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Clockwise and counter clockwise 6 & 9 year geostrophic MC (& Rossby) waves in centre of the mechanism of geomagnetic jerks and relevant LODs

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Abstract: Versus the theory of fully stochastically mechanism of geomagnetic jerks based on the buoyant force driven Quasi-Geostrophic (QG) dynamo, the torsional waves in realistic condition of the Earth's core evolve in the intradecadal time scales. Geostrophic slow MC (& Rossby) waves as entanglement of inertial and Alfvén waves are the source of 6 & 9 year geomagnetic secular variations inferred with intradecadal variations in the Earth's rotation rate defined by length of day. From MHD equations in the Earth's liquid metal core, we find a suit of equations equivalent with Hall-MHD in plasma physics with variables and coefficients defined merely in the system of Earth's core dynamo. On reductive perturbation theory, it is deduced derivative nonlinear Schrödinger (DNLS) equation which describes torsional Alfvén waves. In nonlinearity, Modulational and decay instabilities of torsional Alfvén waves in the Earth's core maintain and control occurrences of the geomagnetic jerks and relevant LODs via perturbation theory. Instability induced from a small amplitude perturbation of the plane Alfvén wave can lead to an exponential growth or decay of nonlinear structures to maintain large amplitude turbulences, reasonable to produce the geomagnetic jerks and relevant LODs. Then interplanetary tiny electromagnetic inductions on the Earth's core dynamo via perturbation theory in nonlinearity can produce the jerks and relevant LODs. Also the first-order perturbation of 6-yr Alfvén wave for modulational instability yields to the localized wave-packets called Kuznetsov-Ma breather coincided to 14-yr periodicity for jerk's reports in the years 1902, 1916, 1930, 1944, 1958, 1972, 1986, 2000, 2014. We don't deny the random turbulences but we find that the random driven jerks have lower energies.

Key words: geological and geophysical evidences of deep processes, core dynamics, geodynamo, Alfvén wave, Hall-MHD model, numerical simulation, geomagnetic jerk, LOD

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variation, perturbation theory, modulational instability, decay, rogue wave, nonlinear Schrödinger equation, Alfvénic turbulences

1. Introduction

As defined in the Treatise on Geophysics (*Olson, 2015*):

"Geomagnetic jerks are sudden changes in the otherwise smoothly changing secular variation of the geomagnetic field (Mandea et al., 2000) and variations in the Earth's rotation (defined in terms of length of day) arise from external tidal torques, or from an exchange of angular momentum between the solid Earth and its fluid components. The occurrence of geomagnetic jerks appears to be correlated with sudden changes in the time rate of change of LOD (e.g. Holme and de Viron, 2005; Mandea et al., 2010; Olsen and Mandea, 2008),. . . "

Geomagnetic jerks are believed to be caused by changes in the flow patterns of the liquid outer core of the Earth (*Mandea et al., 2000*) as for instance carried by hydromagnetic waves such as torsional oscillations (*Braginsky, 1984; Bloxham et al., 2002; De Michelis et al., 2005*) which our mechanism here is following it.

Newly *Aubert and Finlay (2019)* on the numerical simulations of the core dynamo (*Aubert et al., 2017; Aubert, 2018; Wicht and Christensen, 2010; Teed et al., 2014; Schaeffer et al., 2017*) have claimed to reproduce the characteristics of well documented jerks on the fully chaotic processes. By monitoring the control parameters, *Aubert and Finlay (2019)* have found that running the simulation along the thousands years results stochastically arrangements of the impulses concerning to relevant geomagnetic acceleration so that the recurrence time is longer for larger energies on a scaling law. In these simulations (*Aubert et al., 2017; Aubert, 2018*), the jerks are derived theoretically by quasi-geostrophic (QG) Alfvén waves emitted inside the outer core and focusing at the core surface as noted by *Aubert and Finlay (2019)* that in their Midpath model sequence, a localized, intense and temporally alternating pulse of azimuthal flow acceleration is observed in the vicinity of the jerk time.

The theory claims geomagnetic jerks sourced by density anomaly in the outer core (*Aubert and Finlay, 2019*) that:

"The source of this perturbation in the simulation can be traced back to a sudden buoyancy release from the tip of an isolated density anomaly plume at mid-depth in the core 25 years before the event."

Strongly *Aubert and Finlay (2019)* have based their mechanism on the assumption that the QG dynamics is dominant in the Earth's outer core referring to the $T-\ell$ regime diagram (*Schaeffer, 2015*) for turbulence in the Earth's core.

Aubert and Finlay have proposed a mechanism for origin of the jerks on the buoyancy force that arises from small variations of density ρ, written using a codensity c which is completely chaotic process whereas here we refer in our mechanism to the non-chaotic geostrophic torsional waves. Aubert and Finlay have inferred a short patch of simulated occurrences with real geomagnetic jerks among the years 2008–2016 such as the 2014 event whereas that at the year 2018 we have published the interplanetary external driven geomagnetic jerks (*Lutephy, 2018*) and correlation between Jovian alignments and a family of geomagnetic jerks in clear whether size or phase.

Here we consider reductive perturbation theory (*Washimi and Taniuti, 1966; Kakutani et al., 1968; Taniuti and Wei, 1968; Taniuti and Yajima, 1969*) to show nonlinearity of the torsional waves and we find a relation between the space path parameter ε in the geodynamo simulations and small parameter ε applied in the expansion series of the perturbation theory. We find the derivative nonlinear Schrödinger (DNLS) equation which allows torsional waves to be possible in the regions where QG waves are only possible in linearity for codensity dependency. Derivative nonlinear Schrödinger (DNLS) equation also is reduced in long wave perturbations to nonlinear Schrödinger equation (NLSE) first derived independently in hydrodynamics by Benney and Newell in 1967 for wave packet envelopes and longtime behavior of weakly interacting waves (*Benney and Newell, 1967*) and in 1968 by Zakharov for deep-water waves using a spectral method (*Zakharov, 1968*).

For confrontation of analytical studies and numerical simulations it is argued that instead extrapolation of the diagrams in geodynamo simulations to the end-path model (*Aubert and Finlay, 2019*), for nonlinearity of the system it needs the program to be continued up to end-path actually. We show here correlation of the Alfvénic perturbations in nonlinearity with geomagnetic jerks and relevant LODs. We see that modulational and decay instability of Alfvén waves in the outer core is the center of mechanism of geomagnetic jerks and relevant LODs.

2. The discrepancies of the numerical simulation of the jerks

The generation/formation of geomagnetic jerks according to the computer simulations is on the MHD model in liquid metal core. But yet there are inconsistencies with realistic conditions of the Earth core which we see these discrepancies even in the quotations by Aubert and Finlay themselves as noted by *Aubert and Finlay (2019)* that:

"According to the mechanism described here, the duration and alternation time scale of jerk events are expected to scale with τ_A, which is about seven times shorter in Earth's core (Gillet al., 2010) than in our Midpath simulation. (Supplementary Table 1). Yet the observed geomagnetic acceleration changes are only two or three times faster than those simulated by the Midpath model. This discrepancy is probably related to the limited temporal resolution of geomagnetic field models, which prevents the true, potentially sub-annual variations associated with jerks from being retrieved at present."

Another discrepancy is secular acceleration pulses of alternating sign as noted by *Aubert and Finlay (2019)* that:

"...another difference involves the sequence of secular acceleration pulses of alternating sign that has been observed in relation with recent jerks (Figs. 1c and 2a). Such features can be explained in our models by the arrival of successive quasi-geostrophic Alfvén wavefronts (Figs. 1d and 2c). However, in the simulation presented in detail here, we only see two significantly weaker jerks at (−6 yr and + 6 yr, Fig. 1d) on each side of the main jerk. This difference is a consequence of the wave damping factor, which also weak in the Midpath model (as evidenced by the ratio $\tau_A/\tau_\eta \approx 10^{-4}$) is still seven times stronger than expected in Earth's core."

Other discrepancy is related to the scaling law of simulation as noted by *Aubert and Finlay (2019)* that:

"The model outputs follow scaling laws (Aubert et al., 2017) depending on ε that also closely approach the conditions expected in Earth's core as we

progress along the path (Supplementary Table 1). Once the magnetic diffusion time $\tau_\eta = D^2/\eta$ *is set to an Earth-like value (see Rescaling section), the end of path simultaneously matches the Earth's core rotational time* $\tau_\Omega = 2\pi/\Omega$*, convective overturn time* $\tau_U = D/U$*, and Alfvén time* $\tau_A = \sqrt{\rho\mu}D/B$ *(here B is the dynamo-generated magnetic field in the fluid shell). This confirms the continuous physical progression of our suite of models towards Earth's core conditions."*

Then simulations of the geodynamo don't run on a fixed space path parameter but the end-path model is more matched with decadal and intradecadal recurrences as confirmed by *Aubert and Finlay (2019)* correlated to the references (*Brown et al., 2013; Finlay et al., 2016*).

Then for agreement with real jerks, the geodynamo simulations need an extended domain of models with scaling law. However the preferred model is considered by Aubert and Finlay at the mid-path model.

These discrepancies are revealing that the assumption of mid-path model isn't correct and linear extrapolation to find the end-path model isn't also matched with reality. We use here the reductive perturbation theory on MHD model as a cause for such a discrepancy for mid-path model assumption in simulations of the core dynamo. The realistic end-path model seems matched with analytical results, of course not via virtual extrapolation but the end-path should be processed actually. The agreement of a few numbers of consequent simulated impulses with real jerks in a short interval of time may be a selection effect, where along the thousands years we have thousands variant arrangements of the simulated impulses; as noted by *Aubert and Finlay (2019)* in a discrepancy that:

"One limitation of the paper is that theory model does not necessarily fit with geomagnetic jerks recorded in earlier periods, like the one that took place in 1969, for example."

Also as noted by *Aubert and Finlay (2019)*:

"The energy concentration mechanism can be understood by noting that quasi-geostrophic Alfvén wavefronts are both guided along, and bounded by, a strongly heterogeneous distribution of magnetic field lines (Duan et al., 2018)."

But inversely, heterogeneous magnetic field will break the long Alfvén waves.

Also for a long Alfvén wave comparable to the width of the outer core, the scale of the concentration event should be larger than the length of wave whereas *Aubert and Finlay (2019)* in their interpretations speak out about a local anomaly in the density.

3. Slow MC (& Rossby) waves as the source of fast intradecadal geomagnetic secular variations

The Earth's liquid metal core is a medium to produce the Alfvén waves on the electromagnetism and hydrodynamics (*Alfvén, 1942*). The experimental confirmation of Alfvén wave was found several years later in studies of waves in liquid mercury (*Lundquist, 1949*). Alfvén waves arise in a liquid metal permeated by a magnetic field because the Lorentz force tends to oppose the curvature of magnetic field lines. In a rotating system, such as planetary core, the Coriolis force inhibits motions that vary along the rotation axis which is called the Proudman-Taylor theorem.

In a sphere, axisymmetric motions that are purely azimuthal and invariant along the rotation axis obey Proudman-Taylor's constraint in which called geostrophic motions. Geostrophic Alfvén waves are thus favored in a rotating system (*Lehnert 1954; Braginsky, 1970; Jault, 2008; Jault and Finlay, 2015*). They are called also torsional Alfvén waves (*Schaeffer et al., 2012*). When the Earth's relative rotation inhibits Alfvén waves, it is possible geostrophic torsional waves to be driven by balance between the Coriolis and Lorentz forces (*Cardin and Olson, 2015*), observationally detected in the Earth's core (*Gillet et al., 2010*).

On extended version of a lecture "Waves in the presence of magnetic field, rotation and convection" by Christopher C. Finlay given during the August 2007 (*Cardin and Cugliandolo, 2008*), we continue here mathematics of the torsional Alfvén waves in the Earth's liquid core. The Alfvén wave equation is obtained as:

$$\frac{\partial^2 \boldsymbol{\zeta}}{\partial t^2} = \frac{1}{\rho\mu} \left(\boldsymbol{B}_0 \cdot \nabla\right)^2 \boldsymbol{\zeta}\,, \tag{1}$$

where the fluid velocity is defined by $\boldsymbol{\zeta} = \nabla \times \boldsymbol{u}$.

The term on the right hand side arises from the restoring force caused by the stretching of magnetic field lines.

Substituting plane wave solution of the form $\boldsymbol{\zeta} = \mathrm{Re}\left(\widehat{\boldsymbol{\zeta}}\mathbf{e}^{\boldsymbol{i}(\boldsymbol{k}\cdot\boldsymbol{r}-\boldsymbol{\omega}\boldsymbol{t})}\right)$ in Eq. (1), the dispersion relation for angular frequency is deduced as:

$$\omega = \pm\, v_A\Big(\boldsymbol{k}\cdot\widehat{\boldsymbol{B}}_0\Big)\,, \tag{2}$$

where $\widehat{\boldsymbol{B}}_0 = \boldsymbol{B}_0/\left|\boldsymbol{B}_0\right|$ and $v_A = \boldsymbol{B}_0/\sqrt{\rho_0\mu_0}$ is the Alfvén velocity.

Lehnert (1954) deduced that rapid rotation of a hydromagnetic system leads to the splitting of Alfvén waves into the two circularly polarized, transverse waves. He realized these so called MC waves (Magnetic Coriolis) would have very different timescales if the frequency of inertial waves was much larger than that of pure Alfvén waves. To derive the MC waves for a rapidly-rotating fluid permeated by a strong magnetic field in the absence of viscous and magnetic diffusion, the starting point is the linearized momentum equation including Coriolis, Lorentz and inertial acceleration and, also the frozen flux induction equation. This is known as equation for Alfvén-Inertial wave (or magnetostrophic waves) (*Acheson and Hide, 1973; Davidson, 2001; Lehnert, 1954*) which reads four solutions:

$$\omega_{MC} = \pm\frac{\boldsymbol{\Omega}\cdot\boldsymbol{k}}{k} \pm \sqrt{\frac{(\boldsymbol{\Omega}\cdot\boldsymbol{k})^2}{k^2} + \frac{(\boldsymbol{B}_0\cdot\boldsymbol{k})^2}{\rho\mu}}\,, \tag{3}$$

When two signs are the same polarity, then 2 fast MC waves (with Lorentz and Coriolis forces reinforcing each other) travelling in opposite directions are obtained. When two signs are of different polarity, 2 slow MC waves (with Lorentz and Coriolis forces opposing each other) travelling in opposite directions are obtained. Then superposition of two-ways propagating MC waves yields to the oscillatory standing wave as follows:

$$\overline{\boldsymbol{\zeta}} = \overline{\boldsymbol{\zeta}_0}\,\sin(kx)\cos(\omega t)\,, \tag{4}$$

and periodicities $2\pi/\omega$ concern to oscillation of standing oscillatory Alfvén waves in the outer core. As argued previously by scientists that the possible Alfvén waves in realistic condition of the Earth core are stationary Alfvén waves.

For slow MC waves, dispersion relation yields to the below equation:

$$\omega^s_{MC} \approx \pm\frac{k\,(\boldsymbol{B}_0\cdot\boldsymbol{k})^2}{2\,(\boldsymbol{\Omega}\cdot\boldsymbol{k})\,\rho\mu}\,. \tag{5}$$

Coriolis force is proportional with the radius and then the excitation of MC waves in the outer core is limited to the North Pole and South Pole spherical cylindrical regime with lower length scales. In the regime of strong Coriolis force, the Alfvén waves are instable for latitude variant of the Coriolis force by distance. If we substitute $l \approx \sqrt{2} \times 10^5$ m into the Eq. (5) it is deduced 8.6-yr slow MC wave.

There are two reports for intradecadal secular variation of the geomagnetic field related to LOD changes. First so called 6-yr secular variation initially reported by *Abarca del Rio et al. (2000)* and *Chao et al. (2014)* and the second, newly discovered 8.6-yr (~ 9) secular variation (*Duan and Huang, 2020*).

In slow MC waves, the circularly polarized motion of velocity perturbation occurs in a clockwise direction and then the Coriolis force acts in opposite direction to the restoring Lorentz force. Then the Alfvén wave is fainted by increment of the ambient magnetic field. Then the slow MC wave is matched with the 8.6-yr geomagnetic secular variation. This effect answers to the question why trend of 8.6-yr secular geomagnetic field is increased while ambient magnetic field is decreasing as questioned by *Duan and Huang (2020)* that:

"Why amplitude of 8.6-year oscillation in LOD shows a secular increasing trend during the past several decades?"

On the other hand, the effect of variation of Coriolis force with latitude in a spherical shell is essential ingredient needed for Rossby waves (*Rossby, 1939; Pedlosky, 1987*). Hide's waves (*Hide, 1966*) arising from the magnetic modification of Rossby waves are referred to as MC Rossby waves. Interestingly it is deduced a dispersion relation for MC Rossby waves similar to the slow MC waves but referred to the β-plane that:

$$\beta = -\frac{2\Omega \cos \theta_{lat}}{D}\,, \tag{6}$$

and D is the outer radius of spherical shell and θ_{lat} is the latitude angle.

Dispersion relation is extracted in Hide's β-plane model of MC Rossby waves (*Hide, 1966*) as follows:

$$\omega = -\frac{\beta}{2k} \pm \frac{\beta}{2k}\sqrt{1 + \frac{4B_0^2 k^4}{\rho_0 \mu_0 \beta^2}}\,. \tag{7}$$

Then it is extracted angular velocity ω^s_{MC} for slow MC Rossby wave similar to the slow MC wave found in a rotating plane layer, but inversely proportional to β rather than Ω so that:

$$\omega^s_{MC(Rossby)} = \frac{B_0^2 k^3}{\mu_0 \rho_0 \beta}\,. \tag{8}$$

The scale of slow MC Rossby wave is about the same for slow MC wave that is $\ell \sim \sqrt{2} \times 10^5$ m. Referring to the *Hide's (1966)* analysis, the slow MC Rossby wave has two equal components along the North-south and east-west directions. Then equilibrium point of the MC Rossby wave is at $\theta_{lat} = 45°$. The slow MC Rossby wave coincides with 6-yr geomagnetic secular variation (*Abarca del Rio et al., 2000; Chao et al, 2014*). In slow MC Rossby wave, Coriolis and Lorentz forces reinforce each other and increment of ambient magnetic field increases the trend of slow MC Rossby wave reported and verified previously (*Duan et al., 2015; Lutephy, 2018*).

4. Modulational and decay instabilities of the slow MC (& Rossby) waves in the center of mechanism of the geomagnetic jerks and relevant LODs

In a rotating magneto-hydrodynamics (MHD) of an incompressible fluid in a reference frame in the center of the Earth with angular rotation rate Ω_0, using the Boussinesq approximation (e.g., *Gubbins and Roberts, 1987*) and treating the density ρ as constant, except in the buoyancy force that arises from small variations of ρ, written using a codensity c, the equations governing evolution of the velocity field $\boldsymbol{u}$, the perturbation magnetic field $\boldsymbol{b}$ as the composite of Navier-Stokes equation and magnetic induction equation (*Schaeffer, 2015; Teed et al., 2015*) read:

$$\partial_t \boldsymbol{u} + (2\Omega_0 + \nabla \times \boldsymbol{u}) \times \boldsymbol{u} = -\nabla p^* + \nu \Delta \boldsymbol{u} + (\nabla \times \boldsymbol{b}) \times \boldsymbol{b} + cg\,, \tag{9.1}$$

$$\partial_t c + \boldsymbol{u} \cdot \nabla\,(c + C_0) = \kappa\,\Delta c\,, \tag{9.2}$$

$$\partial_t \boldsymbol{b} = \nabla \times (\boldsymbol{u} \times \boldsymbol{b}) + \eta\,\Delta \boldsymbol{b}\,, \tag{9.3}$$

$$\nabla \cdot \boldsymbol{u} = 0\,, \tag{9.4}$$

$$\nabla \cdot \boldsymbol{b} = 0\,. \tag{9.5}$$

Noting that the Eq. (9.1) is the so-called Navier-Stokes equation and the Eq. (9.3) is the so-called magnetic induction equation. The important fluid properties are its kinematic viscosity ν, its magnetic diffusivity $\eta = (\mu_0 \sigma)^{-1}$ (where σ is its conductivity), and the diffusivity of the codensity κ (the thermal diffusivity in the case of thermal convection). p^* is a dynamic pressure (including terms that can be written as a gradient, such as centrifugal force, hydrostatic gravity,...) and C_0 is the imposed base codensity profile.

For a given length scale L, typical velocity U and magnetic field B (in velocity units), the model is controlled by four main dimensionless parameters, the flux-based Rayleigh $Ra_F = g_0 F / 4\pi\rho\Omega^3 D^4$, Ekman $E = \nu/\Omega D^2$, Prandtl $Pr = \nu/\kappa$ and magnetic Prandtl $Pm = \nu/\eta$ numbers (*Aubert and Finlay, 2019*).

Here g_0, ρ, ν, κ and η are the gravity at core–mantle boundary, fluid density, viscosity and thermochemical and magnetic diffusivities ($\eta = 1/\mu\sigma_c$, where μ is the fluid magnetic permeability). However from a geophysical point of view, some are difficult to determine for the Earth's core.

Aubert and Finlay (2019) for interpretation of the geomagnetic jerks have referred to the density anomaly and relevant codensity c in the Eq. (9.2). But despite the claimed theory in (*Aubert and Finlay, 2019*), the density anomaly is not the source of jerks unless very less marked events may be concerned.

We can consider complete equation of Navier-Stokes in compressible fluid which is matched with equation of continuity (11.2) written as follows:

$$\partial_t (\rho \boldsymbol{u}) + (2\rho \boldsymbol{\Omega}_0) \times \boldsymbol{u} + \rho \boldsymbol{u} \cdot \nabla \boldsymbol{u} = -\rho \nabla p^* + \nu\rho\, \Delta \boldsymbol{u} + \rho\, (\nabla \times \boldsymbol{b}) \times \boldsymbol{b}\,. \quad (10)$$

The equation of continuity is hidden in the body of complete equation of Navier-Stokes and then in MHD of incompressible fluid described by the Eqs. (9.1–9.5) in which the Eq. (9.1) is the Navier-Stokes equation, it is not added independently the equation of continuity to the equations.

But here for our required results we use independently the equation of continuity and then the Eq. (10) can be written in the version of two below equations:

$$\rho \partial_t \boldsymbol{u} + \rho\, (2\boldsymbol{\Omega}_0) \times \boldsymbol{u} + \rho \boldsymbol{u} \cdot \nabla \boldsymbol{u} = -\nabla p^* + \nu\, \Delta \boldsymbol{u} + (\nabla \times \boldsymbol{b}) \times \boldsymbol{b}\,, \quad (11.1)$$

$$\partial_t \rho + \nabla \cdot (\rho \boldsymbol{u}) = 0\,. \quad (11.2)$$

The equations (10) and (11.1) are coincident where we consider the variable density in an independent equation as the so called equation of continuity beside the Eq. (11.1).

Interestingly, the equations (11.1) and (11.2) are identical mathematically with the so called Landau-Fluid model in plasmas (*Passot and Sulem, 2003; 2004; Goswami et al., 2005*). In reality the Landau-Fluid model is a version of Navier-Stokes in compressible fluid in which the equation of continuity is written in a separate equation and it is natural to write the equation of continuity separately as a pure equation.

For geostrophic flows in which describe the balance between Coriolis force and pressure, here included to the Lorentz force (see e.g. *Greenspan, 1968*) we have:

$$2\rho\boldsymbol{\Omega}_0 \times \boldsymbol{u} = -\nabla p^* \,. \tag{12}$$

The equation (9.4) is not valid for compressible fluid (*Batchelor 1967, p. 75*) whereas the Eq. (9.5) is valid as the absence of free monopoles. Then by considering Eq. (12) for compressible fluid ultimately we find out:

$$\partial_t \boldsymbol{u} + (\nabla \times \boldsymbol{u}) \times \boldsymbol{u} = \nu\,\Delta\boldsymbol{u} + (\nabla \times \boldsymbol{b}) \times \boldsymbol{b}\,, \tag{13.1}$$

$$\partial_t \rho + \nabla \cdot (\rho\boldsymbol{u}) = 0\,, \tag{13.2}$$

$$\partial_t \boldsymbol{b} = \nabla \times (\boldsymbol{u} \times \boldsymbol{b}) + \eta\,\Delta\boldsymbol{b}\,, \tag{13.3}$$

$$\nabla \cdot \boldsymbol{b} = 0\,. \tag{13.4}$$

In absence of terms related to viscosity $\nu\,\Delta\boldsymbol{u}$ and magnetic diffusivity $\eta\Delta\boldsymbol{b}$, the equations in (13.1–13.4) mathematically are the same equations of linear MHD.

Applying $\nabla\times$ on the Eq. (13.1) we deduce:

$$\partial_t (\nabla \times \boldsymbol{u}) + \nabla \times [(\nabla \times \boldsymbol{u}) \times \boldsymbol{u}] = \nu\,\Delta\,(\nabla \times \boldsymbol{u}) + \nabla \times (\nabla \times \boldsymbol{b}) \times \boldsymbol{b}\,. \tag{14}$$

We know that $\boldsymbol{\zeta} = \nabla \times \boldsymbol{u}$ and then the Eq. (14) is written as follows:

$$\partial_t \boldsymbol{\zeta} + \nabla \times (\boldsymbol{\zeta} \times \boldsymbol{u}) = \nu\,\Delta\boldsymbol{\zeta} + \nabla \times [(\nabla \times \boldsymbol{b}) \times \boldsymbol{b}]\,. \tag{15}$$

Also in torsional MC waves, the magnetic field of Alfvén wave is a vortex aligned to the inertial vortex ξ so that torsional Alfvén waves are derived by vortex ξ (Eq. (1)). The perturbation $\boldsymbol{b}$ is a circularly clockwise or counter clockwise polarized magnetic vortex which mimics left-hand and right hand circularly polarized Alfvén waves in the plasma physics under the tension of ambient magnetic field. Then we can consider a scale coefficient k_s as:

$$\boldsymbol{b} = k_s \boldsymbol{\zeta} \,. \tag{16}$$

Rescaling the dimensions until to obtain $k_s = 1$ and substituting Eq. (17) into the Eq. (16) it is deduced:

$$\partial_t \boldsymbol{b} + \nabla \times (\boldsymbol{b} \times \boldsymbol{u}) = \nu \, \Delta \boldsymbol{b} + \nabla \times ((\nabla \times \boldsymbol{b}) \times \boldsymbol{b}) \,. \tag{17}$$

If we neglect the kinetic viscosity, the equation is identical mathematically with the induction equation with Hall-effect (*Passot and Sulem, 2003; 2004*) which is derived by Faraday's law in Maxwell equations.

But here from magnetic induction Eq. (13.3) and Eq. (17) we obtain:

$$\eta \, \Delta \boldsymbol{b} = \nu \, \Delta \boldsymbol{b} + \nabla \times ((\nabla \times \boldsymbol{b}) \times \boldsymbol{b}) \;. \tag{18}$$

Then we find out that the term $(\eta - \nu) \, \Delta \boldsymbol{b}$ in the Eq. (18) plays mathematically the role of the Hall term $\nabla \times ((\nabla \times \boldsymbol{b}) \times \boldsymbol{b})$ in the outer core of the Earth. Then magneto-hydrodynamics in Earth's outer core produces nonlinearity as well as the Hall term in plasma physics.

In linear MHD, we have a suit of equations included to the continuity, force, equation of state, Ampére's law, Faraday's law, Ohm's law which the Ohm's law in one dimensional mode is written by $\partial_t \boldsymbol{b} - \nabla \times (\boldsymbol{u} \times \boldsymbol{b}) = 0$. In generalized Ohm's law it is added a Hall-term $\nabla \times ((\nabla \times \boldsymbol{b}) \times \boldsymbol{b})$ to the right side of the Ohm's law which it causes linearity transferred to the nonlinearity and it is appeared instabilities reasonable to grow exponentially the small amplitude perturbations in the plane waves. We understand that via liquid metal MHD we arrive to similar term in the equations responsible to create nonlinearity of Alfvén waves in the earth outer core. In long-wave, weakly nonlinear scaling regime from one-dimensional magnetohydrodynamics (MHD) equations in the presence of Hall Effect, by reductive perturbation theory, applying slow variables it is derived DNLS equation. The DNLS equation was first derived by *Rogister (1971)*, who used kinetic theory. Its two-fluid version was subsequently derived by *Mjølhus (1976)* and *Mio et al. (1976)*. In vanishing boundary condition (parallel propagation), the DNLS equation conveniently is written as one single equation (*Spangler et al., 1985; Kennel et al., 1988; Mjølhus and Hada, 1997; Ruderman, 2002*) that:

$$\partial_\tau b - i\mu\,\partial_{\xi\xi} b + \alpha\,\partial_\xi\left(|b|^2\,b\right) = 0\,, \tag{19}$$

where $\mu = \pm\frac{1}{2}$ corresponds to left $(-)$ and right-hand $(+)$ polarized mode and $\alpha = \frac{1}{4}\,(1-\beta)^{-1}$ (β is the ratio of the kinetic to the magnetic pressure). Of course we need to notice that here the parameters are being in normalized mode.

By the way if we repeat expansion of the parameters in the manner of reductive perturbation theory, the equations (13.1, 13.2, 13.3, and 13.4) result such a derivative nonlinear Schrödinger equation in similar way of the plasma physics with difference that the coefficients here are related to the kinetic viscosity ν and magnetic diffusivity η.

Basic model equations for weakly nonlinear dispersive MHD waves in plasma were derived using reductive perturbation theory (*Washimi and Taniuti, 1966; Kakutani et al., 1968; Taniuti and Wei, 1968; Taniuti and Yajima, 1969*) applying slow variables ξ and τ produced by small parameter ε used for mathematical expansion series as:

$$\xi = \varepsilon\,(x - v_A\,t)\;, \tag{20}$$

$$\tau = \varepsilon^2\,t\,. \tag{21}$$

In simulation of geodynamo (*Aubert et al., 2017; Aubert, 2018; Schaeffer, 2015*) the control parameters follow a unidimensional path (*Aubert et al., 2017*) in parameter space ε to connect conditions of previous coupled Earth (CE) model (*Aubert et al., 2013*) to those of Earth's core. A single variable $\widetilde{\varepsilon}$ controls four parameters through the following rules:

$$\begin{cases} Ra_F = \varepsilon\,Ra_F\,(CE)\,, \\ E = \varepsilon E\,(CE)\,, \\ Pr = 1\,, \\ Pm = \sqrt{\varepsilon}\,Pm\,(CE)\,. \end{cases} \tag{22}$$

As noted by *Aubert and Finlay (2019)* that:

"We have shown (Aubert et al., 2017) that parameters, that realistically describe Earth's core conditions, can be obtained by setting $\varepsilon = 10^{-7}$*, which defines the end of the path. Our main model cases are defined in refs. (Aubert et al., 2017; Aubert, 2018) and in Supplementary Table 1 by the values* $\widetilde{\varepsilon} = 10^{-2}, 3.33 \times 10^{-3}, 10^{-3}$ *and* 3.33×10^{-4}*, respectively corresponding to 29%, 36%, 43% and 50% of the path (the Midpath model)".*

By MHD equations in geodynamo simulations (*Aubert et al., 2017; Aubert, 2018*), real variables time t and space x parallel to the magnetic field are transferable by small parameter ε to slow variables τ and ξ so that here we find a relation between small parameter ε in reductive perturbation expansion and space path parameter ε in the numerical simulation of the geodynamo. If we use $\widetilde{\varepsilon}$ instead space path parameter ε to discriminate with small parameter ε in the reductive perturbation expansion (space path parameter: $\widetilde{\varepsilon} \to \varepsilon$), this relation is:

$$\varepsilon^2 = \sqrt[4]{\widetilde{\varepsilon}}\,. \tag{23}$$

Then for Eq. (21), the Alfvén time τ_A is rescaled by $\sqrt[4]{\widetilde{\varepsilon}}$ so that:

$$\tau_A\left(\widetilde{\varepsilon}\right) = \tau_A\left(\widetilde{\varepsilon} = 10^{-7}\right) \times \sqrt[4]{\widetilde{\varepsilon}}\,|\tau_A\left(\widetilde{\varepsilon} = 1\right) = 106\ . \tag{24}$$

Writing in the log form $(X = -\log\left(\tau_A\right), Y = \log\left(\widetilde{\varepsilon}\right))$ yields to:

$$\log\left(\tau_A\right) = 2 + \log\left(\widetilde{\varepsilon}\right). \tag{25}$$

Substituting numerical results (*Aubert and Finlay, 2019*) in this logarithmic equation we obtain a straight line as observable in the Fig. 1 and then experimental reports are verifying Eq. (23).

Then the authors of the simulation of geodynamo (*Aubert et al., 2017; Aubert, 2018; Schaeffer et al., 2017*; etc.) unaware have applied the transformation used in the reductive perturbation theory with small parameter of the expansion as rescaling factor $\varepsilon = \sqrt[8]{\widetilde{\varepsilon}}$. Then we can use the space path parameter in the manner of the slow variables used in the reductive perturbation theory to derive nonlinear Schrödinger equation for Alfvén wave in the liquid metal core as well as the derivation of nonlinear Schrödinger equation for plasma on the expansion of the parameters via small parameter

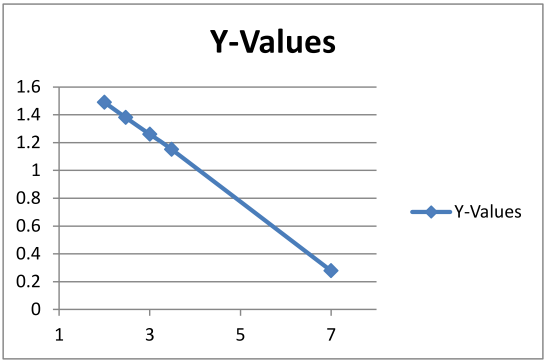

Fig. 1. The correlation of the space path parameter with the small parameter in the reductive perturbation theory.

εin reductive perturbation theory. Then we find out that linear extrapolation of the diagrams in the simulations to the end-path model is an error for nonlinearity of the MHD model in the Earth's core.

The Coriolis driven inertial waves are intrinsically stable (*Batchelor, 1967; Tritton, 1987*). Then instability of the MC geostrophic waves is controlled by instability of the magnetostrophic mode.

We see morphological identity between the circularly polarized Alfvén waves in the plasma physics and circularly polarized torsional Alfvén waves in the Earth's core and then instability of the geostrophic torsional Alfvén waves is identical with the instability of the circularly polarized Alfvén waves in plasma physics.

Hall-MHD model for a carrier plane wave of small amplitude b_0 (e.g., *Wong and Goldstein, 1986; Kennel et al., 1988*) shows that for right-hand polarization, the instability is modulational for $\beta > 1$ and of decay type for $\beta < 1$. For left-hand polarization, the wave is stable for $\beta > 1$, while modulational and decay instabilities coexist for $\beta < 1$. Of course for $\beta < 1$ the decay instability of the left-hand polarized Alfvén wave is weak.

Nonlinearity of the geostrophic torsional waves results instability of the plane waves, whether modulational or decay and instabilities of the torsional waves is reasonable for Alfvénic rouge waves and large turbulences as the intermediate for occurrences of the geomagnetic jerks and relevant LODs.

The compressible perturbations induced in straight untwisted and nonrotating magnetic flux tubes by weakly-nonlinear long-wavelength torsional waves have been discussed previously by *Vasheghani Farahani et al. (2011).* Interestingly they have discussed the perturbations induced by standing torsional waves and standing torsional waves induce growing compressible perturbations, similarly to standing shear Alfvén waves (*Tikhonchuk et al., 1995; Verwichte et al., 1999; Litwin and Rosner, 1998*) and standing kink modes of coronal loops (*Terradas and Ofman, 2004*).

The perturbations in the MC (& Rossby) plane waves can be naturally provided internally and externally. The internal small amplitude perturbation is originated by different ways such as the self-focusing (*Zakharov and Shabat, 1972; Hasegawa, 1970*) and wave steepening (*Cohen and Kulsrud, 1974*) and externally interplanetary tiny effects enable to provide small amplitude perturbations in the torsional Alfvén waves in the Earth's core, reasonable to derive the geomagnetic jerks and relevant LODs.

Observationally modulational instability is matched for MC Rossby wave and decay for MC wave which is verifying morphological identity of circularly polarized Alfvén waves in the plasma physics with circularly polarized torsional Alfvén waves in the Earth's core. We see here that 6-yr torsional Alfvén wave is an intermediate media to provide the external driven Alfvénic perturbations via modulational instability and 8.6-yr torsional Alfvén wave has decay instability to provide the Alfvénic perturbations which in nonlinearity of the outer core can generate high amplitude fluid turbulences and then while the 6-yr torsional wave amplitude is in the low amplitude mode, the jerks are possible to be derived by decay of the 8.6-yr torsional wave.

Of course, the 6-yr periodicity of magnetic field secular variation is matched also with the Sun magnetic field fluctuation at Jupiter position (*Lutephy, 2018*). But this coincidence seems to be accidental, where the 6-yr signals are sourced by Alfvén wave. Interplanetary electromagnetic inductions are very small at the earth position and then in usual condition these tiny effects are neglected but on the modulational and decay instabilities of geostrophic MC waves, the small perturbations can grow up to a rogue wave or high amplitude turbulences. This looks like the rouge waves in the oceans (*Benjamin and Feir, 1967; Onorato et al., 2013*, etc.) and in the optical fiber (*Tai et al., 1986; Agrawal, 2013*; etc.). Where a small perturbation of the torsional wave in the liquid metal core could to grow exponentially or decay, then the interplanetary tiny electromagnetic effects are not underestimated for the source of geomagnetic jerks. When the interplanetary tiny effects focus on the geostrophic 6 & 9 MC (& Rossby) waves in the Earth's core, the instability would cause to transfer tiny perturbations in these waves to the large turbulences which would be observable ultimately in the format of the geomagnetic acceleration impulses which is called geomagnetic jerks. In low amplitudes of the geostrophic 8.6-yr MC wave, the wave is firm upon the perturbations but in extremes of the 8.6-yr MC wave, like the tall buildings which the wind can destroy it, the decay is appeared easier which is verified observationally by *Duan and Huang (2020)* with occurrences of the jerks in the extremes of 8.6-yr magnetic secular variation. Then we find here that occurrences of the external driven geomagnetic jerks is controlled by 6-yr torsional Alfvén wave and also controlled by extremes of the 8.6-yr torsional Alfvén wave. Then the amplitude of a typically geomagnetic jerk should be controlled by sinusoidal term of 6-yr geomagnetic secular variation, verified previously by *Lutephy (2018)* and of the extremes of 8.6-yr (~ 9) geomagnetic secular variation discovered by *Duan and Huang (2020).* Duan and Huang in a figure has shown the correlation of some geomagnetic jerks to the extremes of the 8.6 year periodic secular variation of the geomagnetic jerks in which in this paper we have correlated it to a torsional oscillatory Alfvén wave.

Via looking in the Jovian alignments (*Lutephy, 2018*), we find out that the Jovian alignments does reveal the jerks unless where the amplitude of 6-yr geomagnetic secular variation is very low or jerks are revealed at the extremes of 8.6-yr (~ 9) secular variation. In reality the modulation and decay instabilities of these 6 & 9 year periodic Alfvén waves are an intermediate environment to transfer tiny external interplanetary inductions to the large turbulences which ultimately its signature is observable as the geomagnetic jerks. In reality the amplitude of the external driven perturbations is related to the amplitude of the interplanetary electromagnetic inductions and also to the amplitude of the oscillating Alfvén wave. Then growth rate of the modulation instabilities can be signed by the intensity of the electromagnetic inductions and amplitude of the oscillating Alfvén waves. On the arguments in the paper (*Lutephy, 2018*), the Eq. (7) does show theoretical correlation of the amplitude of external driven geomagnetic jerks to the phase of Alfvén waves α and length of the relevant Jovian alignments L, that is, the equation $O_3 \propto \beta \sin(\alpha)/L^2$. We see in Fig. 2, perfect proportionality between amplitude of 6-yr geomagnetic secular variation and

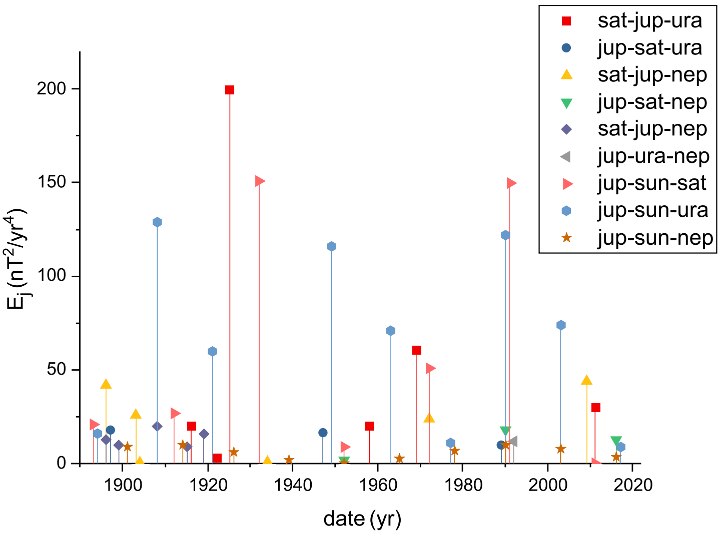

Fig. 2. Geomagnetic jerks reported dates one related to Jovian alignments.

amplitude of the jerks O_3 externally driven by relevant Jovian alignments (*Lutephy, 2018*) and also with inverse square decrement by length of Jovian alignments. Of course we can use the squared geomagnetic acceleration, that is:

$$E_j \propto \beta^2 \sin^2(\alpha)/L^4 \,. \tag{26}$$

As we see in the below table (Table 1), the Jovian alignments occurred in lower phases of 6 year torsional Alfvén wave doesn't show any relevant geomagnetic jerks completely verifying the mechanism proposed here.

Generally the proportionality of the geomagnetic jerks energy to the phase of 6-yr Alfvén wave is experienced in the Fig. 3 (*Nevanlinna, 2004; Balasis et al., 2016; Qamili et al., 2013; Duka et al., 2012; Pinheiro and Travassos, 2010; Alexandrescu et al., 1996; De Michelis et al., 2005; Ducruix et al., 1983; Chambodut and Mandea, 2005; Cafarella and Meloni, 1995; Macmillan, 1996; Brown et al., 2013; Chulliat and Maus, 2014*).

Our prediction about a geomagnetic jerk in the year 2017 (*Lutephy, 2018*) has been verified and reported previously so that SWARM satellite data (*Hammer, 2018*) showed a jerk occurred in 2017. As noted in the paper (*Lutephy, 2018*):

Table 1. Jovian alignments with low phase in 6-yr TAW with no report for jerks.

Jovian Alignments	**Dates**	**Phase (degrees)**	**L (AU)**	**Reports**
Saturn-Jupiter-Uranus	1922.08.02	8°	28	No
Saturn-Jupiter-Neptune	1904.05.01	6°	39	No
Saturn-Jupiter-Neptune	1933.11.30	10°	39	No
Jupiter-Saturn-Neptune	1915.12.18	15°	35	No
Jupiter-Saturn-Neptune	1952.08.06	19°	35	No
Jupiter-Sun-Saturn	2011.02.18	2°	15	No
Jupiter-Sun-Uranus	1934.08.24	12°	25	No
Jupiter-Sun-Neptune	1939.05.12	26°	35	No
Jupiter-Sun-Neptune	1952.02.01	0°	35	No
Jupiter-Sun-Neptune	2015.11.03	39°	35	No

"Eq. (7) above and for alignment at 2017.07.07, the ideal formula shows 11 nT/yr^2"

Alfvénic perturbations produce hydromantic turbulences and geomagnetic jerks are signature of core fluids in the CMB. Of course for similar events,

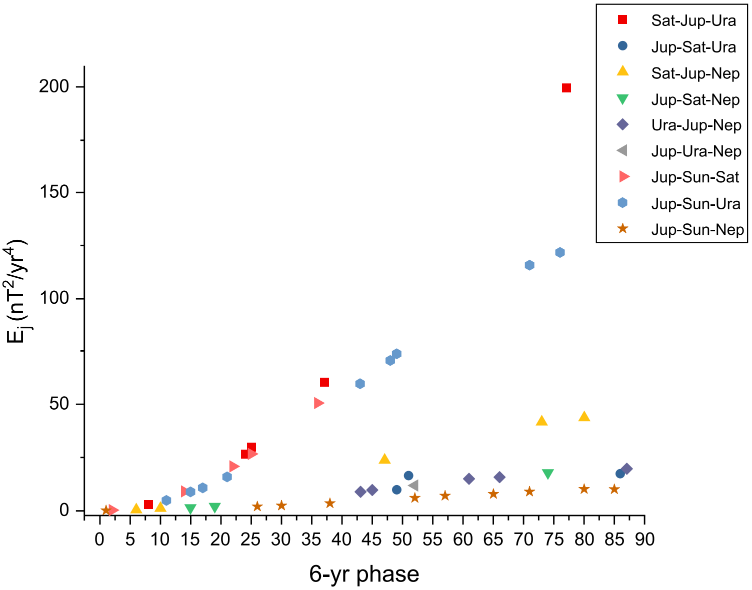

Fig. 3. Correlation of the energy of external driven geomagnetic jerks to the phase of 6-yr Alfvn wave.

the quality and quantity of the turbulences can be different. For example, the number of the jerks for similar Jovian alignments can be different or the position of occurrences in the Earth can be different and the scales too. Then uncertainty is expected ever in the geomagnetic jerks recurrences. Also we need to notice that the coefficient β in the Eq. (26) is different for different planetary alignments.

On the observations, the secular variations occur in different scales such as the 60-yr timescale inferred from the analysis of the decadal length of day changes since the first half of the 19$^{\text{th}}$ century (*Jordi et al., 1994*) and of the geomagnetic secular variation after 1900 (*Braginsky, 1984*) and 300-yr secular variation coincident with timescale of observed wave-like geomagnetic secular variation signals (*Finlay and Jackson, 2003*).

We see that at the year 1995, the 6-yr and 8.6-yr geomagnetic secular variations are in their maximum amplitude but no jerks observed and then despite the proposed theory by *Duan and Huang (2020)*, the secular variations are not directly the source of geomagnetic jerks and the LODs, but the nonlinear media is intermediate to derive the perturbations by external and internal magneto hydrodynamic effects. In fact it was predicted by Duan and Huang for a jerk occurrence in the year 2020 but any jerk was not revealed. By the way where there is no provider for perturbations in the background geostrophic torsional Alfvén waves in the outer core, there is no geomagnetic jerks observable. And jerk in the 1991 is simultaneous with minimum amplitude of the 6-yr geomagnetic secular variation crossed with maximum amplitude of 8.6-yr geomagnetic secular variation and this reality verifies that the 1991 jerk is a decay driven event in which confirms the decay instability of the 8.6-yr torsional wave.

On this mechanism that the torsional waves are intermediate by modulational and decay instabilities for jerks generations, we answered to unsolved questions mentioned by *Duan and Huang (2020)* such as a question that:

"Here, a scientific question arises, i.e., whether the amplitude increasing of the 8.6-year oscillation is related to the physical sources which can cause the jerks?"

In long-wave modulations, the derivative in nonlinear term of the DNLS can be simply replaced by $i\boldsymbol{k}_0$ ($\boldsymbol{k}_0$ being the wave number for the carrier wave) and then DNLS is transferred to the nonlinear Schrödinger equation

(NLSE). Also the slow modulation of an Alfvén wave envelope with small (but finite) amplitude is obtained by a standard multiple-scale analysis in general nonlinearity, defining the slow variable and expanding parameters (*Champeaux et al., 1997*) which results NLSE. NLSE is written in dimensional form as follows:

$$i\,\partial_t b + \frac{1}{2} P\,\partial_{\xi\xi} b + Q\,|b|^2\,b = 0\,. \tag{27}$$

A small perturbation of the wave's amplitude is considered (*Ali Shan, 2018*) as:

$$b = [b_0 + \delta b\,(\eta)]\,\mathrm{e}^{-i\,\Delta\tau}\,, \tag{28}$$

where $\eta = (K_{MI}\xi - \omega_{MI}\,\tau)$ is the phase of the modulation with $K_{MI} \ll k$ and $\omega_{MI} \ll \omega$ are respectively the wavenumber and frequency of the modulation, $b_0$is the constant (real) amplitude of background plane wave of the NLSE, $\delta b \ll b_0$ is the small amplitude perturbation, and $\Delta = -Q\,b_0^2$ is nonlinear frequency shift.

Mathematically for an assumed perturbation embedding in the nonlinear Schrödinger equation it is derived typical wave solutions and a so called type is breather solution. Breather is a nonlinear wave in which energy concentrates in a localized and oscillatory fashion and most breathers are localized in space and oscillate. The instability condition depends on whether sentence PQ in the Eq. (27) is positive or negative (*Hasegawa 1975, 1989; Remoissenet, 1994*). Corresponding to $PQ > 0$, the instability induced from small perturbation of a plane wave can lead to an exponential growth of nonlinear structures of high amplitudes such Peregrine soliton (rogue waves), Akhmediev breather (AB) and the Kuznetsov-Ma breather (KM) (*Akhmedieva et al.,1985; Kibler et al., 2012, Chin et al., 2016; El-Tantawy et al., 2017; Peregrine, 1983*):

$$b\,(\xi,\tau) = b_0\left[1 + F\right]\mathrm{e}^{iP\tau}\,, \tag{29}$$

where $b_0 = \sqrt{P/Q}$, R is free real number and:

$$F = \frac{2(1-2R)\cos\left[\sqrt{8R(2R-1)}P\tau\right] - i\sqrt{8R(2R-1)}\sin\left[\sqrt{8R(2R-1)}P\tau\right]}{\sqrt{2R}\cosh\left(\sqrt{4\,(2R-1)}\xi\right) - \cos\left[\sqrt{8R\,(2R-1)}P\tau\right]}\,.$$

For $0.5 < R < \infty$, the solution reduces to the Kuznetsov-Ma (KM) breather. This means that the first-order perturbation of the background plane Alfvén wave does generate a rogue Alfvén wave. Plane Alfvén wave is periodic about the 6-yr (T_{plane} = 6-yr) and then the Kuznetsov-Ma breather is periodic in time only with temporal period as:

$$T_{KM} = \frac{T_{plane} = 6yr}{\sqrt{8R\,(2R-1)}}\,. \tag{30}$$

In comparison with reported amplitude for Kuznetsov-Ma breather in the fiber optics (*Kibler et al., 2012*) if we assume approximately that the maximum amplitude of the Kuznetsov-Ma breather here to be 4 times larger than its carrier wave amplitude, then we obtain $R \sim 0.522$ and substituting it into the Eq. (30) it is deduced that:

$$T_{KM} \approx 14\text{yr}\,. \tag{31}$$

Interestingly we find out 14-yr periodic geomagnetic jerk reports, that is, the geomagnetic jerks occurred at the years 1902 (*Alexandrescu et al., 1995*); 1916 (*Qamili et al., 2013*); 1930 (*Alexandrescu et al., 1997*); 1944 (*Alexandrescu et al., 1996*); 1958 (*Mandea et al., 2000; De Michelis et al., 2005*); 1972 (*Qamili et al., 2013; Chambodut and Mandea, 2005*); 1986 (*Mandea et al., 2000; De Michelis et al., 2005*); 2000 (*Mandea et al., 2000*); 2014 (*Torta et al., 2015; Kotzé, 2017*).

The geomagnetic jerks at the dates 1972 and 2014 coincide in the series of 14-yr periodic jerks derived by perturbation of 6-yr Alfvén wave. Then we answer to the question noted by *Duan and Huang (2020)* that:

"Interestingly, Fig. 4 shows that almost all the above jerk timings coincide with the extremes of 8.6-yr signal very well within ∼1 year (or less). There are nine jerk epochs leading the extremes of the 8.6-year signal < 1 year, except the 1972 jerk and 2014 (Torta et al.,2015; Kotzé, 2017). Here, the question that why these two jerks did not occur at the corresponding extremes of the 8.6-year signal are worthy to be discussed later."

We find here that the geomagnetic jerk at the year 2014 despite interpretations by *Aubert and Finlay (2019)* it is not a sudden buoyancy concentrated QG Alfvén wave which rise up along the 25 years but it is exact recurrence of the Kuznetsov-Ma breather which is done ever consequently and will be done also at the year 2028. What they have found it is false alarm of the numerical dynamo.

5. Conclusions

We find that the clockwise and counter clockwise torsional MC (& Rossby) waves are the source of 6 & 9 year geomagnetic secular variations. We find a correlation between the space path parameter ε in numerical simulation of the geodynamo and small parameter ε in reductive perturbation theory which yields to the derivative nonlinear Schrödinger (DNLS) equation in the core dynamo to describe long torsional Alfvén waves under the ambient magnetic field. We find that the oscillatory standing geostrophic MC (& Rossby) waves in the core are monitoring the geomagnetic jerks and relevant LODs via the perturbation theory. Modulational and decay instabilities of the torsional Alfvén waves does transfer small perturbations to the rogue waves and intense turbulences, reasonable for appearance of the geomagnetic jerks and relevant LODs. Then we have found a mechanism in which the tiny interplanetary electromagnetic effects can produce geomagnetic jerks and relevant LODs via the perturbation theory and nonlinearity. Also we find that on the modulational instability, the plane geostrophic MC Rossby wave produces internally the perturbation wave-packet impulses such as the Kuznetsov-Ma breather, revealing 14-yr periodic geomagnetic jerks. We find also the mechanism of the variable trend of the intradecadal geomagnetic secular variations via competition between the Lorentz and Coriolis forces.

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