ. For the determination of the quasigeoid it would not be necessary to know the topo-density as all the computations are done not on the geoid surface but on the surface of the Earth (or at an almost identical surface to it, called the telluroid –

see the definition below) and not on the geoid. Molodenskij theory deals throughout with the gravity potential external to the surface of the Earth.

Molodenskij's approach does not require any knowledge of topo-density, as it deals only with the external field and needs only to know the geometry of the external field. On the other hand, as the approach is based on geometry, it requires integration over the surface of the Earth, or more precisely over the telluroid. The telluroid is a surface that looks like the Earth surface except that it is displaced from the Earth surface by the quasi-geoidal height, which is a smoothly varying quantity that ranges, as its cousin the geoidal height does, between −100 and +100 metres.

For the quasigeoid to have some use in practice, it has to have a meaningful system of heights associated with it. This system is called **normal heights** and it is used in the countries of the former Soviet Union and 9 other European countries (France, Germany, Sweden, Poland, Czech Republic, Slovak Republic, Hungary, Romania and Bulgaria). The normal height of a point on the topographical surface is defined as the height of the corresponding point on the telluroid above the reference ellipsoid, measured along the normal plumbline. However, normal heights may equivalently be seen as heights of the topographical surface above the quasigeoid, also measured along the normal plumbline.

The relation among the normal height H^N, height anomaly and geodetic height is exactly the same as that among orthometric height, geoidal height and geodetic height (cf. Figs. 2 and 6). Normal heights and orthometric heights at open sea are exactly the same, while they may differ by up to one and a half metres on land. The relation between normal height and geodetic height is mediated by the height anomaly:

$$H^N \approx h - \varsigma. \tag{3}$$

The difference between the two surfaces – the geoidal surface and the telluroid – over which the integration for the geoid or quasigeoid determination respectively is carried out is as follows: The geoid is a fairly smooth surface without any kinks, edges or other irregularities as seen in Fig. 1; while the telluroid, or the Earth surface for that matter, is much rougher. So much so that it does not satisfy Lipschitz's conditions for an integrable surface (*Jeffreys and Jeffreys, 1988* §1.15). Roughly speaking, the Earth surface is not sufficiently smooth to allow us to integrate on.

Vertical rock faces represent locations where the Earth surface and the telluroid are discontinuous, which is an additional complication. Even worse, there are locations where neither the surface of the Earth nor the telluroid can be described as mathematical functions of horizontal position. These are the rock overhangs. In these locations Molodenskij's mathematical apparatus fails. To paste over these rather fatal difficulties, a "regularization process" of some kind is required to smooth/change the topographical surface, introducing unpredictable error in the result.

5. The discussion

Can a way be found whereby the physical approach can be improved so as to assuage the main objection against it? At UNB, we have chosen to use what we call Stokes-Helmert's method, the crux of which is shown in Fig. 7.

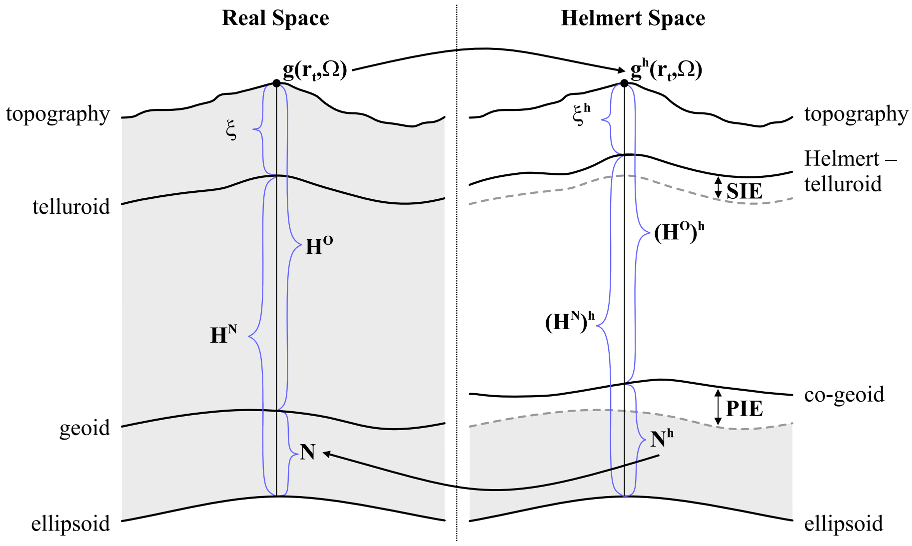

Fig. 7. The concept of the Stokes – Helmert method.

This method employs Helmert's 2nd condensation technique in conjunction with Stokes's theory, and we have shown that it works reasonably well as a candidate for lessening the impact of poor knowledge of topo-density on the accuracy of geoid determination (*Vaníček and Martinec, 1994; Tenzer et al., 2003*). How come this approach can deliver sufficiently accurate results without an accurate knowledge of topo-density?

Helmert's 2nd condensation technique, which we use in our approach, replaces the effect of topographical masses at the Earth surface by the effect of the condensed mass layer on the geoid (*Martinec and Vaníček, 1994*). The mass layer on the geoid is naturally referred to as the "condensed topography". The reason for this approach to deliver sufficiently accurate results without an accurate knowledge of topo-density is that only the effect of the difference between topography and condensed topography, called Helmert's Direct Topographical Effect (DTE), has to be considered. As an example, assuming the topographical density to be equal to the Earth's average crustal density of $\rho = 2.67\text{g cm}^{-3}$, the DTE is responsible only for a few metres of contribution to the geoid over the fairly high Canadian Rocky Mountains, as can be seen in Fig. 8. This must be compared with the effect of total topography, which amounts to many hundreds of metres. The consequence of this is that we have to evaluate the DTE only to an accuracy of 1% to get a one-centimetre uncertainty in the computed geoid, which under ordinary circumstances does not constitute a problem.

But this does not say anything about the effect of density anomalies within topography, which was perceived as being the main problem with this physical approach to geoid determination in the first place. It turns out that realistic topo-density anomalies (differences between the actual densities and the normal density of $\rho = 2.67\ \text{g}\,\text{cm}^{-3}$) contribute usually less than 5% and at most 10% on top of the constant density effect, i.e., up to a maximum of a few decimeters under normal circumstances. This is shown in Fig. 9, in which the Direct Density Effect (DDE) in the Canadian Rockies is plotted. If the real topo-density were known to an accuracy of 10%, the geoid accuracy would be on a centimetre level, a good enough accuracy for most applications. Can this be realistically achieved?

In the first attempts to model the topo-density effect, by *Martinec (1993)*, only lateral (horizontal) anomalies were modelled. In the first practical application of this idea, *Huang et al. (2001)* obtained lateral topo-density values in Western Canada shown in Fig. 10. In the cited work, the following approach to modelling the effect was used:

1. surface density from geological maps was extended vertically down to the geoid;
2. probabilistic confidence intervals of the densities were converted to standard deviations;
3. effects of lateral density anomalies on the DTE and, subsequently, on the geoid were computed;
4. standard deviations of the resulting effects were evaluated.

The result showed that the effect of lateral density variations on the geoid is at most a few decimeters with a standard deviation of less than 2 centimetres (*Huang et al., 2001*).

In the next step we have considered the vertical variations in density. Can the effect of vertical density variations on the geoid reach centimetres and more? In other words: do we have to consider vertical density variations when compiling the geoid? The first relevant results we are aware of are

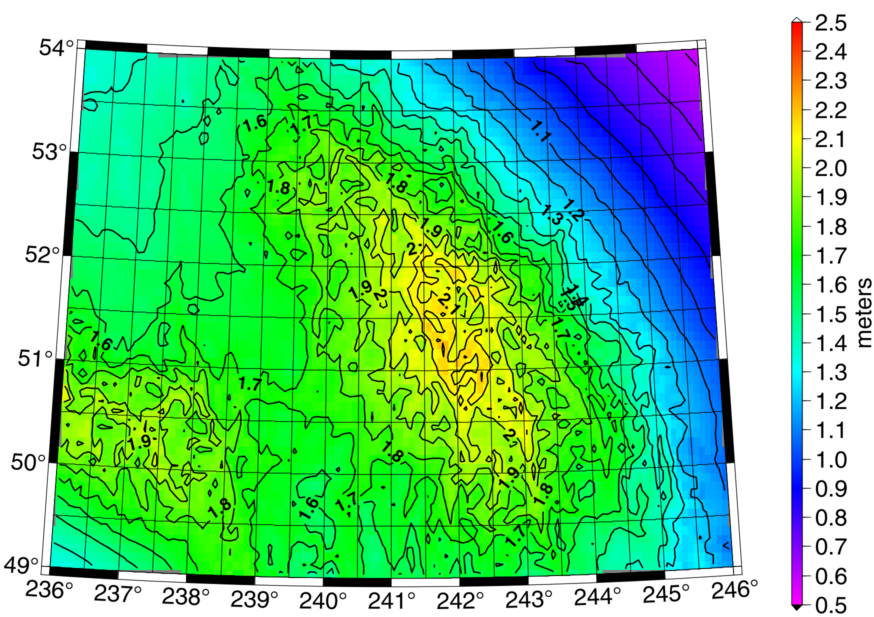

Fig. 8. DTE on geoid in Canada's Rockies computed on 5′ by 5′ grid (min = 0.583 m, max = 2.223 m, mean = 1.564 m and standard deviation = 0.324 m, contour interval 0.1 m).

those of the effect on the geoid of Lake Superior in Canada, due to *Martinec et al. (1995)*. Martinec constructed a horizontal density model incorporating vertical density variations. He found that even the effect of the drastic density contrast given by the lake bottom (a jump from $\rho = 1$ g cm^{-3} to $\rho = 2.67$ g cm^{-3}) ranges only between -1.1 and 1.3 centimetres. A later study in the same area, using a vertical density model, confirms the findings of Martinec *(Kingdon et al., 2009a)*. The vertical density effect of Lake Superior is shown in Fig. 11.

Our studies using vertical density models have also shown that the effect of vertical density variation is less than 5 centimetres even under very extreme conditions *(Kingdon et al., 2009b)*, and under more realistic conditions is unlikely to ever exceed 2–3 centimetres *(Kingdon et al., 2009a)*. Thus, while the vertical density effect is small, it is significant in some areas. Its modelling should be attempted when possible, especially in the vicinity of large lakes, sedimentary basins, or mountain ranges where significant

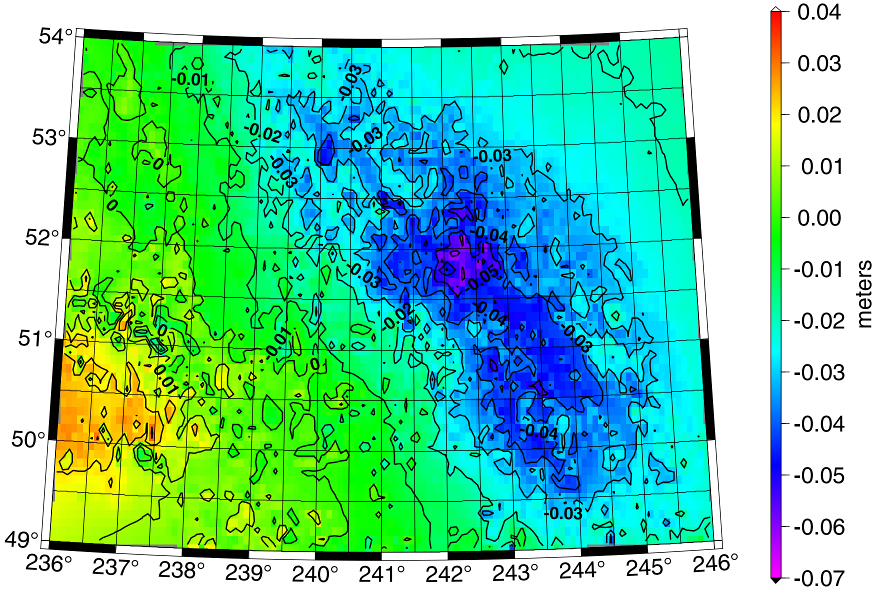

Fig. 9. DDE on geoid in Canada's Rocky Mountains computed on 5′ by 5′ grid (min $= -0.063$ m, max $= 0.031$, mean $= -0.016$ m and standard deviation $= 0.018$ m, contour interval 0.01 m).

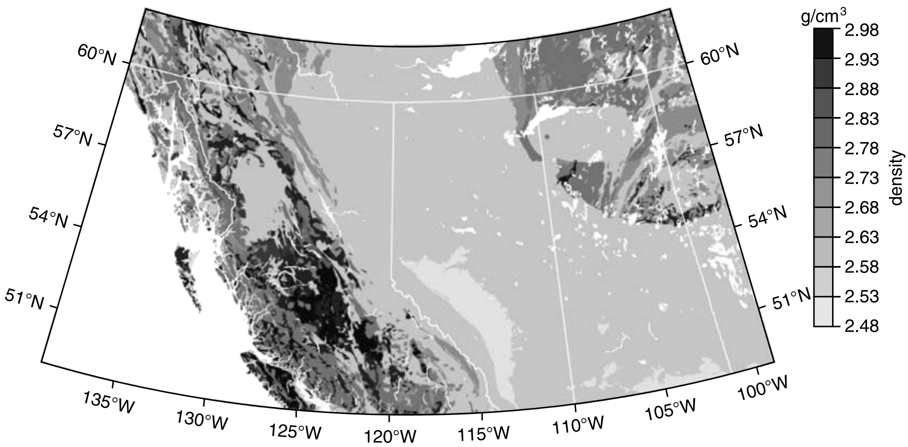

Fig. 10. Lateral density variation for Western Canada as derived from geological maps.

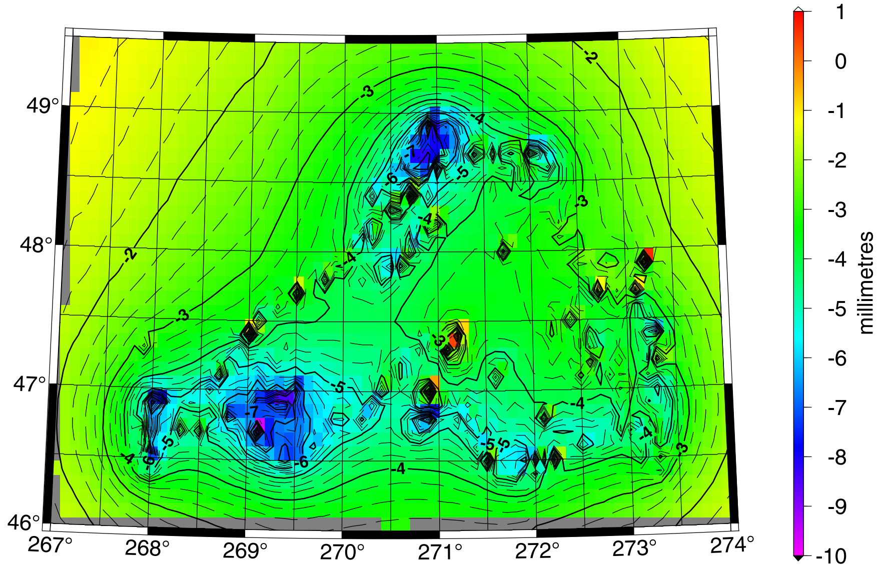

Fig. 11. Effect on geoid of vertical density variations of Lake Superior (after *Kingdon et al., 2009b*).

density contrasts occur. If the geological structure of the Earth's crust is reasonably well known, such modelling is a straightforward problem, and considering the relatively low magnitude of the vertical density effects need not be carried out with great rigour to achieve accuracy within the one centimetre range *(Kingdon et al., 2009a)*. In most areas, where vertical density contrasts are small, the vertical density effect will remain well under the 2–3 centimetres. Thus even a rough modelling of vertical variations together with the horizontal density modelling described above will be sufficient to achieve 1–2 centimetre accuracy in the computed geoid.

6. Conclusions

Topo-density is a problem, but it can be resolved to an accuracy of a few centimetres if the geological formation of the crust is reasonably well known. To get the total geoid accuracy the uncertainty in the effect of irregular topo-density must be added to the uncertainty in geoid determination that comes from the employed approximations in the theory and the numerical computations. Was Molodenskij right, then?

Yes, Molodenskij was right 50 years ago, but he would not be right any more today. Improvements in the theory of the geodetic boundary value problems and the substantial increase in the knowledge of topographical density distribution have changed the situation substantially. We have shown that nowadays, the geoidal heights can be determined to an accuracy of 1 to 2 centimetres, perhaps 2 to 3 centimetres in regions of extreme difficulty.

There is an additional aspect to be considered. Molodenskij's approach does not require any knowledge of topo-density, but it requires the surface integration to be carried over the surface of the Earth, or over the telluroid to be more accurate. Yet, the Earth surface does not satisfy Lipschitz's conditions of integrability as it is not sufficiently smooth and nobody knows the magnitude of the error caused by this failure. Moreover, in some places, the Earth surface/telluroid are not continuous or cannot be even described by a mathematical function.

It has to be concluded that we have managed to eliminate, or at least considerably reduce the well-understood physical difficulties encountered when solving the classical geodetic boundary value problem (with the Stokes-Helmert technique and topo-density modelling), while the geometrical difficulties associated with Molodenskij's theory remain. Our guess is that Molodenskij's approach cannot offer any better accuracy than the Stokes-Helmert's approach does. Additionally, the geoid, being an equipotential surface, is a physically meaningful entity while the quasigeoid is not.

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