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Integrating altimetry derived gravity anomalies with ship-borne gravity data for Africa

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Abstract: The most comprehensive and precise gravity data for the sea region must be obtained for the Sub-Commission of gravity and geoid for Africa (The African Gravity and Geoid sub-commission (AGG) belongs to the Commission 2 of the International Association of Geodesy (IAG)). There are two gravity data sets available for the marine region. The first is the ship-borne gravity data set, which has large gaps but has good precision. The second is the regularly covered and less accurate gravity anomalies generated from satellite altimetry. The most effective fusion of the previous two gravity data sets is examined in this research. First, each data set has passed a gross-error detection scheme. Points with differences of more than 4.5 mGal between estimated and observed gravity anomalies were eliminated because they were deemed to have a blunder. The base has been entirely taken from the ship-borne gravity points since they are more precise (after the gross-error removal). At the altimetry data points, assessments have been made of the discrepancies between gravity anomalies generated by altimetry and ship-borne gravity anomalies. In most places, the employed ship-borne data and the used altimetry data exhibit an acceptable level of consistency. If altimetry-derived gravity anomalies deviate by more than 20 mGal from ship-borne gravity anomalies, they are disregarded. For the majority of nations, especially those with oceanic and maritime borders like Egypt, a mix of land gravity data and shipborne and altimetry data is necessary for exact regional geoid modelling. In nations where there are significant gaps in the terrestrial gravity anomalies, the shipborne and altimetry free-air anomalies are significant. Moreover, the smoothness of gravity database resulting from this investigation is used significantly in geophysical interpretation. Finally, the used method sought to produce a residual gravity field as smooth as feasible because the interpolation errors are inversely related to the level of smoothness of the field.

Key words: ship-borne-gravity-anomalies, altimetry gravity anomalies, Africa, gross-error detection

1. Introduction

Wessel and Watts (1988) noted that marine gravity measurements from shipborne platforms are notoriously difficult. On the other hand, it is de-

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batable if multi-mission satellite radar altimetry, which produces almost homogeneous, nearly global coverage of marine gravity anomalies, is a better source of gravity data (*Haxby et al., 1983*). Several distinct altimetry-derived gravity anomaly data, computed by multiple parties using slightly different data combinations and computational philosophies, are currently accessible in the public domain. Therefore, the question of which, if any, is better arises. This question can be addressed by contrasting the gravity anomalies calculated from shipborne gravimetry with those determined from altimetry (e.g., *Rapp and Bašić, 1992; Rapp, 1992; Rapp, 1998; Olgiati et al., 1995; Zhang, 1998*).

2. Ship-borne free-air gravity anomaly data

The Marine Trackline Geophysics database gives enables access to bathymetry, magnetic, gravity, and seismic reflection data that have been gathered on maritime expeditions from 1939 to the present. A global audience is being reached. Both the United States and non-United States oceanographic institutions, governmental agencies, and universities are mainly the sources for data. Utilizing an interactive ArcGIS map viewer, data are made available online. It is possible to conduct searches by location, cruise year, institution, platform, date, or other criteria. The area under consideration, parameter, and format of data is possible in the case of downloading the data. The observations were recorded in several formats using various (gravity and horizontal) datums. Because no attempt was made to preserve uniformity among individual trips, there is a significant amount of variability between different cruises as a result of Instrumental defects (drift, cross-coupling, off-leveling), navigational problems, and other issues. Digital data files include a documentation header record and some data archives and can be exported in a number of formats, including the MGD77 Exchange Format. The structure and content of the data archives are described in header archives. Geophysical data (bathymetry, magnetics, gravity, and seismic shot-point ids) with time and location can be found in data archives.

For the African window, ship-borne free-air gravity anomalies are retrieved ($-40° \leq \varphi \leq 42°$ N and $-20° \leq \lambda \leq 60°$ E). They are shown in Fig. 1. The data set available for this area is 1,233,381 stations. The average ship-borne free-air anomaly for Africa is -4.60 Gal, with a standard deviation of −58.5 mGal. The ship-borne free-air anomalies extend from −996.0 mGal to 998.0 mGal.

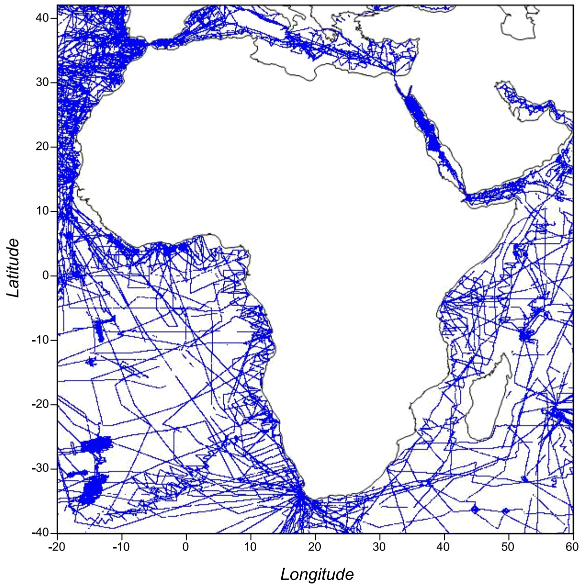

Fig. 1. Distribution of the ship-borne data point/tracks for Africa.

3. Altimetry-derived free-air gravity anomaly data

The Marine Trackline Geophysics database provides access to free-air anomalies discovered through altimetry. Using an average of 44 GEOSAT repetition cycles, this database of geoid and gravity anomaly profiles was produced. The information is split across two files:

1. geo44asc.bin (987,755 records) contains the ascending profiles which run southeast to northwest between 72° S and 72° N,
2. geo44des.bin (991,313 records) contains all of the descending profiles.

The data parameters are geoid height, gravity anomaly, and uncertainty in gravity in addition to time and location. The data are grouped by passes which consist of continuous profiles having no gaps greater than 5 seconds (approximately 33 km). The first record of each pass is slightly different from the other records; the time offset field is 0, the gravity anomaly field contains the number of records in the past (as a 2-byte integer), and the uncertainty field contains 1000 (as a 2-byte integer).

Figure 2 shows the altimetry-derived gravity anomalies for Africa. This data set consists of 188,715 points. The altimetry-derived anomalies range between −318.2 mGal to 320.0 mGal with an average of 6.1 mGal and a standard deviation of 22.4 mGal.

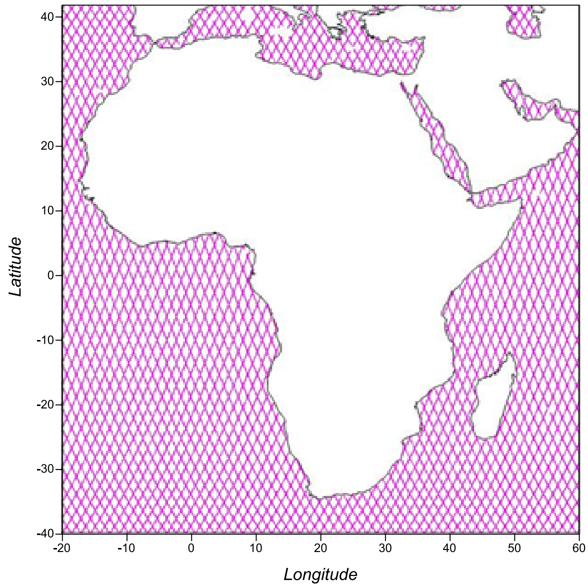

Fig. 2. Distribution of the altimetry-derived gravity anomaly data for Africa.

4. Methodology

The used data set contained roughly 2000 duplicated points, which were found and eliminated first for the ship-borne anomalies. Two models, the EGM2008 and the EIGEN-6C2 global geopotential models, have been employed to calculate the gravity anomalies at the data points. Calculations have been made to determine the discrepancies between the observed anomalies and those that were created by these models' geopotential. The data set has been cleaned up by removing any spots with differences of more than 50 mGal in magnitude. The value of 50 mGal depends on practical background information (*Featherstone, 2009; Denker and Roland, 2005*). Using that technique, approximately 3000 points were removed from the data points.

Utilizing the least-squares prediction method, the gravity anomaly at each place is calculated using the nearby points while leaving out the computational point. Ten to fifteen points have been chosen as the number of points that are considered in the area surrounding the computational point. It found that this amount was realistically adequate (*Abd-Elmotaal and El-Tokhey, 1997; Kraiger, 1988*).

The procedure for removing major errors in error detection is described by *Moritz (1980); Tscherning (2002); Fashir and Kadir (1998)*:

$$\Delta g_p = \Big(C(p,p_1)\ C(p,p_2)\ ...\ C(p,p_n)\Big) \times \\ \times \begin{pmatrix} C(p_1,p_1) & C(p_1,p_2) & ... & C(p_1,p_n) \\ C(p_2,p_1) & ... & ... & C(p_2,p_n) \\ ... & ... & ... & ... \\ C(p_n,p_1) & ... & ... & C(p_n,p_n) \end{pmatrix}^{-1} \begin{pmatrix} \Delta g_{p_1} \\ \Delta g_{p_2} \\ ... \\ \Delta g_{p_n} \end{pmatrix}, \tag{1}$$

where $C(P, P_i)$ is the covariance between the point under consideration and the running nearby points and $C(P_i, P_j)$ represents the covariance between the running nearby points.

In the current study, the Hirvonen generalized covariance model has been identified and detected. This model is provided by *Moritz (1980), p. 179*:

$$C(P_i, P_j) = C(s) = \frac{C_0}{(1 + A^2 s^2)^p}, \tag{2}$$

where s is the distance between the two points under consideration, and the variable A is determined by *Abd-Elmotaal (1992)*:

$$A = \frac{1}{\xi}\left(2^{\frac{1}{P}} - 1\right)^{\frac{1}{2}}, \tag{3}$$

where C_0 is the variance that determined empirically and ξ represents the correlation length. The nature of gravity anomalies affects on the determination of the parameter p. In this investigation, a numerical quantity amount of $p = 0.25$ has been proposed (*ibid.*).

Equation (2) demonstrates that the covariance function depends mainly on the inverse square distance, and it leads to severely ill conditional covariance matrices. Therefore, it has been replaced by the following local covariance function (*Fashir and Kadir, 1998, Eq. 3*) as:

$$C(P_i, P_j) = C(s) = C_0 \left(1 + \frac{s}{R}\right)^{-1}, \tag{4}$$

where R represents the Earth's mean radius. After calculating the gravity anomalies at all data points using the aforementioned least-squares prediction method, the difference between the computed and observed values is calculated. The points would be eliminated if the differences exceeded three times the standard deviation. Repeatedly using the least-squares error detection and elimination method until the square root of the variance of the discrepancies is smaller than 1.5 mGal (Fig. 3).

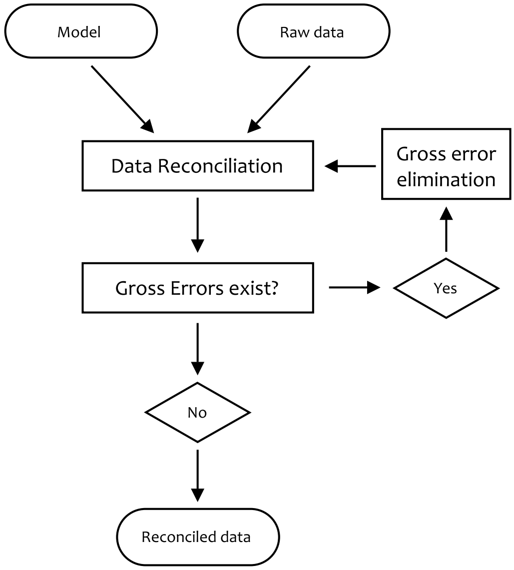

Fig. 3. Gross error detection procedures.

The range of the measured and anticipated ship-borne free-air anomalies is −1183 mGal to 952 mGal, with an average of roughly zero and a standard deviation of 10.3 mGal. These findings were obtained from the initial trial. The points with variations exceeding three times the standard deviation have been eliminated. Subsequently, for the remaining points, the least-squares method was used. Up till the standard deviation was smaller than 1.5 mGal, this procedure was iterated through six times. A total of 141,485 points are removed as errors as a result of this operation. This accounts for 11% of the entire data set. Eliminating the gross errors revealed that the differences in magnitude between the observed blunder-free and anticipated ship-borne free-air anomalies ranged from −3.475 mGal to 3.475 mGal, with a mean of nearly zero and a standard deviation of 0.93 mGal. For additional information, see (*Abd-Elmotaal and Makhloof, 2013*).

The error-free ship-borne gravity anomalies for Africa are displayed in Fig. 4. These error-free gravity anomalies (1,091,896 points) vary from −238.3 mGal to 364.8 mGal. These values have a mean of around 5.08 mGal and a standard deviation of 39.65 mGal. Gravity anomalies with a magnitude less than 10 mGal are indicated by the white color pattern in Fig. 4.

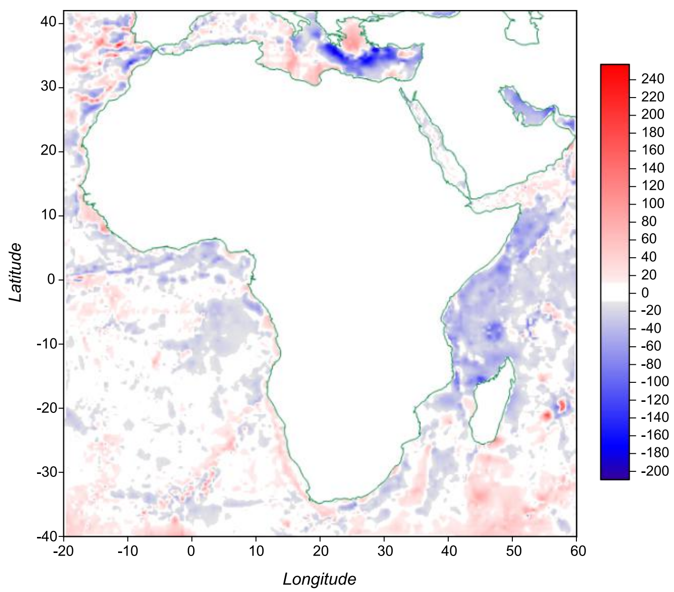

Fig. 4. Blunder-free ship-borne free-air gravity anomalies for Africa. Units in [mGal].

The altimetry data have undergone the same gross-error identification technique. 5,379 points are removed as errors as a result of this method. This is equivalent to 2.85% of the overall data set. The gross-error detection process has gone through three times. The discrepancy between the observed blunder-free and expected altimetry-derived free-air anomalies varied from −4.44 mGal to 4.45 mGal after the errors have been removed, with a mean of around zero and a standard deviation of only 0.76 mGal.

Africa's error-free gravity anomalies using altimetry are shown in Fig. 5. These error-free gravity anomalies have a mean of 5.84 mGal and a standard deviation of 21.57 mGal, and they range from −281.95 mGal to 251.63 mGal. Gravity anomalies of a magnitude smaller than 10 mGal are indicated by the white pattern Fig. 5.

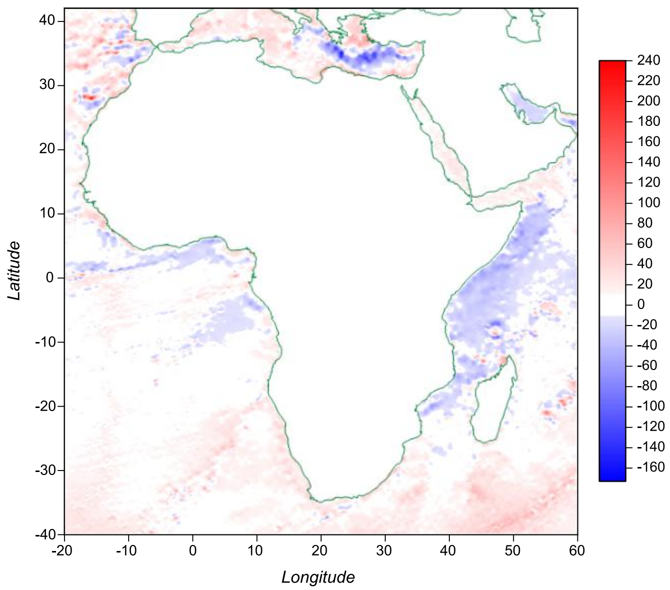

Fig. 5. Blunder-free altimetry-derived free-air anomalies for Africa. Units in [mGal].

Comparing Figs. 4 and 5 shows that the ship-borne and altimetry data generally match in most of the area. However, significant differences are remarkable in some spots. A thorough comparison between both data sets is discussed in the following section.

5. Comparison of ship-borne and altimetry anomalies

The difference between the blunder-free ship-borne and altimetry data sets is computed. The ship-borne data, being directly measured, are more accurate, and hence they have been considered as a base. The differences between the two data sets have then been computed at the altimetry points. They are illustrated in Fig. 6. These differences range between −227.8 mGal and 236.6 mGal with an average of 9.5 mGal and a standard deviation of 19.5 mGal. The white pattern in Fig. 6 signalizes differences fewer than 20 mGal in magnitude. Figure 6 reveals that the two data sets match, within a discrepancy up to 20 mGal, at most of the area under consideration. However, there are still some spots where the discrepancy attains large values.

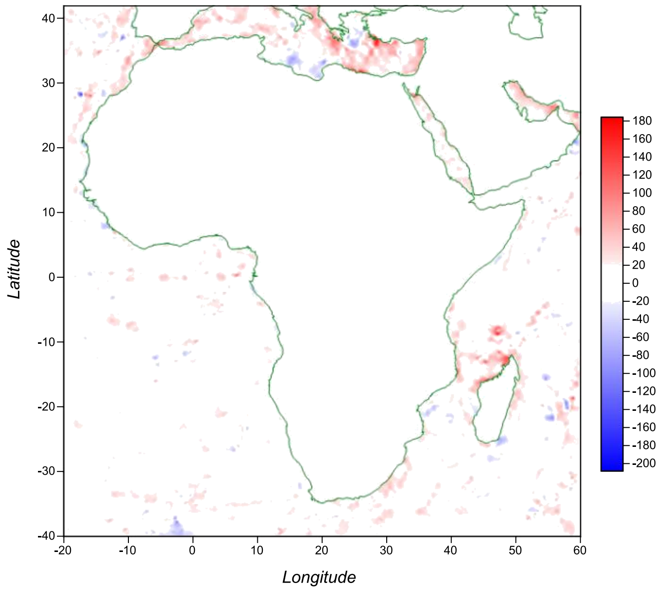

Fig. 6. Difference between blunder-free-ship-borne and altimetry-derived free-air anomalies for Africa. Units in [mGal].

Two subsets are created. The first subset, ALT20, is created by removing all altimetry data points having differences larger than 20 mGal to the ship-borne points. The distribution of the ALT20 altimetry subset (119,257 points), as well as the ship-borne data, is illustrated in Fig. 7. The second subset, ALT10, is created by removing all altimetry data points having differences larger than 10 mGal in magnitude to the ship-borne data. The distribution of the ALT10 altimetry subset (70,728 points), as well as the ship-borne data, is illustrated in Fig. 8.

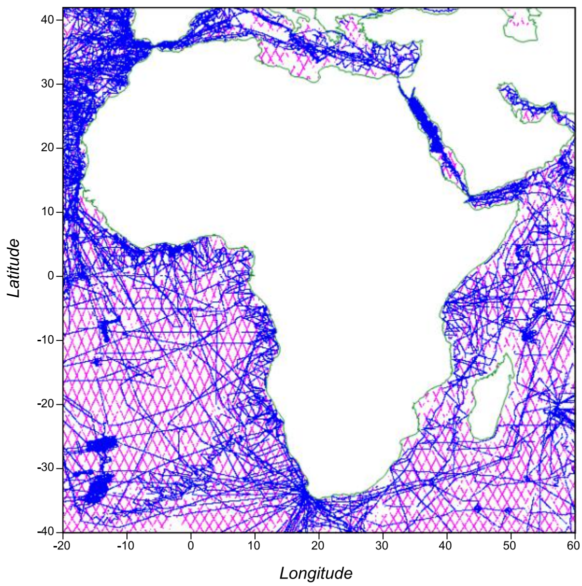

Fig. 7. Distribution of the altimetry data station having discrepancy fewer than 20 mGal to the ship-borne data (subset ALT20).

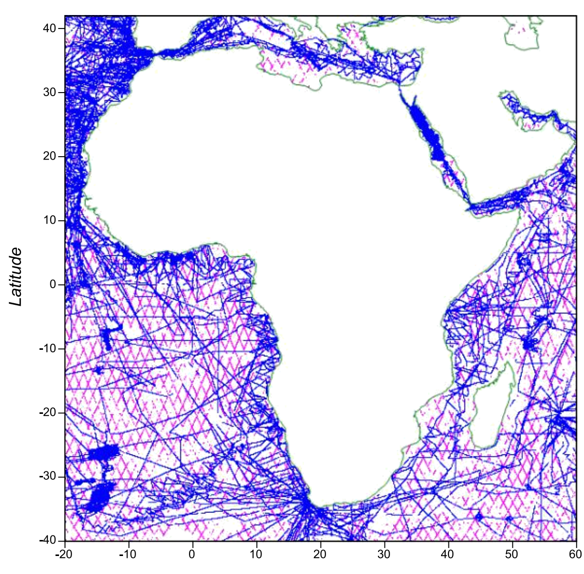

Fig. 8. Distribution of the altimetry data points having discrepancy smaller than 10 mGal to the ship-borne data (subset ALT10).

The statistic for the differences between the ship-borne and the two created subsets ALT20 and ALT10 are given in Table 1.

Table 1. Statistics of the differences between the ship-borne data and the two altimetry subsets. Units in [mGal].

anomaly difference	min	max	mean	std
shipborne – ALT20	−19.99	19.99	5.29	8.89
shipborne – ALT10	−9.99	9.99	2.38	5.17

The combined free-air anomalies for shinbone and altimetry data for differences lesser than 20 mGal are shown in Fig. 9. These free-air anomalies for the combined two data (shipborne and ALT20) range between −238.3 mGal and 364.8 mGal with a mean of −4.18 mGal and a standard deviation of 38.19 mGal. Also, the combined free-air anomalies for shinbone and altimetry data (ALT10) are shown in Fig. 10. These free-air anomalies in this case range between −238.3 mGal and 364.8 mGal with a mean of −4.59 mGal and a standard deviation of 38.73 mGal. From Figures 9 and 10 and the previous statistics it follows that it is not necessary to consider differences smaller than 20.0 mGal. Also, Figures 9 and 10 show that the ship-borne and altimetry data generally match in all of the areas. However, significant differences are not remarkable for the two figures. It is important to note that this gravity database is utilized for more than only geoid calculation; it is also a standalone product used in earth sciences since it reflects intriguing geophysical signals. Then, scientists can be focused on their research interests in geophysical interpretations from the results of this research.

For validation, the obtained results reveal that by going down to a discrepancy of less than 10 mGal, we lose about 50,000 more points from the altimetry data. However, the distribution still looks good (this is, however, a false indicator as the plots have a very small scale). To decide on whether we use either ALT20 or ALT10 altimetry subsets, another check has been carried out. The check is performed by computing the difference between using either ALT20 or ALT10 altimetry subsets with the shipborne data. These differences are plotted in Fig. 11. These differences range between −42.2 mGal and 46.5 mGal with a mean of −1.4 mGal and a standard deviation of 4.3 mGal. Figure 11 shows that most of the region has differences below 5 mGal (the white pattern). Therefore, it has been decided

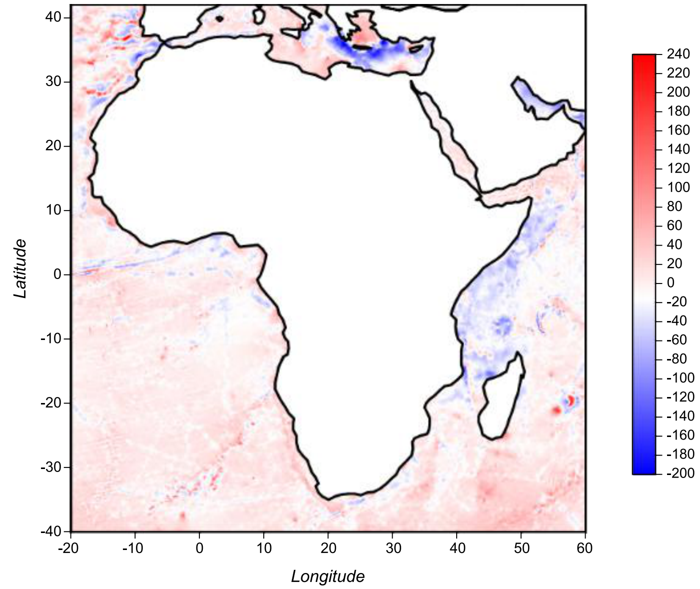

Fig. 9. Combination of altimetry anomalies (ALT20) and shipborne anomalies.

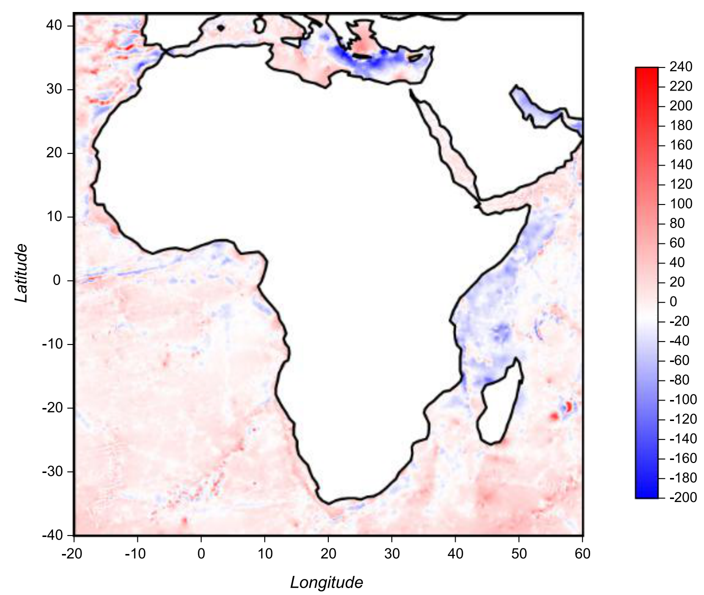

Fig. 10. Combination of altimetry anomalies (ALT10) and shipborne sanomalies.

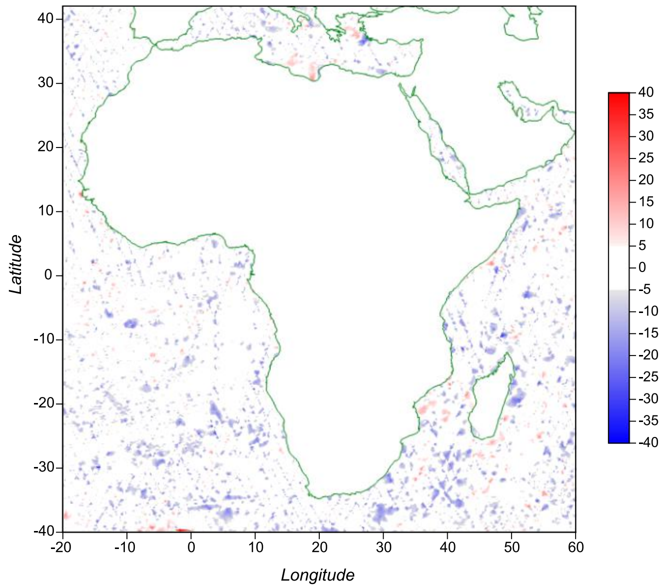

Fig. 11. Difference between using ALT20 and ALT10 altimetry sub-sets. Units in [mGal].

to approve the altimetry data having discrepancy up to 20 mGal to the ship-borne data (the ALT20 altimetry subset) in order to save more needed data points for the Sub-Commission of gravity and geoid for Africa without significantly affecting the data precision.

The primary results of this investigation have been used to determine a Precise Geoid Model for Africa: AFRgeo2019 (*Abd-Elmotaal et al., 2020*) and to create a new gravity database for Africa. The DIR_R5 GOCE model was used to de-trend the generated geoid model for Africa. In this research the interpolation technique used for residuals did not cause aliasing effects, particularly where there were gaps in the point data. Thus, they provide reduced interpolation errors, particularly in the significant gaps in the gravity data. The development in estimating the African height reference surface is apparent when compared to the earlier model AGP2003 (the geoid model for Africa "AGP2003" which has been carried out by *Merry (2003)*). The preliminary results of our research have led to a significant improvement in the African geoid model. The difference between the de-trended AFRgeo2019 and the AGP2003 geoid models is approximately in the round of

1 m in magnitude. The large differences over the Atlantic Ocean arise from the fact that the AGP2003 didn't include ocean data in the solution.

The compilation of a regional gravity database and the determination of a regional geoid for Egypt is important for our research. Then, the results from this investigation have been tested for determining the gravimetric geoid for Egypt in *Abd-Elmotaal et al. (2023)*. The total count of the gravity stations is 102,418. The maximum and minimum values of free-air anomalies for Egypt are −210.6 and 315.0 mGal respectively. The results of this research revealed that the gravimetric geoid for Egypt was significantly improved. The discrepancy in geoid heights between GPS/leveling and gravimetric geoid decreased by around a multiplier of decimeters.

6. Conclusion

In this investigation, a gross-error detection process is performed for ship-borne and altimetry-derived gravity anomalies for Africa. Our technique has been applied iteratively till the standard deviation of the discrepancies (differences) between observed and predicted gravity anomalies' was reduced to smaller than 1.5 mGal. Then, the blunder-free altimetry-derived anomalies are compared with blunder-free ship-borne anomalies. The results prove that the altimetry-derived anomalies are compatible with ship-borne anomalies up to a certain limit. The altimetry points having differences of more than 20 mGal to ship-borne anomalies are recommended to be extracted from the altimetry data set. The ship-borne data set and the altimetry data having differences less than 20 mGal (the ALT20 altimetry subset) thus form a homogenous data set, which is going to be part of the data, used for the computation of the African geoid. Since interpolation errors are inversely correlated with the field's degree of smoothness, the used method sought to produce a residual gravity field as smooth as possible.

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