3D analytical and numerical modelling of the regional topography influence on the surface deformation due to underground heat source

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Abstract: Thermo-elastic strains and stresses play a considerable role in the stress state of the lithosphere and its dynamics, especially at pronounced positive geothermal anomalies. Topography has a significant effect on ground deformation. In this paper we describe two methods for including the topographic effects in the thermo-viscoelastic model. First we use an approximate methodology which assumes that the main effect of the topography is due to distance from the source to the free surface and permits to have an analytical solution very attractive for solving the inverse problem. A numerical solution using Finite Element Method (FEM) is also computed. The numerical method allows to include the local shape of the topography in the modelling. In the numerical model the buried magmatic body is represented by a finite volume thermal source. The temperature distribution is computed by the higher-degree FEM. For analytical as well as numerical model solution only the forces of thermal origin are considered. The comparison of the results obtained using both analytical and numerical techniques shows the qualitative agreement of the vertical displacements. In the numerical values small differences were obtained. The results show that for the volcanic areas with an important relief the perturbation of the thermo-viscoelastic solution (deformation and total gravity anomaly) due to the topography can be quite significant. In consequence, neglecting topography could give erroneous results in the estimated source parameters.

Key words: ground deformation, analytical model, numerical model, finite element method, thermoelasticity

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1. Introduction

Extensive studies of ground deformation on volcanoes have been developed during the last decades. Recording the surface displacements and gravity changes that occur before, during and after the events allows to learn about the physics of active volcanoes. Owing to this and to the high levels of precision attainable, geophysical and geodetic techniques are proving to be a powerful tool in the monitoring of volcanic activity.

It is well known that thermoelastic strains and stresses play a considerable role in the stress state of the lithosphere and its dynamic especially in localities with pronounced geothermal anomalies *(Combs and Hadley, 1977; Teisseyre, 1986)*. *Hvoždara and Rosa (1979)* carried out a theoretical analysis of thermo-elastic deformations of a homogeneous half-space due to a point or linear source of heat located at a particular depth in the half-space. They showed that thermo-elastic stresses are expansive and that considerably disturb the normal lithostatic stress, especially near the surface of the half-space. *Hvoždara and Brimich (1991)* presented the basic formulae and numerical results of two important effects due to magmatic bodies in the Earth's lithosphere: a) static thermoelastic deformations, b) static elastic deformations due to upward pressure. The magmatic body was represented by a finite volume source of heat in the first model and by a concentrated vertical force in the second one. The formulae for gravity anomaly due to non-uniform extension connected with thermo-elastic deformations was derived by *Hvoždara and Brimich (1995)*.

Theoretical models can be used to optimally design the volcano monitoring, especially when no information exists about prior episodes of ground deformation, since the pattern and rate of surface displacement reveal the depth and physical phenomena that are taking place within the magma reservoir. The models frequently used to interpret the geodetic data measured in volcanic areas, typically compute the deformation field and gravity changes at the surface of an elastic half-space due to a buried point source and assume that topography does not significantly affects the results, but volcanoes are commonly associated with significant topographic relief. The approximation of Earth's surface as flat can lead to erroneous interpretation of the deformation data (e.g., *Cayol and Cornet, 1998; Williams and Wadge, 1998, 2000; Folch et al., 2000*). *Williams and Wadge (1998, 2000)* and *Cayol and Cornet (1998)* pointed out that topography has a large effect on predicted surface deformation by elastic models in regions of significant relief. The interpretation of ground-surface displacements with flat half-space models can lead to erroneous source parameter determination. *Cayol and Cornet (1998)* found that the steeper the volcano, the flatter the vertical displacement field. *Folch et al. (2000)* showed that this result is dramatically emphasized in the visco-elastic case, where topography changes in a very important way both the magnitude and the pattern of the displacement field. Besides they showed that neglecting the topographic effects may in some cases introduce an error greater than the implicit in the point source hypothesis. In summary, several studies reach the conclusion that the topography has a significant effect on deformation field that can lead to erroneous estimation of source parameters.

Few works take into account the topographic effects on gravity changes (e.g., *Charco et al., 2007; Currenti et al., 2007; Trasatti and Bonafede, 2009*). *Charco et al. (2007)* compute displacements and gravity changes produced by volcanic loading by using an Indirect Boundary Element Method. They carried out several theoretical tests and applied the numerical methodology to take into account the real topography of Teide volcano (Tenerife, Canry Islands). The magnitude and pattern of the gravity changes are significantly different from those of half-space solutions. Applying a Finite Element method *Currenti et al. (2007)* study the effect of both topography and medium heterogeneities. They found that perturbations on gravity changes are more evident in the presence of severe heterogeneities as in the case of volcano summit.

Previously, *Charco et al. (2002)* study the topography effects in thermo-viscoelastic surface deformation and gravity changes by using the approximate methodology proposed by *Williams and Wadge (1998)* that assumes a different source depth at each point for which the solution is desired. This methodology provides an approximate analytical solution for a heat source, very attractive for solving the inverse problem. On the other hand, in this work we carried out a numerical modelling based on the finite element computations to compute the thermoelastic deformations due to an underground heat source. Two identical models, with and without cone-shaped topographical feature, are processed. The influence of the topography is obtained from the comparison of both numerical and analytical solutions.

2. Analytical model

The solution of displacements time evolution and stresses due to a sudden action of a point source of heat buried in a visco-elastic half-space (with the flat surface) was presented in *Hvoždara (1992)*. Fundamental equations for the uncoupled thermo-visco-elastic problem for a point heat source located at depth ζ are given in *Nowacki (1962)*. Thermo-visco-elastic gravity anomaly on the surface is given by *Brimich (2000)*.

Charco et al. (2002) propose a simple method for including topographic effects in a 3D thermo-visco-elastic model that allows source depth to vary with the relief. This methodology was introduced by *Williams and Wadge (1998)* and permits to get analytical approximate solutions (quasi-analytical solutions) even if we relaxed the restriction of a free flat surface. To study the topographic effect, the relief of an area may be represented by a volcanic axi-symmetrical cone with height H and average slope of the flanks α. The main effect of topography is a reduction of vertical displacement and gravity change magnitude in regions of higher relief, as other authors have pointed out. Thus, neglecting the topography may lead to a misinterpretation of the volume change and depth of the source. It also showed a change in the pattern of total gravity changes.

Varying depth methodology gives a reasonable approximation of the topographic effect if it is due primarily to the distance of the free surface to magma chamber rather than the local shape of free surface *(Williams and Wadge, 1998; 2000)*. *Charco (2007)* develops a 3D indirect boundary elements method for the analysis of the elastic deformation field and gravity changes. Vertical displacement computed by varying depth methodology matched the numerical solution as *Williams and Wadge (1998)* showed. Although topographic effects on radial displacements decrease with increasing source depth, which is consistent with the idea that the effect is due to differences in distance between the free surface and source location, radial displacements are also affected by some other factors. The effects of considered ground relief obtained with the varying depth methodology are not very important for thermo-visco-elastic radial displacements. These results should be tested using a numerical method that permits to include local shape of topographic relief and circumvent the approach made by *Charco et al. (2002)*.

3. Numerical model

Displacement and gravitational anomalies approach their static values slowly, i.e., thermo-visco-elastic problem and thermo-elastic problem become the same for large characteristic. Therefore at such times the thermo-visco-elastic quasi-analytical solution described before can be used to compare with the static numerical solution computed by FEM. In this work, we have used the finite element method to include the topography effect in the thermoelastic solutions. The principles and basics of finite-element method are generally known and are described in numerous monographs (e.g. *Irons and Ahmad, 1986; Babuška and Szabo, 1990*). All the computations are obtained by the COMSOL Multiphysics® software (`http://www.comsol.com`). Although numerical methods are time consuming, their results are more precise than the analytical approximate solutions since they allow to include structural characteristics of the medium as the topography.

The models are homogeneous, isotropic, axi-symmetric with respect to the vertical axis. In this way, the 3D rock massif has been modelled by an axi-symmetric section with respect to the vertical axis passing through the heat source (see Fig. 1) with 2 versions – with and without topographical feature modelling the volcano cone (of 2 km height). The summit of the volcano is located over the thermal source. The domain horizontal length is 120 km and the vertical span is from +2 to –38 km in order to minimize the influence of the external boundaries. The heat source is modelled by the spherical body in the 5 km depth. In the computation only the forces of thermal origin are considered.

At the first step, the domain was divided into finite elements. The mesh corresponding to each plane section is formed by 19568 triangular elements. In the neighborhood of the thermal source, the mesh is refined into smaller elements due to the large gradients of computed fields in this area. Figure 1 shows the mesh detail. We compute the temperature field by solving the static heat transfer equation:

$$\nabla(\lambda_T \cdot \mathrm{grad}\, T) = w, \tag{1}$$

with T as the unknown temperature field and w as the thermal source power density. Table 1 shows the physical parameters of the media and source for the computation of temperature field. The center of the source is located at

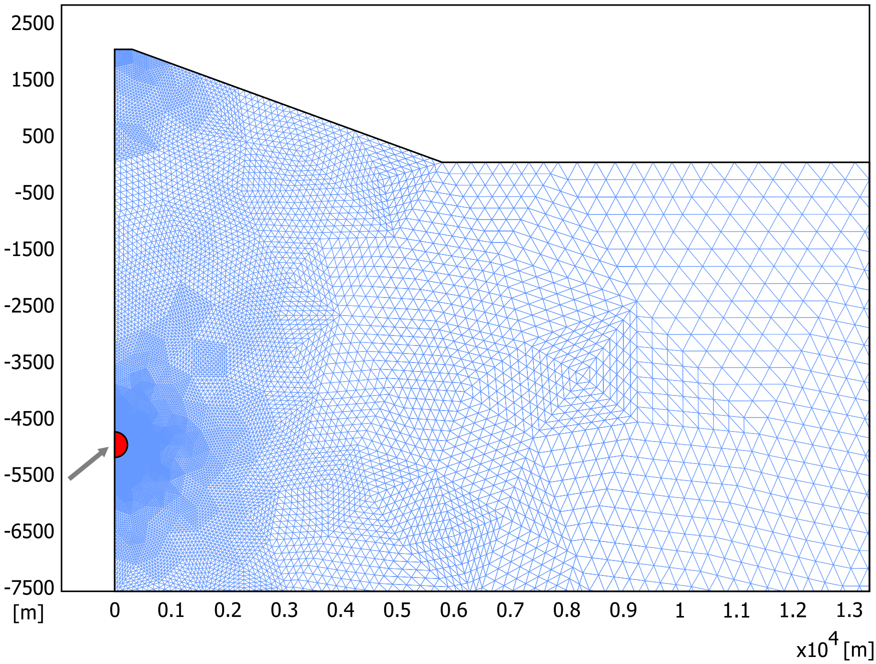

Fig. 1. Detail of the mesh for numerical model with topography (topography height 2000 m over the flat upper surface) close to the source area. The heat source is identified by the arrow.

Table 1. Physical parameters of the media and source for the computation of temperature field

Heat Conductivity (λ_T)	**Total power of thermal source** (P_t)	**Cross section area of source**
3 $W.K^{-1}.m^{-1}$	0.28	0.16 km^2

5 km depth (i.e., 7 km if we consider the volcanic cone). Surface topography would be more realistic if a thermal flux was prescribed by using a boundary condition of Neumann type but for simplicity, we impose a boundary condition of Dirichlet type at the upper model boundary (Fig. 1). The temperature on the lower horizontal boundary is also constant. We solved heat equation in static case, therefore, it is not necessary to involve the specific heat of material.

Table 2. Physical parameters used for the solution of the elasticity equations in a homogeneous media

Young Modulus (E)	**Poisson ratio** (ν)	**Density** (ρ)	**Coefficient of linear thermal expansion** (α_T)
130 GPa	0.28	2500 kg.m^{-3}	10^{-6} K^{-1}

The input physical parameters used for the subsequent solution of the elasticity equations are given in Table 2. These parameters specified in Tables 1-2 were taken for typical volcanic rock – granite. The solution region is a homogeneous medium, thus the parameters have been considered constant over the domain without taking into account their temperature dependence. Theoretical expressions for Young Modulus, E, and Poisson ratio, ν, as the functions of temperature are given for example in *Obetková et al. (1990)*.

The Lamé system of equations (with thermoelastic terms) for displacement field for axi-symmetric case (e.g. *Seremet, 2010*) can be written as:

$$
\begin{aligned}
&\mu\left(\nabla^2 u_r - \frac{u_r}{r^2} - \frac{2\partial u_\varphi}{r^2\partial\varphi}\right) + (\lambda+\mu)\frac{\partial\theta}{\partial r} - \gamma\frac{\partial T}{\partial r} = 0\\
&\mu\left(\nabla^2 u_\varphi - \frac{u_\varphi}{r^2} + \frac{2\partial u_r}{r^2\partial\varphi}\right) + (\lambda+\mu)\frac{\partial\theta}{r\partial\varphi} - \gamma\frac{\partial T}{r\partial\varphi} = 0\\
&\mu\nabla^2 u_z + (\lambda+\mu)\frac{\partial\theta}{\partial z} - \gamma\frac{\partial T}{\partial z} = 0
\end{aligned}
\tag{2}
$$

where λ, μ are Lamé elastic paramaters, $\theta = \partial u_r/\partial r + r^{-1}u_r + \partial u_\varphi/r\partial\varphi + \partial u_z/\partial z$ is the thermoelastic volume dilatation due to the temperature field T, $\nabla^2 = \partial^2/\partial r^2 + r^{-1}\partial/\partial r + r^{-2}\partial^2/\partial\varphi^2 + \partial^2/\partial z^2$ is Lapalce's differential operator. The unknown displacement field is $u = (u_r, u_\varphi, u_z)^{\mathrm{T}}$. The term, proportional to temperature gradient, represents the forces of thermal origin, no other external forces are considered. $\gamma = (2\mu + 3\lambda)\alpha_T$ is the thermoelastic coefficient, α_T is the coefficient of the linear thermal expansion of medium – shown in Table 2 for our case.

The bottom part of the boundary is considered fixed (zero displacements), the vertical part of the margin is fixed only in horizontal direction. The upper part of the boundary is set free. The fixed part of domain boundary may be considered sufficiently far away since the area of main interest is immediately over the thermal source.

4. Results and discussion

The complete set of temperature and subsequent displacement fields were performed by a FEM modelling and simulation software COMSOL Multiphysics®. The postprocessing procedure gives the distributions of the strain tensor components ε_{rr}, ε_{zz}, $\varepsilon_{\phi\phi}$ and ε_{rz} but the goal of this work is to estimate the topographic effect on the displacement field and strain.

Figures 2–3 illustrate the horizontal and vertical components of the displacements of the model with topography. Figures 4–7 display the components of the strain tensor. The differences of the surface displacements between models with and without topography in the neighborhood of the topography are shown in Figures 8–9.

We have compared the results for both quasi-analytical and numerical models in the static case at the level $z = 0$. The amplitude and location of the maximum differences of the vertical displacement have been computed. In both cases, maximum difference points are located near the cone summit that represents the topographic relief. For quasi-analytical model – *Charco et al. (2002)* – the maximum difference between varying depth and flat half-space vertical displacement solution is 0.07 m at 3.25 km horizontal distance from source. The maximum difference of 0.2 m is located at 3.5 km horizontal distance from source for the FEM solution.

Disagreement between numerical and quasi-analytical results could be caused by the following effects:

– While quasi-analytical solution is for 3D axi-symmetric and point heat source model, the numerical (FEM) solution was modelled for finite volume heat source.

– As it has been shown by other authors *(Williams and Wadge, 1998, 2000; Charco, 2007)* the elastic vertical displacement computed by varying depth method includes the topographic effects in an approximate way without considering the local shape of the relief.

Taking into account the heat source power (Table 1) and the elastic and thermoelastic parameters of rock (Table 2), we can roughly estimate the thermo-elastic force density caused by the heat source at the summit of the volcano. It is around 7×10^3 N.m^{-3}, which could fairly correspond to the 20 cm vertical displacements difference.

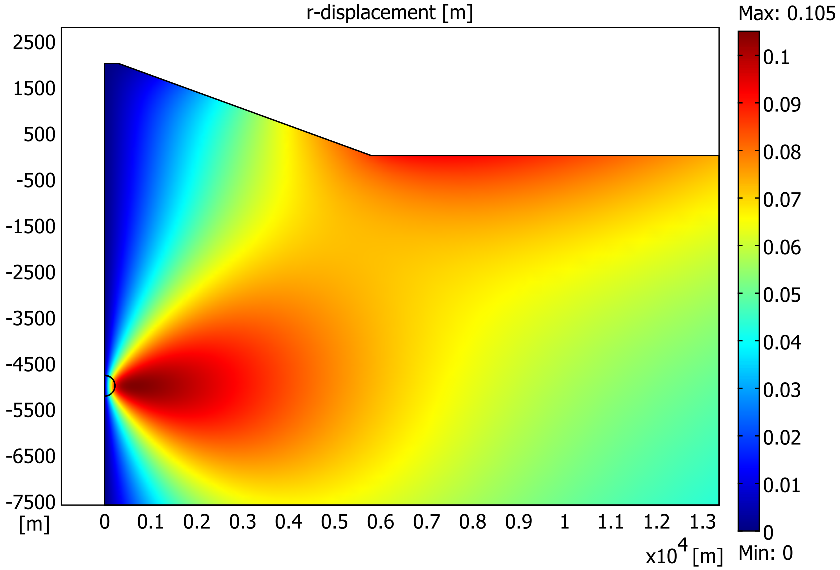

Fig. 2. The computed horizontal component of the displacement field (m).

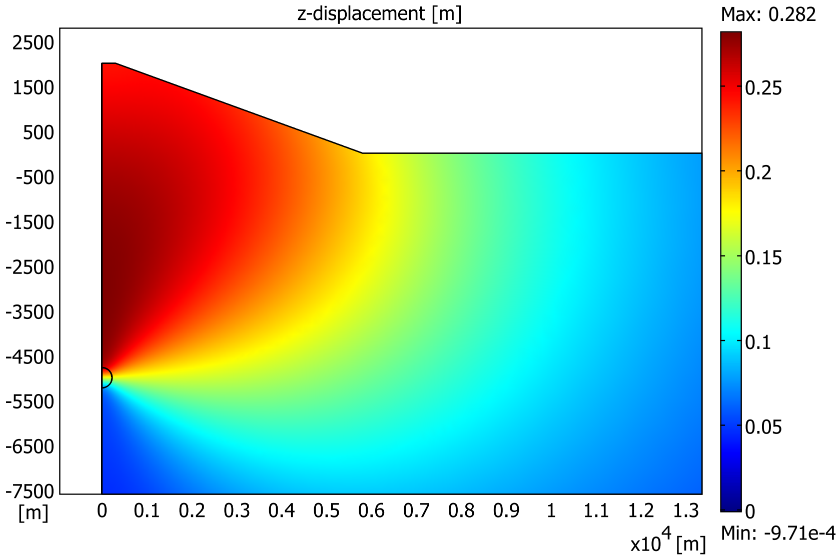

Fig. 3. The computed vertical component of the displacement field (m).

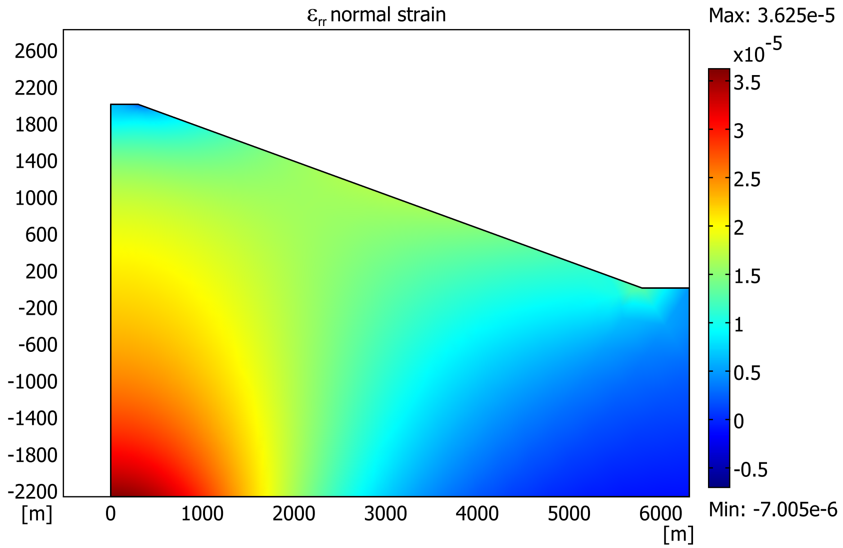

Fig. 4. The computed normal ε_{rr} – component of the strain tensor.

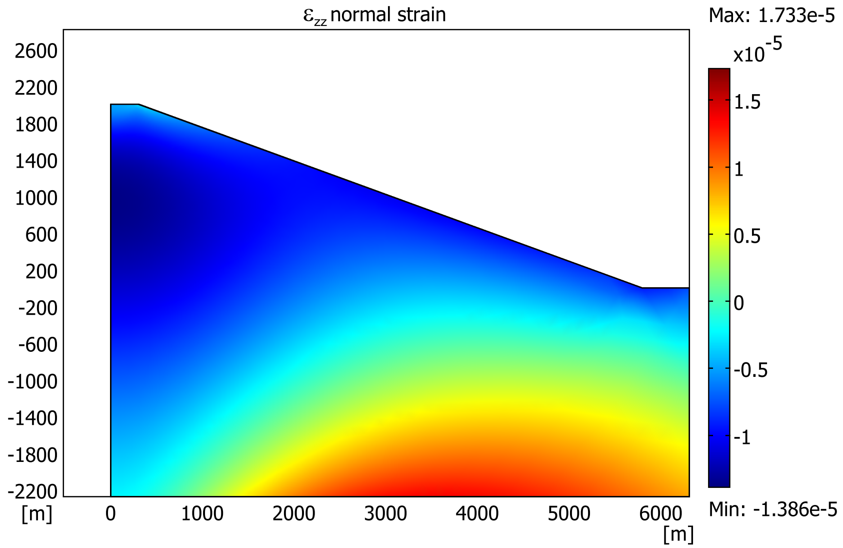

Fig. 5. The computed normal ε_{zz} – component of the strain tensor.

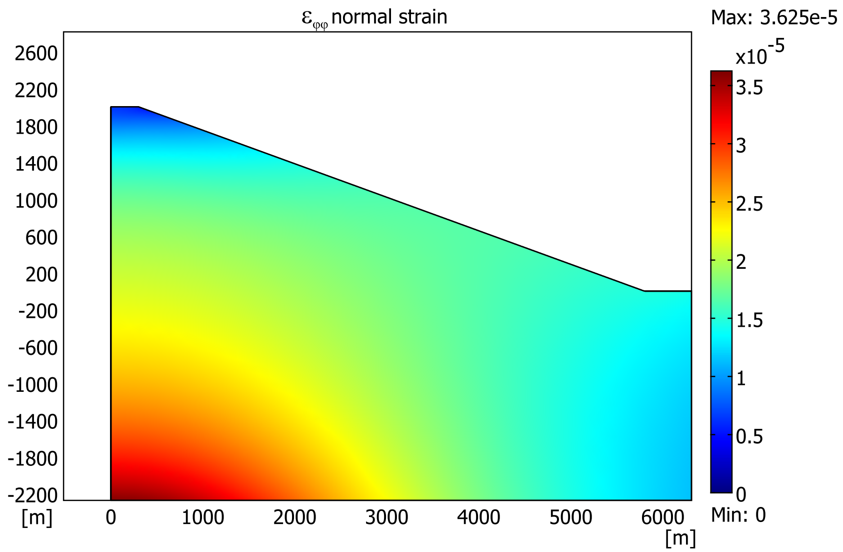

Fig. 6. The computed normal $\varepsilon_{\phi\phi}$ – component of the strain tensor.

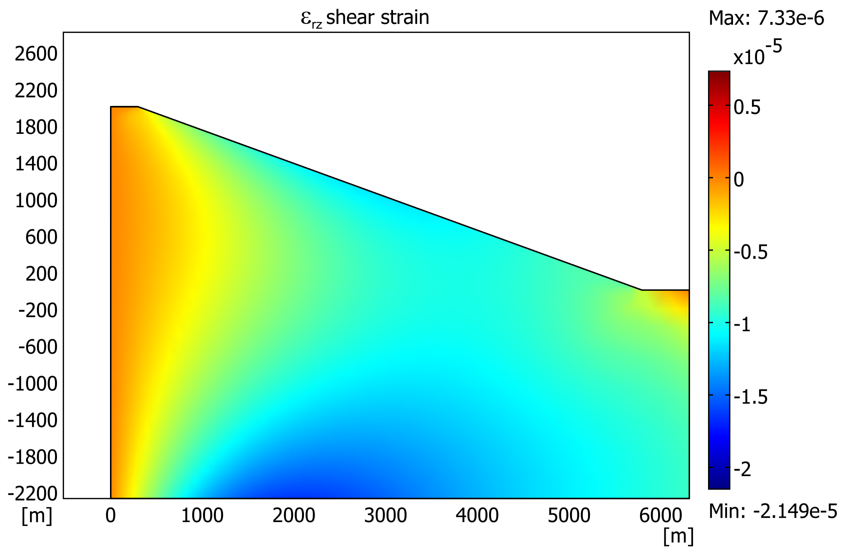

Fig. 7. The computed normal ε_{rz} – component of the strain tensor.

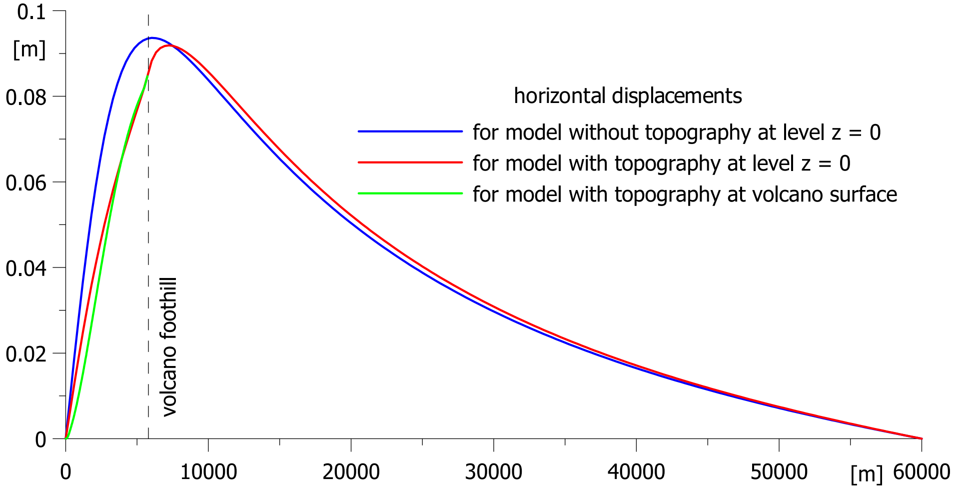

Fig. 8. The comparison of the horizontal displacements for both variants of the model for the level z = 0 (m). The "foothill" marks the horizontal distance where the volcano topography starts.

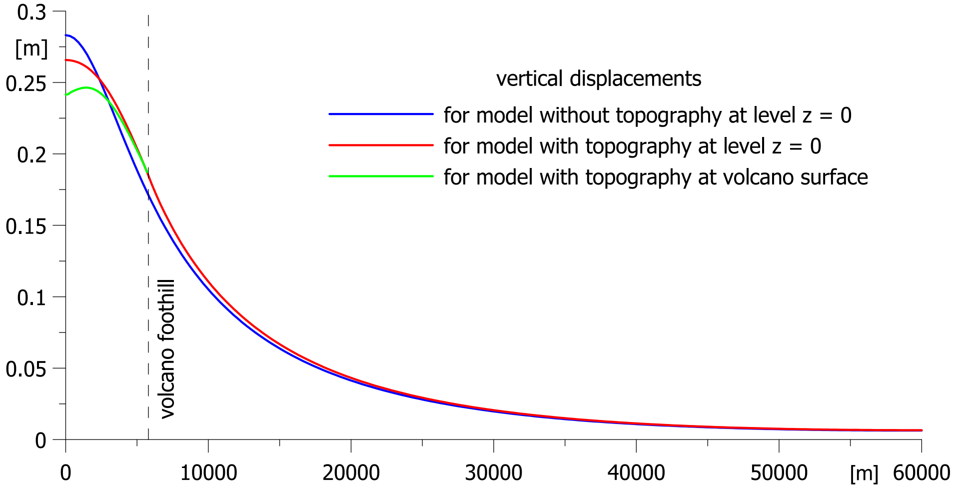

Fig. 9. The comparison of the vertical displacements for both variants of the model for the level z = 0 (m). The meaning of foothill print is the same as in Fig. 8.

The cone slope angle used in the model is 20°. The corresponding value of vertical displacement computed by analytical method *(Charco et al., 2002)* for the topography of the same slope is according to Fig. 2 in *Charco et al., (2002)* approximately 7 cm (in the surface point over the heat source).

In Fig. 9 we can see that the difference in the vertical displacements for both variants of the model (with and without topography) computed by FEM procedure is about 1.74 cm (over heat source). This difference could be attributed to the geometrical difference of the models and to the different application of boundary conditions resulting from different nature of used methods.

5. Conclusions

For the thermo-viscoelastic case the vertical and radial surface displacements were computed for various topography patterns. In the thermoelastic case the temperature and heat flux fields for the models with and without topography were computed. Following this, the differences of displacements and deformation tensor components were determined.

Figures 2–4 in *Charco et al. (2002)* show the reduction of the displacement field and gravity changes magnitude in regions with higher topography due to the greater distance from the source of heat to the free surface. In volcanic areas of greater relief the perturbation of the thermo-viscoelastic solution (deformation and total gravity anomaly) due to topography can be quite significant. Vertical displacements and gravity changes are strongly influenced by the topographic effect if we use the analytical approximation, but this effect in radial displacements is less important. This fact should be tested using a numerical method that permits to include the local shape of the topography, the larger effect in the radial displacements. Therefore the topography may significantly affect the surface displacements and gravity changes computed for magma chamber represented by a heat point source and neglecting the topography may produce erroneous source parameter determination.

Thus, we can conclude that any model that neglects the topographic effect could cause same error in the estimation of surface displacements and gravity changes or in the determination of the characteristics of the intrusion if we use the model to solve the inverse problem. The methods described in this work can be very suitable to more complex models that consider sources of different geometries and allow elastic properties of the medium to vary with depth. While the analytical approximate methodology can be very attractive for solving the inverse problem, the numerical method described above may be used to include the topography when accurate solution is desired since it permits the consideration of non-uniform elastic and thermal properties of the medium and the local shape of the Earth's surface.

The aim of our work was to estimate the influence of the surface topography on the thermoelastic (thermoviscoelastic) deformations. Modeling of the real volcano was out of scope of this contribution. Therefore the mechanical forces due to initial emplacement of magmatic body were not taken into account as well as considering the fractures and other mechanical nonlinearities.

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