Practical comparison of formulae for computing normal gravity at the observation point with emphasis on the territory of Slovakia

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A b s t r a c t: Modern theory in gravimetry requires the computation of normal gravity at the point of observation of actual gravity. Closed formulae for normal gravity above the reference ellipsoid, making use of Jacobi-ellipsoidal and geodetic coordinates are reviewed. Various systematic errors in computing the normal gravity are assessed at the topographical surface on the territory of Slovakia. First the systematic error committed by using only the free-air gradient term as the height-term in computing normal gravity is calculated and displayed. Second the systematic error caused by using local geographical latitude not referred to the mean earth ellipsoid, but to a local (non-geocentric) reference ellipsoid, is evaluated for two reference ellipsoids commonly used in Slovakia – the Bessel and the Krassovsky ellipsoids. Third the systematic error in computing normal gravity introduced by failing to use the geodetic heights, and using the "sea-level" heights instead, is assessed. This systematic error is also known among geophysicists as the "free-air geophysical indirect effect".

Key words: international gravity formula, reference ellipsoid, equipotential ellipsoid, Somigliana-Pizzetti gravity field, geophysical indirect effect

1. Introduction

Earth's gravity measured at the topographical surface varies with latitude as much as 5 Gal, due to the oblateness and spin of the Earth, and varies with height as much as 2.7 Gal. The purpose of the *reference gravity field* is to mathematically describe the bulk of the earth's gravity field, in order to work with (or to study) only the anomalous field. The reference field can be chosen according to a variety of schemes, such as taking a

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truncated series of the spherical (or ellipsoidal) harmonic expansion of the actual gravity field (e.g., *Heiskanen and Moritz, 1967*, sec. 2-12). If the reference field is selected as that of a geocentric equipotential biaxial ellipsoid (e.g., *Heiskanen and Moritz, 1967, sec. 2-21; Vaníček and Krakiwsky, 1986, p. 477*), it becomes the *normal gravity field* of the *mean earth ellipsoid* (cf. *Heiskanen and Moritz, 1967, secs. 2-7 through 2-10, 2-21, 5-11; Vaníček and Krakiwsky, 1986, ch. 20.3*). The use of the mean earth ellipsoid in geophysics is vitally important, as it assures the compatibility between geometric and physical quantities. In the case of the mean earth ellipsoid, the surface of this reference ellipsoid becomes not only the reference coordinate surface at/from which geographical coordinates are reckoned, but also the equipotential surface of the normal gravity field, on which potential has the same value as on the geoid. Currently the GRS'80 reference ellipsoid is adopted as the mean earth ellipsoid.

One of the first attempts to define a normal gravity field that would absorb the effect of the earth's oblateness and spin, and the effect of height, was made by *Bowie and Avers (1914)*. Various other formulae were advocated. In order to unify the definition of normal gravity the International Association of Geodesy (IAG) adopted in 1930 a formula for normal gravity (*Cassinis, 1930*), which became known as the *International Gravity Formula 1930* (IGF'30). In 1967 the IAG approved a new formula, accurate to 4 μGal, called the IGF'67 (*IAG, 1971*). This formula was updated in 1980 to yield the IGF'80 (*IAG, 1980*), which is accurate to 0.7 μGal. The IGF typically gives the value of normal gravity on the ellipsoid.

The formulation of gravity anomalies/disturbances defined at the observation point calls for evaluating normal gravity above the reference ellipsoid. We shall review the formulae for the computation of normal gravity in space, and shall compare various approximate formulae to the rigorous formula with the objective of practical applicability to the territory of Slovakia, whereby we shall consider mainly terrestrial gravity measurements.

2. Normal gravity of mean earth ellipsoid – rigorous formula in (Jacobi-) ellipsoidal coordinates

The most convenient coordinates for deriving the formulae for the potential of the normal gravity field, hence for the *normal gravity vector* and *normal gravity* (as its modulus), are the ellipsoidal ("Jacobi-ellipsoidal") coordinates (u, β, λ), also called "spheroidal", "EL", or "one-parametric ellipsoidal" coordinates, (e.g., *Heiskanen and Moritz, 1967, sec. 1-19; Vaníček and Krakiwsky, 1986, pp. 464–466)*, where u is the semi-minor axis of the ellipsoidal coordinate surface, β is the so called "reduced" latitude, and λ is longitude. The normal gravity field of the (biaxial) mean earth ellipsoid is uniquely described by four parameters – the major and minor semi-axes a and b, mass M identical with the earth's mass, and spin angular velocity ω identical with that of the earth. Normal gravity, expressed in Jacobi-ellipsoidal coordinates is given as $\gamma(u, \beta) = \sqrt{\gamma_u^2(u, \beta) + \gamma_\beta^2(u, \beta)}$, while $\gamma_\lambda = 0$, since the reference field is biaxial. The normal gravity vector components γ_u and γ_β are given by e.g. *Heiskanen and Moritz (1967, Eqs 2–66)*. With an accuracy better than 80 nanoGal anywhere at altitudes up to 9 km above the reference ellipsoid the normal gravity is given (neglecting the contribution of the γ_β component to the modulus of the normal gravity vector) as

$$\gamma(u, \beta) = |\gamma_u(u, \beta)| = \sqrt{\frac{u^2 + E^2}{u^2 + E^2 \sin^2 \beta}} \left[\frac{GM}{u^2 + E^2} + \right. \\ \left. + \frac{a^2 \omega^2 E}{u^2 + E^2} \frac{q'}{q_0} \left(\frac{1}{2} \sin^2 \beta - \frac{1}{6}\right) - \omega^2 u \cos^2 \beta \right], \tag{1}$$

where G is the gravitational constant, E is linear eccentricity (focal length) defined as $E = \sqrt{a^2 - b^2}$, and where

$$\begin{aligned} q' &= 3 \left(1 + \frac{u^2}{E^2}\right) \left(1 - \frac{u}{E} \arctan \frac{E}{u}\right) - 1 \,, \\ q_0 &= \frac{1}{2} \left[\left(1 + 3 \frac{b^2}{E^2}\right) \arctan \frac{E}{b} - 3 \frac{b}{E}\right] . \end{aligned} \tag{2}$$

At the reference ellipsoid $(u = b)$ Eq. (1) is rigorous, as $\gamma_\beta(u = b, \beta) = 0$.

However, the observation (evaluation) point is typically referred in terms of geographical coordinates, namely geodetic ("Gauss-ellipsoidal") coordinates (h, ϕ, λ) (e.g., *Heiskanen and Moritz, 1967, sec. 5-3; Vaníček and Krakiwsky, 1986, p. 325)*, where h is geodetic (now by ISO standards: "ellipsoidal") height and ϕ is geodetic (now by ISO standards: "ellipsoidal") latitude. Longitude is the same in both the Jacobi-ellipsoidal and Gauss-ellipsoidal coordinate systems, identical with the spherical longitude, as the reference ellipsoid is biaxial. More specifically, we are talking about the geocentric geodetic coordinates here. The issue of non-geocentric geodetic coordinates used to position gravity stations will be addressed later. Hence, to evaluate the normal gravity at the given point using Eq. (1), its position referred to in geographical (geodetic) coordinates must be transformed into Jacobi-ellipsoidal coordinates. The transformation between these two types of coordinates is carried out by means of the geocentric Cartesian coordinates

$$\begin{bmatrix} \sqrt{u^2+E^2}\cos\beta\cos\lambda \\ \sqrt{u^2+E^2}\cos\beta\sin\lambda \\ u\ \sin\beta \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} (\eta(\phi)+h)\cos\phi\cos\lambda \\ (\eta(\phi)+h)\cos\phi\sin\lambda \\ (\eta(\phi)(1-e^2)+h)\sin\phi \end{bmatrix}, \tag{3}$$

where

$$\eta\,(\phi) = \frac{a}{\sqrt{1-e^2\sin^2\phi}} \tag{4}$$

is the *prime vertical radius of curvature* of the mean earth ellipsoid, while e is the (first numerical) eccentricity, defined as $e^2 = [a^2 - b^2]/a^2$ or $e = E/a$. The closed form formulae for the above transformation of coordinates can be found in e.g. *Ardalan and Grafarend (2001)*, Eqs (5) through (7).

3. Normal gravity of mean earth ellipsoid – formulae in geodetic coordinates

Alternatively, to avoid the above transformation of coordinates, it is customary to evaluate the normal gravity at the observation (evaluation) point as *normal gravity on the ellipsoid*, γ_0, plus a *height term*, denoted here as $\delta\gamma_h$

$$\gamma(h,\phi) = \gamma_0(\phi) + \delta\gamma_h(h,\phi)\,, \tag{5}$$

as will be described below.

3.1. Normal gravity on reference ellipsoid – rigorous formula in geodetic coordinates

At the surface of the reference ellipsoid ($u = b$) equation (1) can be written in the form known as the *Somigliana formula* (e.g., *Somigliana, 1929; Heiskanen and Moritz, 1967, Eq. 2-76; Vaníček and Krakiwsky, 1986, Eq. (20.82))* using polar and equatorial normal gravity. At the surface of the reference ellipsoid ($u = b$, $h = 0$) we get from Eq. (3) that $a \tan\beta = b \tan\phi$. Then the Somigliana formula can be written in geodetic coordinates as

$$\gamma_0(\phi) = \gamma_e \frac{1 + k \sin^2\phi}{\sqrt{1 - e^2 \sin^2\phi}}, \quad k = \frac{b\gamma_P}{a\gamma_e} - 1\,, \quad e^2 = \frac{a^2 - b^2}{a^2}\,. \tag{6}$$

This formula is rigorous. Here γ_P and γ_e are the normal gravity at the pole and at the equator, respectively. The polar and equatorial normal gravity is expressed in terms of the four defining parameters of the mean earth ellipsoid (such as GRS'80). Instead of giving the formulae, we list their values in Tab. 1 together with the parameters of the GRS'80 reference ellipsoid. Note, that instead of earth's mass M the GM parameter called "geocentric gravitational constant" is used. The Somigliana formula represents the International Gravity Formula in a closed form.

Tab. 1. The IAG reference values of the parameters of the GRS'80 reference ellipsoid (*Moritz, 1980*)

parameter	meaning	GRS'80 value
GM	geocentric gravitational constant	$3\,986\,005 \times 10^8$ m^3 s^{-2}
ω	earth's spin angular velocity	$7\,292\,115 \times 10^{-11}$ rad/s
a	semi-major axis	6 378 137 m
b	semi-minor axis	6 356 752.3141 m
f	geometrical flattening	$f = (a - b)/a$
m	geodetic parameter	$m = (\omega^2 a^2 b)/(GM)$
γ_e	equatorial normal gravity	9.780 326 7715 ms^{-2}
γ_P	polar normal gravity	9.832 186 3685 ms^{-2}

3.2. Normal gravity above reference ellipsoid – height term in geodetic coordinates

The normal gravity can be expanded into a Taylor series in terms of geodetic height. Thus for the height term of Eq. (5) we get

$$\delta\gamma_h(h,\phi) = \frac{\partial\gamma(h=0,\phi)}{\partial h}h + \frac{1}{2}\frac{\partial^2\gamma(h=0,\phi)}{\partial h^2}h^2 + \cdots \tag{7}$$

Depending on how many terms are taken into account, and how are the vertical derivatives of normal gravity evaluated, we may find a variety of formulae for $\delta\gamma_h$. The simplest and less accurate would be the "free-air" height term

$$\delta\gamma_h(h,\phi) = -0.3086\ [\mathrm{mGal/m}]\ h\,. \tag{8}$$

Higher order series expansion formulae can be found in *Hirvonen (1960)*. Here we list the formula given by *Heiskanen and Moritz (1967*, Eq. 2-123), which is widely used in geophysical practice

$$\delta\gamma_h(h,\phi) = -\frac{2\gamma_0(\phi)}{a}\left(1+f+m-2f\sin^2\phi\right)h + \frac{3\gamma_0(\phi)}{a^2}h^2, \tag{9}$$

where $\gamma_0(\phi)$ is given by Eq. (6), and the geometrical flattening f as well as the geodetic parameter m are given in Tab. 1. Expressing the normal gravity on the ellipsoid using Eq. (6) and neglecting the higher order terms (of f^2, fm, m^2, and higher), an approximate formula is obtained (ibid, Eq. 2-124)

$$\delta\gamma_h(h,\phi) = -\frac{2\gamma_e}{a}\left[1+f+m+\left(-3f+\frac{5}{2}m\right)\sin^2\phi\right]h + \frac{3\gamma_e}{a^2}h^2. \tag{10}$$

Equations (5), (6), and (10) give a prescription for computing the normal gravity above the reference ellipsoid in geodetic coordinates. Here we shall call this prescription "SHM formula" (after "Somigliana plus Heiskanen and Moritz") to be able to refer to it. The "SHM" formula systematically deviates from the rigorous formula (Eq. (1)) as much as tens of μGal at altitudes up to 9 km above the reference ellipsoid, cf. Fig. 1.

Within the accuracy threshold of 80 μGal globally, or 30 μGal on the territory of Slovakia, the "SHM" formula can be considered "exact". If a better accuracy is desired, one must use the rigorous formula, as discussed in sec. 2. The rigorous formula expressed in geodetic coordinates in a laborious closed form can be found in *Ardalan and Grafarend (2001), Eq. (74)*.

In geophysical practice the "free-air" height term given by Eq. (8) is often used. By comparing it to the height term of Eq. (10) we portray the systematic error ε_γ^h thus committed in the computation of normal gravity,

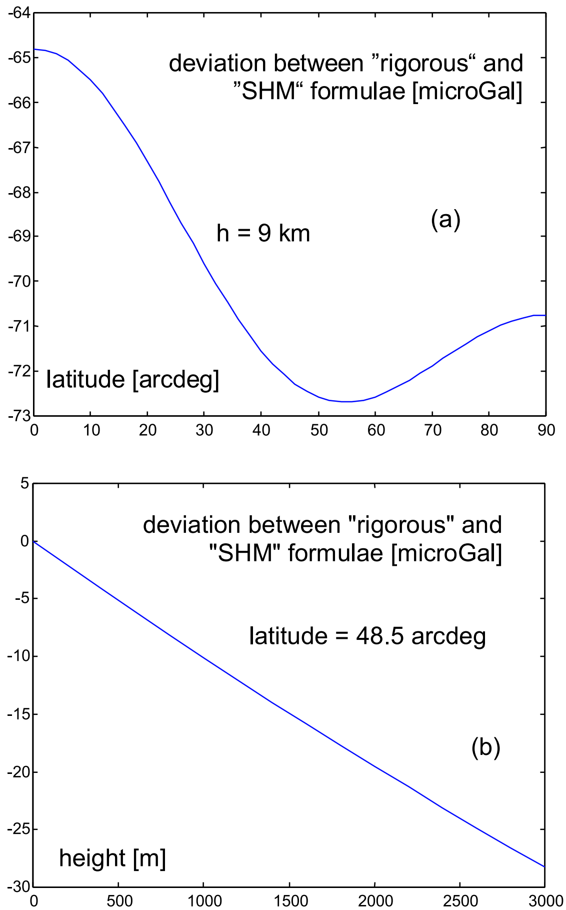

Fig. 1. The systematic deviation of normal gravity given by Eqs (5), (6), and (10) from its rigorous value. Plot (a) shows the deviation as a function of latitude at an altitude of 9 km, while plot (b) shows the deviation as a function of height at the latitude typical for Slovakia.

see Figs 2 and 3. Figure 2 shows such systematic error as a function of geodetic height for several latitudes (one of them being typical for Slovakia), while figure 3 shows this systematic error evaluated at the topo-surface on the territory of Slovakia for heights on a 0.025 × 0.025 [arcdeg] grid. This systematic error is correlated with the topo-surface, and attains values of the order of 0.1 mGal on the territory of Slovakia (0.8 mGal at the top of Mt. Gerlach).

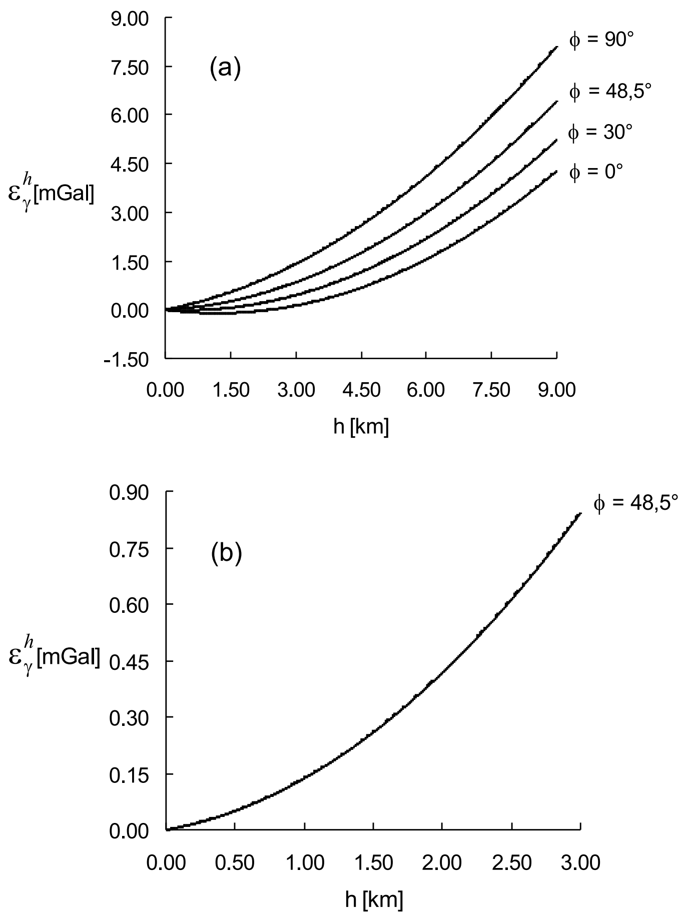

Fig. 2. Systematic error ε_γ^h [mGal] stemming from oversimplified height term in the computation of normal gravity, displayed as a function of height for several latitudes (a), including that typical for Slovakia (b).

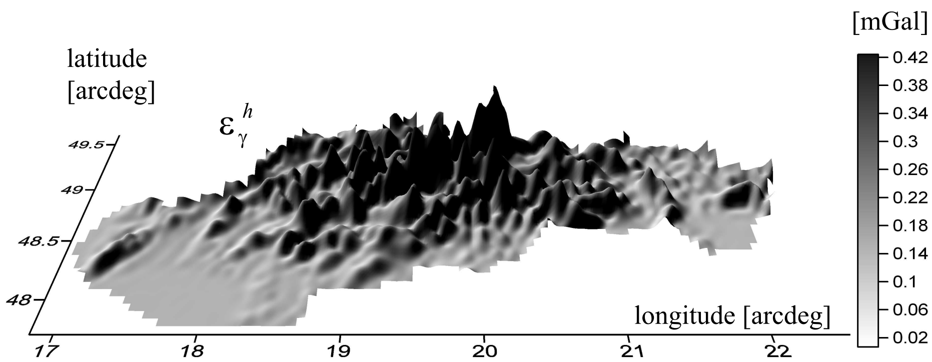

Fig. 3. Systematic error ε_γ^h [mGal] stemming from oversimplified height term in the computation of normal gravity, evaluated at the topo-surface on the territory of Slovakia.

At the accuracy threshold of 1 mGal, normal gravity may be computed on the territory of Slovakia using the "free-air" height term (Eq. (8)) in the "SHM" formula instead of the "HM" height term (Eq. (10)).

4. Effect of geodetic coordinates, that are not referred to mean earth ellipsoid, on normal gravity

The formula for normal gravity requires geodetic coordinates that are referred to the mean earth ellipsoid, such as GRS'80. However, often geographical (geodetic) coordinates are referred to local ("relative") reference ellipsoids. In such cases the exact evaluation of normal gravity requires the transformation of geographical coordinates into those that refer to the mean earth ellipsoid. In Slovakia practically two local ellipsoids are in use: the Bessel ellipsoid to which the S-JTSK coordinates are referred, and the Krassovsky ellipsoid, to which the S-42/83 coordinates are referred.

The transformation, known as Helmert's, is accomplished in three steps (e.g. *Vaníček and Krakiwsky, 1986, ch. 15.4; Featherstone and Dentith, 1998, pp. 1066–1067*):

(1) The local geodetic coordinates (h^*, ϕ^*, λ^*) are transformed to the local Cartesian coordinates (x^*, y^*, z^*) using the r.h.s. of Eq. (3),

(2) the local Cartesian coordinates (x^*, y^*, z^*) are transformed to the geocentric Cartesian coordinates (x, y, z) respective to the mean earth ellipsoid – by translation, rotation, and scaling (ibid) using 7 transformation parameters that are known for the local ellipsoidand can be obtained from national survey authorities. (Accurate transformation parameters between the Bessel and GRS'80 ellipsoids are not available (*Janák, 2005, pers. comm.*));

(3) the geocentric Cartesian coordinates (x, y, z) are transformed to the geodetic coordinates respective to the mean earth ellipsoid (h, ϕ, λ). This transformation is not as straight forward, and is typically carried out in an iterative fashion (*Featherstone and Dentith, 1998, pp. 1066–1067; Jones, 2002; Pollard, 2002*). Laborious closed form formulae can be found in e.g. *Ardalan and Grafarend (2001), Eqs (63) through (65).*

Alternatively, the transformation of latitude and longitude (excluding height) is carried out regionally by means of multiple regression equations (*Mojzeš, 1997*). For Slovakia, the S-JTSK geographical latitude ϕ^{S-JTSK}, referred to the Bessel ellipsoid, is transformed into the geocentric latitude ϕ as follows (ibid):

$$\phi = \phi^{S-JTSK} + a_0 + a_{10}u + a_{11}v + a_{20}u^2 + a_{21}v^2 + a_{22}uv\,, \tag{11}$$

where $u = k\left(\phi^{S-JTSK} - \phi_0^{S-JTSK}\right)$, $v = k\left(\lambda^{S-JTSK} - \lambda_0^{S-JTSK}\right)$, $k = 1$ (*Janák, 2005, pers. comm.*), $\phi_0^{S-JTSK} = 48.65974531°$ and $\lambda_0^{S-JTSK} = 19.33338131°$, and the coefficients are listed in the table below

a_0 [″]	a_{10} [″/°]	a_{11} [″/°]	a_{20} [″/(°)^2]	a_{21} [″/(°)^2]	a_{22} [″/(°)^2]
-1.2799	-0.3851	+0.0677	-0.0244	+0.0092	+0.0124

The S-42/83 geographical latitude $\phi^{S-42/83}$, referred to the Krassovsky ellipsoid, is transformed into the geocentric latitude ϕ as follows (*Mojzeš, 1997*):

$$\phi = \phi^{S-42/83} + b_0 + b_{10}u + b_{11}v + b_{20}u^2 + b_{21}v^2 + b_{22}uv\,, \tag{12}$$

where $u = k\left(\phi^{S-42/83} - \phi_0^{S-42/83}\right)$, $v = k\left(\lambda^{S-42/83} - \lambda_0^{S-42/83}\right)$, $k = 1$ (*Janák, 2005, pers. comm.*), $\phi_0^{S-42/83} = 48.59274122°$ and $\lambda_0^{S-42/83} = 19.22372931°$, and the coefficients are listed in the below table

b_0 [″]	b_{10} [″/°]	b_{11} [″/°]	b_{20} [″/(°)^2]	b_{21} [″/(°)^2]	b_{22} [″/(°)^2]
-1.2300	+0.0544	+0.0911	-0.0064	+0.0056	+0.0195

The use of a local (relative) geodetic latitude ϕ^*, such as the S-JTSK latitude ϕ^{S-JTSK} or S-42/83 latitude $\phi^{S-42/83}$, causes a systematic error in computing the normal gravity at the observation point

$$\varepsilon_\gamma^\phi = \gamma(h, \phi) - \gamma(h, \phi^*). \tag{13}$$

This systematic error, evaluated at the topo-surface on the territory of Slovakia, is plotted, for the two local reference ellipsoids, in Fig. 4. For both the local ellipsoids this systematic error is of the order of magnitude of 10 μGal.

At the accuracy level of 0.1 mGal even the geographical latitude referred to the Bessel or Krassovsky ellipsoids can be used for computing normal gravity on the territory of Slovakia.

5. Effect of using sea-level heights instead of geodetic heights on normal gravity – Free Air Geophysical Indirect Effect

The height term (cf. Eq. (10)) in computing normal gravity represents the upward continuation of normal gravity from the reference ellipsoid to the observation (evaluation) point above the ellipsoid. If "sea level" height H of the observation point is used in the formula instead of geodetic (ellipsoidal) height h, a systematic error is introduced into the normal gravity evaluation

$$\varepsilon_\gamma^{GIE-FA} = \delta\gamma_h(h, \phi) - \delta\gamma_h(H, \phi). \tag{14}$$

This systematic error is also known as the "Free-Air Geophysical Indirect Effect", hence the superscript "GIE-FA", (*Chapman and Bodine, 1979; Vogel, 1982; Jung and Rabinowitz, 1988; Meurers, 1992; Talwani, 1998; Hackney and Featherstone, 2003*), since it can be accurately (on the territory of Slovakia to 90 μGal) estimated (cf. Eq. (8)) as

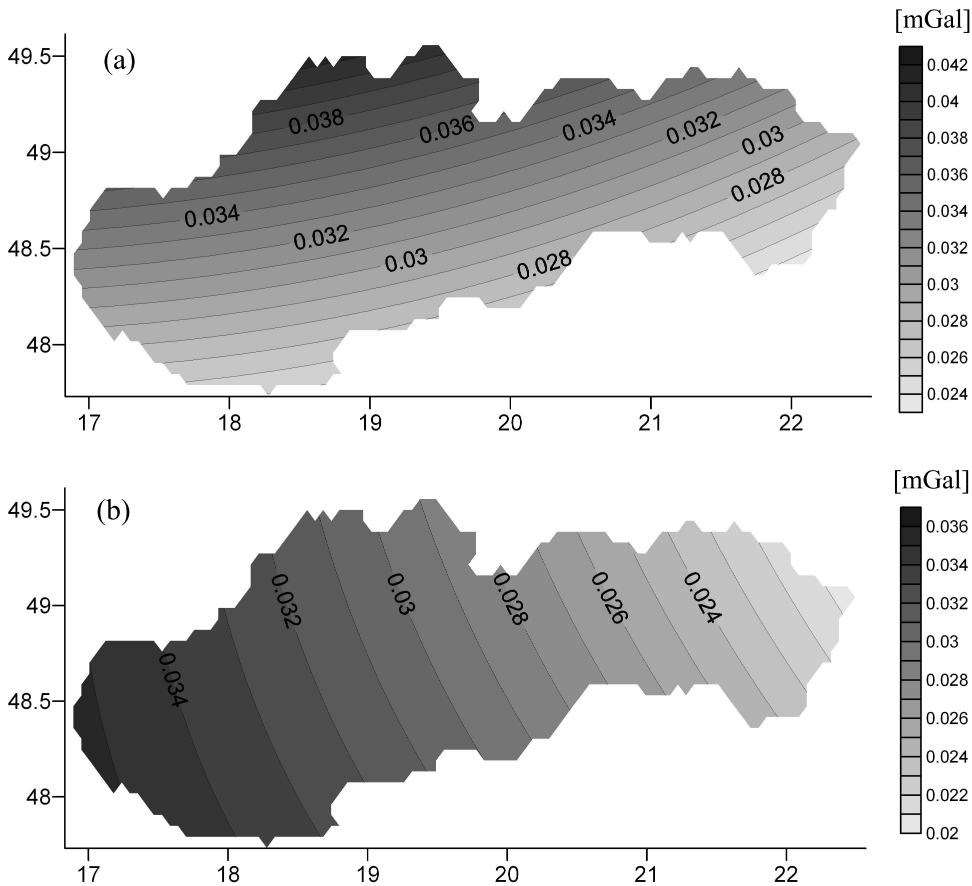

Fig. 4. Systematic error $\varepsilon_{\gamma}^{\phi}$ [mGal], due to the use of local non-geocentric geographical latitude: (a) respective to the Bessel ellipsoid, (b) respective to the Krassovsky ellipsoid. Horizontal coordinates are geocentric geodetic longitude and latitude [arcdeg].

$$\varepsilon_{\gamma}^{GIE-FA} \cong -0.3086 \text{ [mGal/m] } \varsigma \,, \qquad (15)$$

where ς is the separation between "sea level" (quasigeoid in Slovakia) and the reference ellipsoid, i.e. the quasigeoidal height. In Slovakia ,,normal heights" are used as "sea-level heights". They are referred to the quasigeoid as the "height datum". The Free-Air Geophysical Indirect Effect computed at the topo-surface for the territory of Slovakia is displayed as a systematic

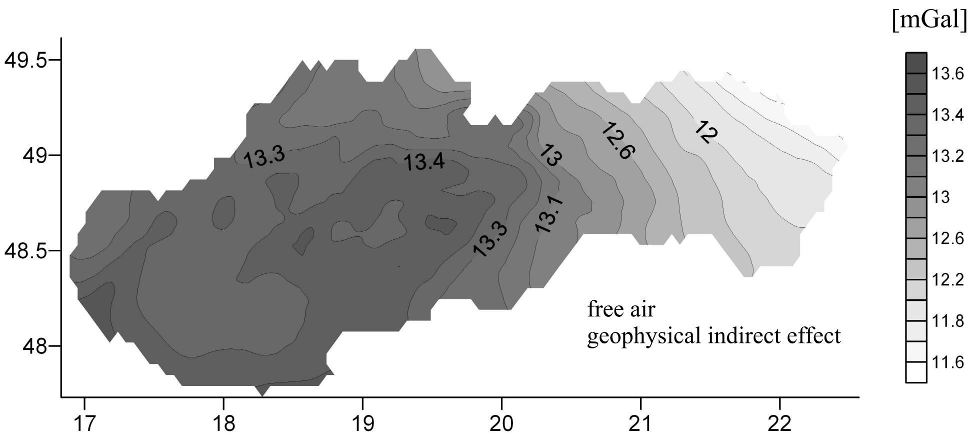

Fig. 5. The systematic error in computing normal gravity above the reference ellipsoid caused by improper use of heights, known also as the Free Air Geophysical Indirect Effect [mGal]. Horizontal coordinates are geocentric geodetic longitude and latitude [arcdeg].

error in Fig. 5. This systematic error is on the territory of Slovakia of the order of magnitude 10 mGal.

When the positions of gravity observation points (stations) are given in sea level heights, these heights must be transformed to geodetic (ellipsoidal) heights, in order to correctly compute normal gravity at these points, simply as

$$h = H + \varsigma \,. \tag{16}$$

The grid of quasigeoidal heights ς can be obtained from the national survey authorities, i.e., in Slovakia from the Geodetic and Cartographic Institute.

6. Conclusions

The compilation of gravity disturbance/"anomaly" at the point where actual gravity is given requires the computation of normal gravity at that point. For terrestrial surveys this point is typically located at the topographical surface, i.e., above the reference ellipsoid. We have reviewed the evaluation of normal gravity – more exactly the modulus of the gravity vector of the Somigliana-Pizzetti gravity field of the equipotential mean earth reference ellipsoid – on and above the ellipsoid. Closed formulae for normal gravity on and above the reference ellipsoid with a sub-nanoGal accuracy are given by *Ardalan and Grafarend (2001)*, expressed in both the Jacobi-ellipsoidal coordinates (*ibid, Eq. (55)*) and the geocentric geodetic (Gauss-ellipsoidal) coordinates (*ibid, Eq. (74)*). These formulae are quite laborious. At the accuracy threshold of 80 nanoGals (for terrestrial evaluation points) the closed formula in Jacobi-ellipsoidal coordinates simplifies to the form of Eq. (1).

At the accuracy threshold of 80 μGal (terrestrial points globally), or 30 μGal (terrestrial points Slovakia), cf. Fig. 1, the "SHM" formula (Eqs (5), (6), and (10)) expressed in geographical (more precisely: geocentric geodetic) coordinates can be considered "exact". The "SHM" formula consists of the Somigliana formula for normal gravity on the reference ellipsoid plus the normal gravity height term given by *Heiskanen and Moritz (1967, Eq. (2-124))*. This analytical prescription is simple and does not require any unaffordable computer time. The "SHM" formula is used by us here as a reference formula. Other formulae commonly used in practice are compared to it to assess systematic errors occurring from the use of such formulae.

The systematic error committed by using only the free-air term as the height-term in computing normal gravity was calculated for the points at the topo-surface on the territory of Slovakia. It is shown in Figs 2 and 3. This systematic error is correlated with the topo-surface, and attains values of the order of 0.1 mGal on the territory of Slovakia (0.8 mGal at the top of Mt. Gerlach). At the accuracy threshold of 1 mGal, normal gravity may be computed on the territory of Slovakia using the "free-air" height term (Eq. (8)) in the "SHM" formula instead of the "HM" height term (Eq. (10)).

The systematic error in computing normal gravity caused by using geographical coordinates not referred to the mean earth ellipsoid, but to a local reference ellipsoid, in Slovakia namely the Bessel and Krassovsky ellipsoids, was calculated for the territory of Slovakia and is displayed in Fig. 4. For both the local ellipsoids this systematic error reaches values around 40 μGal. At the accuracy level of 0.1 mGal even the geographical latitude referred to the Bessel or Krassovsky ellipsoids can be used for computing normal gravity for Slovakia.

The systematic error in computing normal gravity introduced by failing to use the geodetic (ellipsoidal) heights, and using the "sea-level" heights instead (in Slovakia namely normal heights), was assessed for the territory of Slovakia and is shown in Fig. 5. This systematic error is also known among geophysicists as the "free-air geophysical indirect effect". On the territory of Slovakia it reaches values between 11 and 13 mGal. This systematic error can be easily eliminated by adding quasigeoidal heights to sea-level heights when computing normal gravity to be used in constructing "anomalous gravity data", or by adding the "Free Air Geophysical Indirect Effect" as a correction to the "anomalous gravity data" – the GIE-FA correction corrects normal gravity, not actual gravity. The "free-air geophysical indirect effect" is the first of the two terms of the "geophysical indirect effect". For a more detailed discussion of the geophysical indirect effect and its impact on geophysical interpretation we would like to refer the reader to *Hackney and Featherstone (2003); Vajda et al. (2005).*

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