Linearization of the Sobolev and Babeyko's formulae for transformation of P-wave velocity to density in the Carpathian-Pannonian Basin region

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Abstract: The initial density model has to be based on a reasonable geological hypothesis and while the modelling process is non-unique, one of the interpretation aims is to define the robust parameters of the model. It is important at this stage to integrate the seismic and gravity data. One of the possibilities how to integrate these data is transformation of the seismic velocities to densities. The Sobolev and Babeyko's formulae belong to the most available relationships for this transformation. They are very complex and rigorous taking into account the PT conditions. On the other hand its application is relatively complicated. Therefore the main goal of the paper is to try to determine more easily the formula for transformation of the seismic velocities to densities. Based on the analysis of the results obtained using the Sobolev and Babeyko's formula on real data, we found out that in the Carpathian-Pannonian Basin region this formula can be transformed to simpler linear velocity–density relationship with required accuracy.

Key words: density modelling, density, seismic velocity, transformation of seismic velocity to density, Carpathian-Pannonian Basin region

1. Introduction

Density modelling is a very useful method for interpretation of the observed gravity field (e.i., *Lillie et al., 1994; Bielik, 1995, 1998; Alasonati Tašárová*

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et al., 2008, 2009; Grinč et al., 2010; Grabowska et al., 2010). In any case the initial model would have to be based on a reasonable geophysical and geological data. Therefore the input model would incorporate the multiple sources of data (seismic, gravimetric, magnetic, geothermal and other geophysical and geological knowledge) to design the model as realistically as possible. On the other hand the quality of the modelling results also depends on the knowledge about the densities of the anomalous bodies. If we have available information about the seismic velocities in the crust and/or lithosphere then one of the best approaches how to define the most real density for the model is to use the suitable formula for transformation of these velocities to the densities.

During the last 50 years the numerous empirical relations linking seismic velocity and density have been estimated (e.g. *Birch, 1961; Christensen and Mooney, 1995; Sobolev and Babeyko, 1994*). It has been found out that the temperature, pressure, porosity, saturation, mineralogical and petrological composition have the largest influence on the variation of seismic velocities and densities *(Goldberg and Gurevich, 1998; Monsen and Johnstad, 2005)*. From them the temperature and pressure (PT conditions) play probably the most important role (e.g., *Christensen and Mooney, 1995; Sobolev and Babeyko, 1994*). It is well-known that most of the velocity–density relationships do not take into account the PT conditions. Among them it can be ranked the relationships determined by *Nafe and Drake (1957, 1963), Ludwig et al. (1970)* and *Birch (1961)*. The formulae defined by *Christensen and Mooney (1995)* and *Sobolev and Babeyko (1994)* incorporated the PT conditions and belong to one of the most used formulae for the transformation of the P-wave velocities to densities.

The seismic results of CELEBRATION 2000 profiles (e.i., *Grad et al., 2006; Środa et al., 2006; Janik et al., 2011*) brought very good information on the P-wave velocity distribution in the crust and in the upper part of the mantle in the Carpathian-Pannonian region. It allowed us to transform the crustal P-wave velocities to densities. The detailed analysis of the resultant crustal densities in the study region showed a relation which could be used for simplification of the Sobolev and Babeyko's formulae (1994). Moreover, the formulae are relatively complicated for the application. Therefore, the aim of the paper is to try to find a simpler formula, but the one which could have the same quality.

2. Sobolev and Babeyko's formulae

Sobolev and Babeyko's formulae (1994) belong to the most complex velocity–density relationships. These relationships have been developed by very precise laboratory measurements and analyses of the crustal crystalline rocks properties. The formulae allow the transformation of the in situ P-wave velocities to the in situ densities using the PT conditions. The transformation approach consists of three steps.

In the first step the in situ P-wave velocities are transformed by equation (1) to the P-wave velocities at normal conditions which are defined with $P_0 = 0.1\,\mathrm{MPa}$ (normal pressure) and $T_0 = 25\,^\circ\mathrm{C}$ (room temperature):

$$v_{P_0} = v_{P_{(\mathrm{in\,situ})}} - \frac{\partial v_P}{\partial P}\,P - \frac{\partial v_P}{\partial T}\,(T - 25\,^\circ\mathrm{C}). \tag{1}$$

The next step transforms the P-wave velocities at normal conditions to the densities at normal conditions. This step has some restrictions, as the laboratory analysis of the rock properties has been realized only for the crystalline rocks. Therefore, the transformation can be used only when P-wave velocities at normal conditions are falling into the intervals 6.05–6.95 km/s or 6.95–7.80 km/s.

In the final step, the densities at normal conditions are transformed by equation (2) to the in situ densities:

$$\rho_{(\mathrm{in\,situ})} = \rho_0 + \frac{\partial \rho}{\partial P}\,P + \frac{\partial \rho}{\partial T}\,(T - 25\,^\circ\mathrm{C}). \tag{2}$$

The advantage of these formulae is that they take into account also PT conditions in depths, in which the rocks are located. The estimated average error is $\pm 0.05\,\mathrm{g/cm^3}$. For more details see the paper by *Sobolev and Babeyko (1994)*.

In situ pressure can be calculated from equation (3):

$$P = P_0 + \frac{\partial P}{\partial z}\,z = 0.1\,\mathrm{GPa} + (1.1\,\mathrm{GPa}/40\,\mathrm{km})\,z. \tag{3}$$

When the local geotherms are unknown for the determination of the in situ temperature then the depth-temperature relationship (global geotherms) for the single heat-flow (Fig. 1) can be applied *(Ranalli, 1997)*.

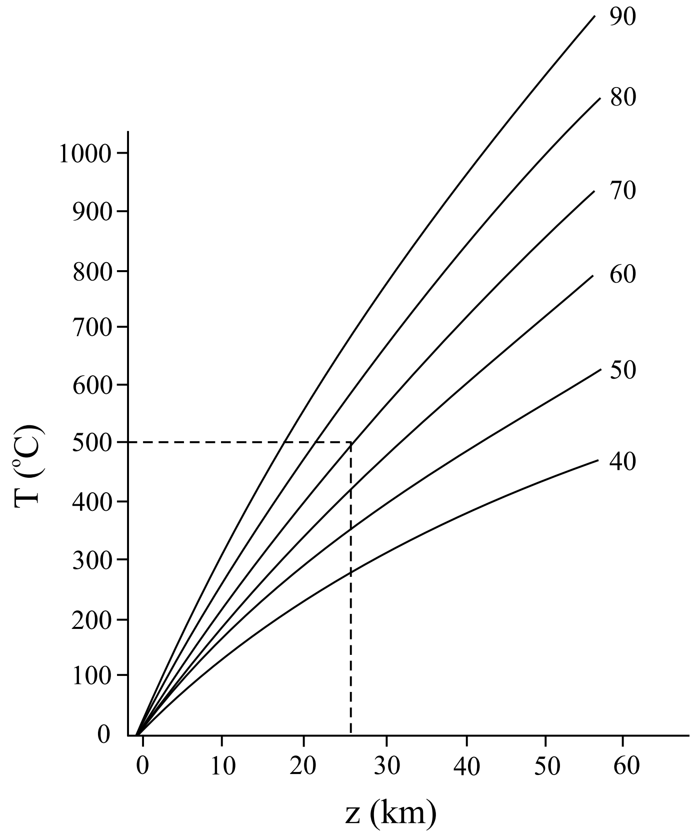

Fig. 1. Depth–temperature relationship (global geotherms) for the single heat-flow (in mW/m^2) after *Ranalli (1997)*.

3. Straight-line approximation of the Sobolev and Babeyko's formulae

3.1. Synthetic data transformation

In the synthetic calculations we considered all densities, which could be calculated by the Sobolev and Babeyko's formulae (1994). The calculations have been performed for the P-wave velocities in the interval 5.00–8.00 km/s with a step 0.1 km/s. The depth interval was 0–50 km with a step 2 km and the global geotherms for the heat flow varying from from 40 to 90 mW/m^2 with a step 10 mW/m^2 *(Ranalli, 1997)*. More than 2500 in situ densities have been calculated at various depth and PT conditions. The linear function approximating all the calculated densities (Fig. 2) can be expressed by equation (4):

$$\rho = -0.09 + 0.455\, v_P. \tag{4}$$

An average error of the approximation is ± 0.05 g/cm^3 in comparison with the Sobolev and Babeyko's formulae (1994). The total average error of the approximation formulae ± 0.1 g/cm^3 is overmuch for potential application in the density modelling.

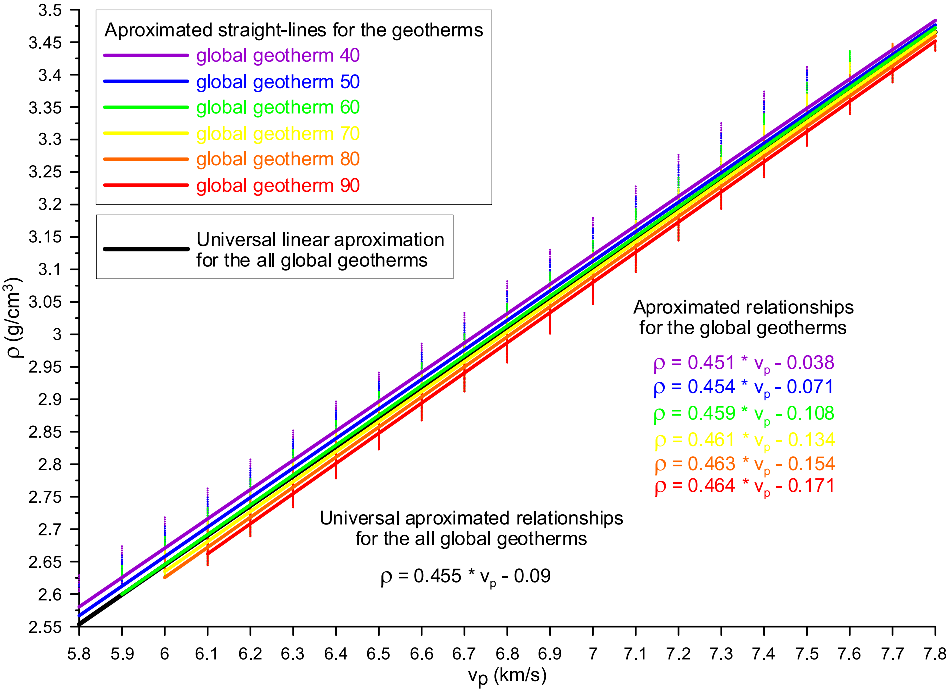

Fig. 2. The approximated straight-lines and their equations for the global geotherms of the synthetic data. Global geotherms are in mW/m^2.

3.2. Real data transformation

The transformation of the crustal P-wave velocities to densities in the Carpathian-Pannonian region has been performed along the refraction CELEBRATION 2000 profiles CEL01, CEL04 *(Środa et al., 2006)* and CEL05 *(Grad et al., 2006).* The study and analysis of the crustal densities, which have been determined by the Sobolev and Babeyko's formulae, indicated that the calculated densities for the same P-wave velocities and depths, but for various geotherms, are very close to each other. The average error of the determined densities is ± 0.005 g/cm^3, which is negligible in comparison with that of the Sobolev and Babeyko's formulae (1994). Based on this result we tried to simplify these formulae. The simplest approximation with required accuracy seems to be linear function.

Based on the application of the global Sobolev and Babeyko's formulae (1994) for the real data in the Carpathian-Pannonian Basin region, a new local, linear velocity–density relationship (5) has been obtained:

$$\rho = -0.315 + 0.489\, v_P. \tag{5}$$

The total average error of this equation is ± 0.06 g/cm^3. It shows a high correlation (the correlation coefficient is 0.998, with an average error of ± 0.01 g/cm^3) with the mentioned global formulae (2) (Fig. 3). The figure shows the negligibly small differences (the heights of the rectangles reflect the size of the differences) for the P-wave velocity interval of 6.35–7.00 km/s and the depth interval of 5–35 km.

The calculations were performed with the local geotherms which have been determinated by *Dérerová et al. (2006).* The depth interval of 5–35 km with a step 5 km and the P-wave velocity interval of 6.00–7.20 km/s were applied in calculations. In the Carpathian-Pannonian region it has been observed that this P-wave velocity interval is valid for the whole crust (e.g. *Grad et al., 2006; Hrubcová et al., 2005; Růžek et al., 2006; Środa et al., 2006*).

4. Conclusion

Our calculations clearly showed that the formulae of Sobolev and Babeyko (1994), in spite of their global character, cannot be simplified into the global

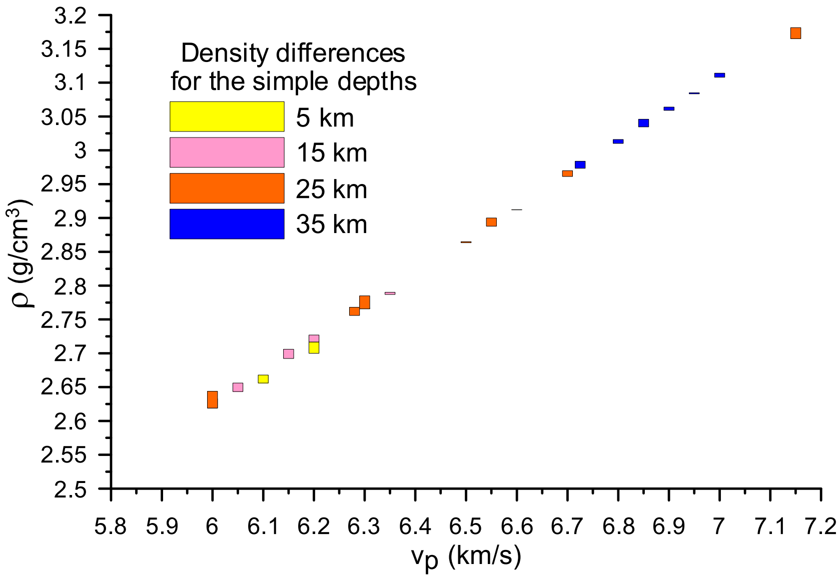

Fig. 3. The floating bar graph shows the differences between the original Sobolev and Babeyko's formulae and its simple linear approximation using the real data.

linear velocity–density relationships. The average error ± 0.1 g/cm^3 for the global velocity–density relationship linearization is large. On the other hand, our attempt to linearize the Sobolev and Babeyko's formulae (1994) in the Carpathian-Pannonian region proved as very useful for application in the density modelling. The transformation of the crustal P-wave velocities to densities, using the resultant formula (5) is for required accuracy of the crustal density determinations simpler and more effective. The formula has been used successfully for the estimation of the lithospheric density models in the study region along the CELEBRATION 2000 profiles CEL01, CEL04 and CEL05. The results indicate very good agreement between density and seismic refraction models *(Bielik et al., 2006; Csicsay, 2010)*.

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