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The quasigeoid modelling in New Zealand using the boundary element method

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Abstract: We compile a quasigeoid model at the study area of New Zealand using the boundary element method (BEM). The direct BEM formulation for the Laplace equation is applied to obtain a numerical solution to the linearized fixed gravimetric boundary-value problem in points at the Earth's surface. The numerical scheme uses the collocation method with linear basis functions. It involves a discretisation of the Earth's surface which is considered as a fixed boundary. The surface gravity disturbances represent the oblique derivative boundary condition. The geocentric positions of the collocation points are determined combining the digital elevation data and the a priori quasigeoid model (onshore) and the mean sea surface topography (offshore). In our numerical realization, we use the global elevation data from SRTM30PLUS_V5.0, the detailed DTM of New Zealand, the EGM2008 quasigeoid heights, and the mean sea surface topography from the DNSC08 marine database. The gravity disturbances are computed using two heterogeneous gravity data sets: the altimetry-derived gravity anomalies from the DNSC08 gravity database (offshore) and the observed ground gravity anomalies from the GNS Science gravity database (onshore). The transformation of gravity anomalies to gravity disturbances is realized using the quasigeoid heights calculated from the EGM2008 global geopotential model. The new experimental quasigeoid model NZQM2010 is compiled at the study area of New Zealand bounded by the parallels of 34 and 47.5 arc-deg southern latitude and the meridians of 166 and 179 arc-deg eastern longitude. The least-squares analysis is applied to combine the gravimetric solution with GPS-levelling data using a 7-parameter model. NZQM2010 is validated using GPS-levelling data and compared with the existing regional and global quasigeoid models NZGeoid2009 and EGM2008. The validation at GPS-levelling testing network in New Zealand shows a similar STD fit of all investigated quasigeoid models with the geometric height anomalies computed from GPS-levelling data between 7 cm (NZGeoid2009) and 8 cm (NZQM2010 and EGM2008). The inaccuracies of the compiled quasigeoid models in New Zealand are expected to be mainly due to the presence of large systematic errors and inconsistencies of levelling networks throughout the country. Another source of the inaccuracy is an insufficient coverage and a low accuracy of gravity data especially over large parts of the South Island.

Key words: boundary element method, fixed gravimetric boundary-value problem, gravity, numerical integration, quasigeoid

1. Introduction

The geodetic vertical reference system in New Zealand was realized by 13 major local vertical datums (LVDs) relative to the mean sea level (MSL) observed at 12 different tide-gauge stations (cf., *Amos and Featherstone, 2003; Amos and Featherstone, 2009*). The LVD Dunedin-Bluff 1960 was defined by fixing the heights of two levelling benchmarks from the LVDs Dunedin 1958 and BLUFF 1955 instead of using the tide-gauge station as the origin. Moreover, additional vertical datums were established for surveying purposes throughout the country. Since gravity was not measured along the precise levelling lines, the LVDs are defined in the system of the approximate normal-orthometric heights. The cumulative normal-orthometric correction to the levelled height differences was defined based on the GRS67 normal gravity formula and computed approximately using a truncated form of Rapp's equation (*Rapp, 1961*). *Amos and Featherstone (2009)* applied the iterative gravimetric approach to unify the LVDs in New Zealand using a regional gravimetric quasigeoid model and GPS-levelling data on each LVD. The principle of this method is based on an iterative quasigeoid modelling where the LVD offsets computed from an earlier model are used to apply additional gravity reductions from each LVD to that model. The result of this procedure was the first detailed regional gravimetric quasigeoid model of New Zealand NZGeoid05. NZGeoid05 was computed jointly by the Land Information New Zealand (LINZ) and the Western Australian Centre for Geodesy - Curtin University of Technology (*Amos and Featherstone, 2009*). NZGeoid05 was calculated from different heterogeneous ground, seaborne and altimetry-derived gravity data using the deterministic modification of the Stokes kernel. NZGeoid05 was complied on a 2×2 arc-min geographical grid over New Zealand and its continental shelf (area bounded by the parallels of 25 and 60 arc-deg southern spherical latitude and the meridians of 160 and 190 arc-deg western spherical longitude). The estimated

LVD offsets relative to the regional quasigeoid model NZGeoid05 are from 26 cm (One Tree Point 1964, Nelson 1955, and Dunedin-Bluff 1960 LVDs) up to 59 cm (Gisborne 1926 LVD). The New Zealand quasigeoid model NZGeoid2009 is the currently adopted official height reference surface for New Zealand. NZGeoid2009 was computed using a similar approach as NZGeoid05 (*Claessens et al., 2009*). The main difference in computing NZGeoid05 and NZGeoid2009 is the use of different global geopotential models (GGMs); NZGeoid05 was computed using EGM96 (*Lemoine et al., 1998*), while EGM2008 (*Pavlis et al., 2008*) was used for the computation of NZGeoid2009. NZGeoid2009 model is provided to users on a 1×1 arc-min geographical grid over the same area as NZGeoid05. GPS-levelling data were used to determine the LVD offsets in New Zealand relative to NZGeoid2009. The estimated LVD offsets relative to NZGeoid09 are within 6 cm (One Tree Point 1964 LVD) and 49 cm (Dunedin 1958 LVD).

With the current development of high-performance computing facilities, numerical methods such us the boundary element method (BEM), finite element method (FEM), and finite volume method (FVM) are used more often in precise global and regional gravity field modelling. The first applications of FEM to the gravity field modelling was given by *Meissl (1981)* and *Shaofeng and Dingbo (1991)*. Recently, FEM and FVM applied in physical geodesy have been discussed in *Fašková (2008)* and *Fašková et al. (2009)*. The first application of BEM in physical geodesy was given by *Klees (1992)*. This approach, based on the indirect BEM formulation and the Galerkin BEM, was further developed by *Lehmann and Klees (1996)*, *Lehmann (1997)*, *Klees (1998)* and *Klees et al. (2001)*. *Čunderlík et al. (2008)* formulated the direct BEM for the fixed gravimetric boundary-value problem based on the collocation with linear basis functions. This approach was later completed by developing an iterative procedure for the elimination of far-zone interactions in *Čunderlík and Mikula (2009)*.

In this study, we apply the BEM approach developed by *Čunderlík et al. (2008)* and *Čunderlík and Mikula (2009)* to determine the gravimetric quasigeoid model at the study area of New Zealand. The mathematical formulation of the direct BEM approach is briefly reviewed in Section 2. Results of the numerical realization are provided in Section 3. The combination of the gravimetric solution with GPS-levelling data is done in Section 4. The new experimental quasigeoid model is validated using GPS-levelling data and compared with the regional and global quasigeoid models NZGeoid2009 and EGM2008 in Section 5. The summary and conclusions are given in Section 6.

2. Direct BEM for the linearized fixed gravimetric boundary-value problem

The linearized fixed gravimetric boundary-value problem represents an exterior oblique derivative problem for the Laplace equation. It is defined as (cf. *Koch and Pope, 1972; Bjernhammar and Svensson, 1983; Grafarend, 1989*)

$$\nabla^2 T(\mathbf{x}) = 0 \qquad \mathbf{x} \in \Re^3 - \Omega, \tag{1}$$

$$\langle \nabla T(\mathbf{x}),\, \mathbf{s}(\mathbf{x}) \rangle = -\delta g(\mathbf{x}) \qquad \mathbf{x} \in \Gamma, \tag{2}$$

$$T(\mathbf{x}) = O(|\mathbf{x}|)^{-1} \qquad \mathbf{x} \to \infty, \tag{3}$$

where T is the disturbing potential (i.e., difference between the actual gravity potential W and the normal gravity potential U) at any point $\mathbf{x}$, and δg is the gravity disturbance. The domain Ω represents the body of the Earth with its boundary Γ given by the Earth's surface. $\langle \nabla T,\, \mathbf{s} \rangle$ is the inner product of two vectors ∇T and $\mathbf{s}$, where the unit vector $\mathbf{s}$ is defined as follows

$$\mathbf{s}(\mathbf{x}) = -\frac{\nabla U(\mathbf{x})}{|\nabla U(\mathbf{x})|} \qquad \mathbf{x} \in \Gamma. \tag{4}$$

Equation (2) represents the oblique derivative boundary condition as the normal to the Earth's surface Γ does not coincide with the vector $\mathbf{s}$ defined in Eq. (4). The direct BEM formulation for the Laplace equation leads to a boundary integral equation (BIE) that can be derived using the Green's third identity or through the method of weighted residuals (cf. *Brebbia et al., 1984; Schatz et al., 1990*). A main advantage arises from the fact that only the boundary of the solution domain requires a subdivision into its elements. Thus, the dimension of the problem is effectively reduced by one. The application of the direct BEM to the linearized fixed gravimetric boundary-value problem (Eqs. 1–3) yields BIE in the following form (cf. *Čunderlík et al., 2008*)

$$\frac{1}{2}T(\mathbf{x})+\int_{\Gamma}T(\mathbf{y})\,\frac{\partial G}{\partial\,\mathbf{n_\Gamma}}(\mathbf{x},\mathbf{y})\,\mathrm{d}\mathbf{y}=\int_{\Gamma}\frac{\partial\,T}{\partial\,\mathbf{n_\Gamma}}(\mathbf{y})\;G(\mathbf{x},\mathbf{y})\,\mathrm{d}\mathbf{y}\qquad \mathbf{x},\mathbf{y}\in\Gamma, \quad (5)$$

where $\mathbf{x}$ and $\mathbf{y}$ are the geocentric position vectors of the computation and moving (integration) points, respectively, $\mathbf{n_\Gamma}$ is the normal to the boundary Γ, and the kernel function G represents the fundamental solution to the Laplace equation

$$G(\mathbf{x},\mathbf{y})=\frac{1}{4\pi\,|\,\mathbf{x}-\mathbf{y}\,|}\qquad \mathbf{x},\mathbf{y}\in\Re^3. \quad (6)$$

In order to handle the oblique derivative problem we use the same simplification as proposed by *Čunderlík et al. (2008).* According to the oblique derivative boundary condition in Eq. (2), the negative value of the gravity disturbance δg is defined as a projection of the vector $\nabla T(\mathbf{x})$ onto the direction of $\mathbf{s}(\mathbf{x})$. The normal derivative term $\partial T/\partial\,\mathbf{n_\Gamma}$ on the right-hand side of BIE in Eq. (5) approximately equals $\partial T/\partial\,\mathbf{n_\Gamma}\cong-\delta g(\mathbf{x})\cos\mu(\mathbf{x})$, where $\mu(\mathbf{x})$ is the angle $\angle(\,\mathbf{n_\Gamma}(\mathbf{x}),\mathbf{s}(\mathbf{x}))$. This term represents the projection of the vector $\delta g(\mathbf{x})\;\mathbf{s}(\mathbf{x})$ onto the normal $\mathbf{n_\Gamma}(\mathbf{x})$. In this way the oblique derivative boundary condition in Eq. (2) is incorporated to the direct BEM formulation in Eq. (5).

The boundary integral equation in Eq. (5) is discretised using the collocation method. It involves a discretisation of the Earth's surface by a triangulation of the topography and approximations of the boundary functions by linear functions on each triangular panel using linear basis functions $\{\,\psi_j:\; j=1,\,2,\,...,\,N\,\}$. This is realized by the piece-wise linear polynomials defined on the planar triangular panels, where vertices of this triangulation represent the collocation points. BIE in Eq. (5) is then rewritten to the following discrete form (*Čunderlík et al., 2008*)

$$c_i\,T_i\,\psi_i+\sum_{j=1}^{N}\int_{\mathrm{supp}\psi_\mathrm{j}}T_j\frac{\partial G_{\mathrm{i,j}}}{\partial\,\mathbf{n_\Gamma}}\;\psi_j\;\mathrm{d}\Gamma_j=$$
$$=\sum_{j=1}^{N}\int_{\mathrm{supp}\psi_\mathrm{j}}\delta g_j G_{\mathrm{i,j}}\,\psi_j\;\mathrm{d}\Gamma_j\quad(i=1,\,2,\,...,\,N)\,, \quad (7)$$

where c_i represents the spatial segment bounded by the panels joined at the i-th collocation point, and N is the total number of nodes. The discretised boundary integral equations in Eq. (7) form the linear system of observation equations

$$\mathbf{M}\,\mathbf{t} = \mathbf{L}\,\delta\mathbf{g}, \tag{8}$$

where $\mathbf{t}$ is the vector of unknown values of the disturbing potential T at the collocation points, and $\delta\mathbf{g}$ is the vector of observed gravity disturbances δg. The elements of matrices $\mathbf{M}$ and $\mathbf{L}$ represent the integrals of the discrete form of BIEs in Eq. (7). The discretisation of the integral operators is affected by the weak singularity of the kernel functions. The integrals with regular integrands which are approximated by the Gaussian quadrature and non-regular integrands (singular elements) require a special treatment, for more details see *Čunderlík et al. (2008)*. In case of the oblique derivative boundary condition given by Eq. (2), or the Neumann boundary condition based on using the aforementioned projection, the matrix $\mathbf{M}$ represents a system matrix, while the known vector $\mathbf{f} = \mathbf{L}\,\delta\mathbf{g}$ is given on the right-hand side of Eq. (8).

3. Numerical study

The BEM approach requires integration over the whole globe allowing local refinements (cf. *Čunderlík et al., 2008*). In our numerical study the global rough triangulation over the whole Earth's surface with the resolution of about 0.2 arc-deg (1,215,002 collocation points) was successively refined until the detailed resolution of 1.5 arc-min over New Zealand and surrounding offshore areas was achieved (see Fig. 1). The total number of all collocation points reached 1,374,658.

The geocentric positions of onshore collocation points were determined from the topographical heights of detailed local and global elevation models and from the quasigeoid heights evaluated using GGM. In our numerical study we used the 30×30 arc-sec global elevation data of SRTM30PLUS_V5.0 (*Becker et al., 2009*) and the 1 × 1 arc-sec detailed DTM of New Zealand. The quasigeoid heights at the collocation points were evaluated using the EGM2008 coefficients complete to degree and order 2160 of spherical harmonics. The geocentric positions of offshore collocation points were determined using the DNSC08 mean sea surface model (*Andersen and Knudsen, 2008*). The ellipsoidal heights of the collocation points at the study

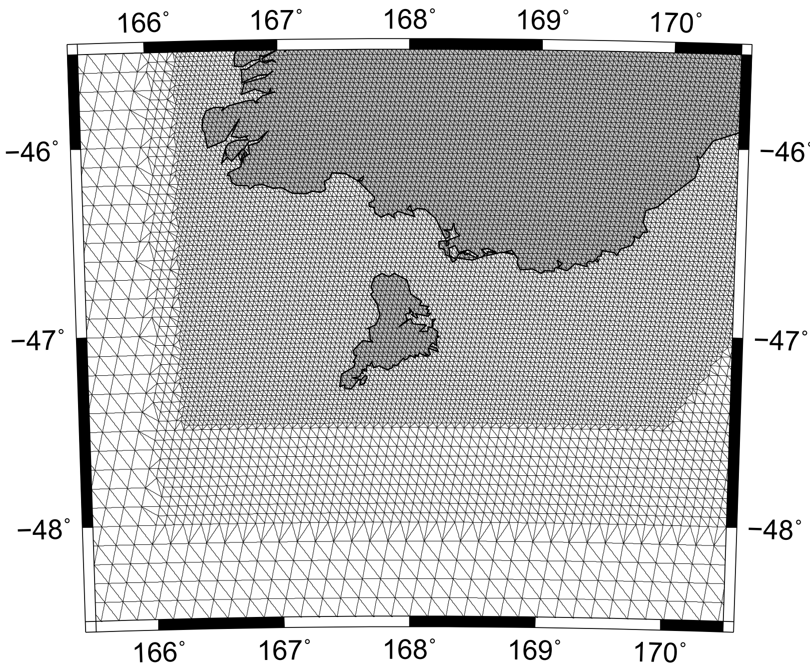

Fig. 1. The local refinement of the triangulation – an example at the southern part of New Zealand.

area of New Zealand are shown in Fig. 2. They vary from –6.1 to 2,774.4 m with a mean of 156.0 m, and a standard deviation is 317.1 m.

The gravity disturbances at the triangulation grid of collocation points were used to determine the detailed gravimetric quasigeoid model. The gravity disturbances δg at the collocation points were computed from the corresponding grid of gravity anomalies Δg using the following well-known expression (see e.g., *Heiskanen and Moritz, 1967*)

$$\delta g = \Delta g - \frac{\partial \gamma}{\partial h} \varsigma, \tag{9}$$

where $\partial \gamma / \partial h$ is the linear normal gravity gradient computed using the parameters of the GRS80 reference ellipsoid. The height anomalies ς in Eq. (9) were calculated using the EGM2008 coefficients complete to degree and order 2160. The gravity anomalies were compiled from the GNS Science gravity data (onshore) and extracted from the DNSC08 marine gravity

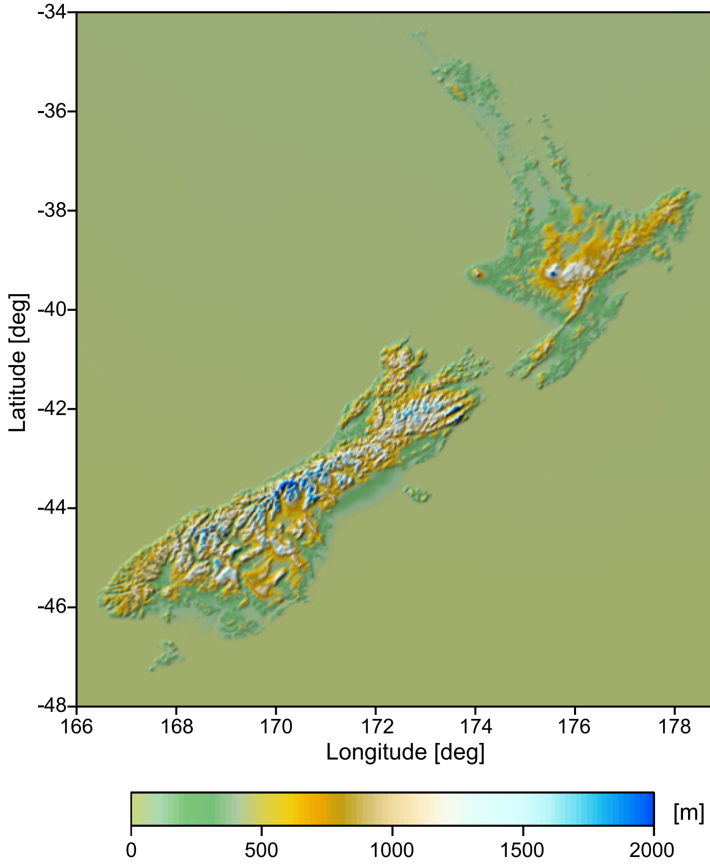

Fig. 2. The ellipsoidal heights of the collocation points at the study area of New Zealand.

database (offshore) provided by the Danish National Space Centre (*Andersen et al., 2009*). The map of the gravity disturbances at the study area of New Zealand bounded by the parallels of 34 and 47.5 arc-deg southern latitude and the meridians of 166 and 179 arc-deg eastern longitude is shown in Fig. 3. The gravity disturbances at the Earth's surface within the study area vary from 204.9 to 307.5 mGal with a mean of 22.2 mGal, and a standard deviation is 48.7 mGal.

The final large-scale parallel computations were performed on the cluster with 16 processors and 128 GB of distributed internal memory using

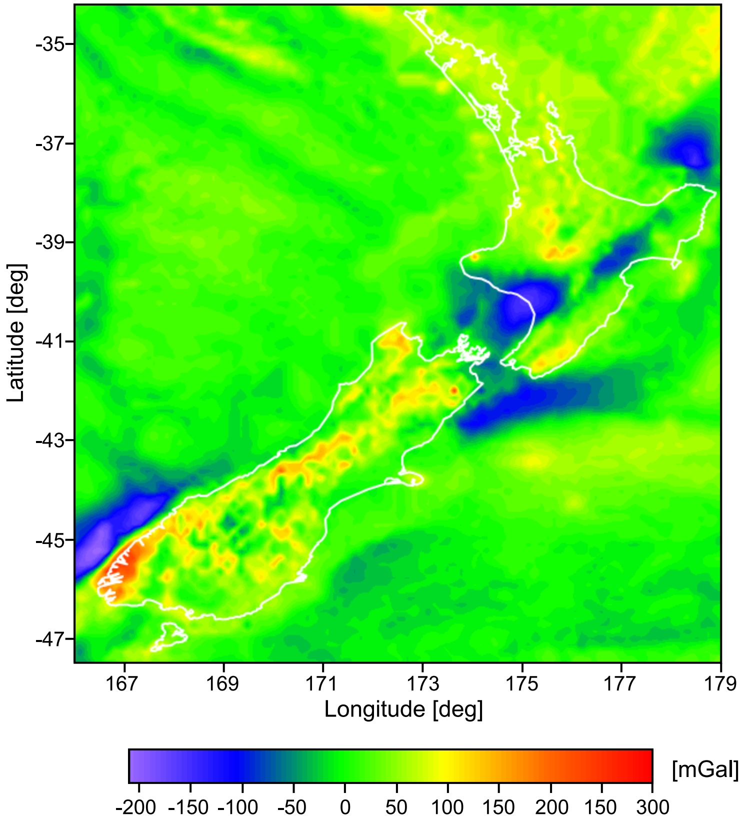

Fig. 3. The gravity disturbances at the study area of New Zealand.

the standard MPI (Message Passing Interface) subroutines (*Aoyama and Nakano, 1999*). In order to reduce the large memory requirements we eliminated the far-zones interactions using the ITG-GRACE03S satellite geopotential model (*Mayer-Gürr, 2007*). The arisen long-wavelength error surface was reduced after using 4 iterations. For more details of this iterative procedure we refer readers to *Čunderlík and Mikula (2009)*.

The new experimental gravimetric quasigeoid model NZQM2010 is shown in Fig. 4. The computed values of the quasigeoid heights vary from –5.91 to 41.85 m with a mean of 18.29 m, and a standard deviation of 11.68 m. Within onshore New Zealand, the quasigeoid heights are everywhere pos-

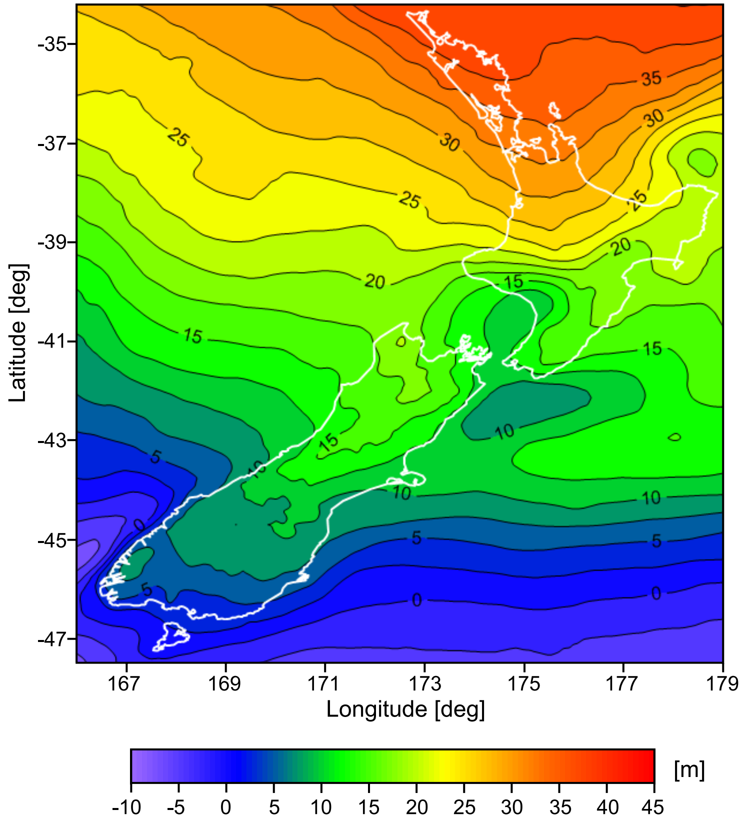

Fig. 4. The new experimental gravimetric quasigeoid model NZQM2010 compiled at the study area of New Zealand.

itive with the minima located at the Steward Island and along the south coast of the South Island and the maxima at the upper part of the North Island.

4. Combination of gravimetric solution with GPS-levelling data

The GPS-levelling testing network in New Zealand consists of 2320 points from the LINZ geodetic database. The ellipsoidal heights above the GRS80 geocentric reference ellipsoid are defined in the New Zealand Geodetic Datum 2000 (NZGD2000). The NZGD2000 is aligned to the International Terrestrial Reference Frame 1996 (ITRF1996) at the reference epoch of January 1st, 2000 (*Blick et al., 2005*). Since the normal-orthometric heights at the points of GPS-levelling testing network in New Zealand are aligned to 18 different LVDs, we utilized the geopotential value approach (cf. *Burša et al., 1999, 2001, 2002*) to estimate the average offsets of LVDs relative to the World Height System (WHS). WHS is defined by the adopted value of the geoidal geopotential $W_0 = 62636856$ m^2s^{-2}. The estimated average offsets of 18 LVDs in New Zealand relative to WHS are summarized in Table 1. The LVD offsets within the South and North Islands of New Zealand are positive and range from 1 cm (Wellington 1953 LVD) to 37 cm (One Tree Point 1964 LVD).

Table 1. The offsets of 18 LVDs in New Zealand relative to WHS

LVD	**LVD offset** [m]
One Tree Point 1964	0.37
Auckland 1946	0.12
Moturiki 1953	0.19
Gisborne 1926	0.10
Napier 1962	0.24
Taranaki 1970	0.12
Wellington 1953	0.01
Nelson 1955	0.20
Lyttelton 1937	0.13
Dunedin 1958	0.07
Dunedin-Bluff 1960	0.23
Bluff 1955	0.17
MSL	0.15
Deep Cove 1960	0.30
Port 1954	0.32
Tarakohe 1982	0.23
Tararu	0.21
Unahi	0.19

The new gravimetric quasigeoid solution was further combined with GPS-levelling data corrected for the average LVD offsets in order to reduce additional systematic distortions between the geometric and gravimetric quasigeoid heights. The systematic distortions were modelled by a 7-parameter model (see *Kotsakis and Sideris, 1999*) formed for the observation equations of differences between the geometric and gravimetric quasigeoid heights at

GPS-levelling points and solved applying the least-squares analysis.

5. Validation of NZQM2010

NZQM2010 was validated at the GPS-levelling testing network in New Zealand. The geometric quasigeoid heights were calculated from the NZGD2000 ellipsoidal heights by subtracting the normal-orthometric heights corrected for the average LVD offsets relative to WHS (see Table 1). The same validation is done for the quasigeoid models NZGeoid2009 and EGM2008. The average LVD offsets relative to WHS (see Table 1) were applied for a validation of EGM2008. The average offsets of 12 major LVDs relative to NZGeoid2009 (adopted from *Claessens et al., 2009*) were applied to the geometric quasigeoid heights for a validation of NZGeoid2009. Statistics of the differences between the geometric and gravimetric quasigeoid heights at the GPS-levelling testing network are given in Table 2. The differences between normal and normal-orthometric heights were not taken into consideration.

Table 2. Statistics of the differences between the geometric and gravimetric quasigeoid heights calculated for NZGeoid2009, NZQM2010 and EGM2008 at the GPS-levelling testing network in New Zealand

Quasigeoid Model	Differences at GPS-levelling points			
	Min [m]	**Max** [m]	**Mean** [m]	**STD** [m]
NZGeoid2009	-0.42	0.38	-0.01	0.07
EGM2008	-0.42	0.40	0.00	0.08
NZQM2010	-0.52	0.44	0.01	0.08

As seen in Table 2 the STD fit of NZGeoid2009 with GPS-levelling data is 7 cm. The STD fit of the quasigeoid models NZQM2010 and EGM2008 with GPS-levelling data is 8 cm.

6. Summary and conclusions

We have applied the direct BEM approach to determine the new experimental quasigeoid model NZQM2010 at the study area of New Zealand.

NZQM2010 was validated at the GPS-levelling testing network and compared with the available regional and global quasigeoid models NZGeoid2009 and EGM2008.

Since the normal-orthometric heights at GPS-levelling testing network in New Zealand are defined in 18 different LVDs, the average offsets of LVDs relative to WHS were estimated and applied to the geometric quasigeoid heights. WHS is defined by the adopted geoidal geopotential value $W_0 = 62636856\ m^2 s^{-2}$. The new gravimetric quasigeoid solution was further combined with GPS-levelling data (corrected for the average LVD offsets) in order to reduce additional systematic distortions between the geometric and gravimetric quasigeoid heights using a 7-parameter model.

The validation of the quasigeoid models NZQM2010, NZGeoid2009 and EGM2008 at 2320 points of the GPS-levelling testing network in New Zealand revealed a similar STD fit with GPS-levelling data of 7 cm for NZGeoid2009 and 8 cm for NZQM2010 and EGM2008. The largest systematic differences between NZGeoid2009, NZQM2010 and EGM2008 up to several decimeters are along the Southern Alps. These large differences are most likely due to an insufficient coverage and a low accuracy of gravity data (especially over large parts of the South Island) and due to large systematic errors and inconsistencies of levelling networks throughout the country.

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