Magnetic anomaly due to magnetic halfspace with buried cylindrical perturbing body

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A b s t r a c t: For purposes of better knowledge of the magnetic anomalous fields in volcanic areas we analyze the mathematical model of relevant properties: wide spread magnetic halfspace of permeability μ_1 (corresponding to the lava field) which contains buried cylindrical body of radius a, permeability μ_2. This problem is solved exactly by means of Laplace equations in bipolar coordinate system. Numerical results show that the presence of cylinder is reflected in the anomaly of ΔT, as well as in the inclination angle I.

Key words: magnetometric models, bipolar coordinates

1. Potential of normal magnetic field

In the practice of magnetometric measurements in regions of solidified volcanic mountains we encounter a situation when the magnetometric measurements are performed on the surface of a halfspace (wide spread volcanic complex) of permeability μ_1, and there occurs perturbing a body in the interior of the halfspace, with permeability μ_2. On the basis of electromagnetic theory *(Stratton, 1941)* it is clear, that if the exciting magnetic field $\boldsymbol{H}_{0e}$ is oblique with respect to the surface plane $z = 0$, then in the interior of the halfspace $z > 0$ there appears refraction of force lines $\boldsymbol{H}_{1e}$, which must be taken into account in theoretical analysis.

Let the exciting magnetic field above the halfspace be $\boldsymbol{H}_{0e} \equiv (H_{0x}, H_{0z})$, and inside of it $\boldsymbol{H}_{1e} \equiv (H_{1x}, H_{1z})$. It is known that potentials of the fields are

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$$V_{0e}(x,z) = -x \cdot H_{0x} - z \cdot H_{0z}, \tag{1}$$

where $H_{0x} = |\boldsymbol{H}_{0e}| \cdot \cos\alpha$, $H_{0z} = |\boldsymbol{H}_{0e}| \cdot \sin\alpha$,

$$V_{1e}(x,z) = -x \cdot H_{0x} - z \cdot D\, H_{0z}, \tag{2}$$

and where constant D is to be determined from boundary conditions at $z = 0$. There must be continuous potential and z-component of the magnetic induction $\boldsymbol{B} = \mu \boldsymbol{H}$, i.e.:

$$V_{0e}\big|_{z=0} = V_{1e}\big|_{z=0}\,, \tag{3}$$

$$\mu_0 \frac{\partial V_{0e}}{\partial z}\bigg|_{z=0} = \mu_1 \frac{\partial V_{1e}}{\partial z}\bigg|_{z=0}. \tag{4}$$

The condition (3) is for (1) and (2) satisfied and (4) $+\mu_0 H_{0z} = \mu_1 D\, H_{0z}$. Then we have:

$$D = \frac{\mu_0}{\mu_1} = \frac{1}{\mu_{1r}} \leq 1 \tag{5}$$

It means, that if the inclination of normal magnetic field above the halfspace is α, with

$$\operatorname{tg}\alpha = \frac{H_{0z}}{H_{0x}}\,, \tag{6}$$

then inside of halfspace we have another inclination angle β with

$$\operatorname{tg}\beta = \frac{D\, H_{0z}}{H_{0x}} = D \cdot \operatorname{tg}\alpha = \frac{\mu_0}{\mu_1} \operatorname{tg}\alpha. \tag{7}$$

Because the rocks of magmatic wide-spread field “1” are magnetic, $D < 1$ and $\beta < \alpha$. This refraction is an important feature of the magnetic “normal” fields of our problem.

2. Effect of the cylindrical intrusions buried in the halfspace

The calculation of the effect of the cylindrical intrusive body is another important step in our analysis. This body is considered in the form of infinitely long circular cylinder, radius a permeability μ_2, infinitely extended

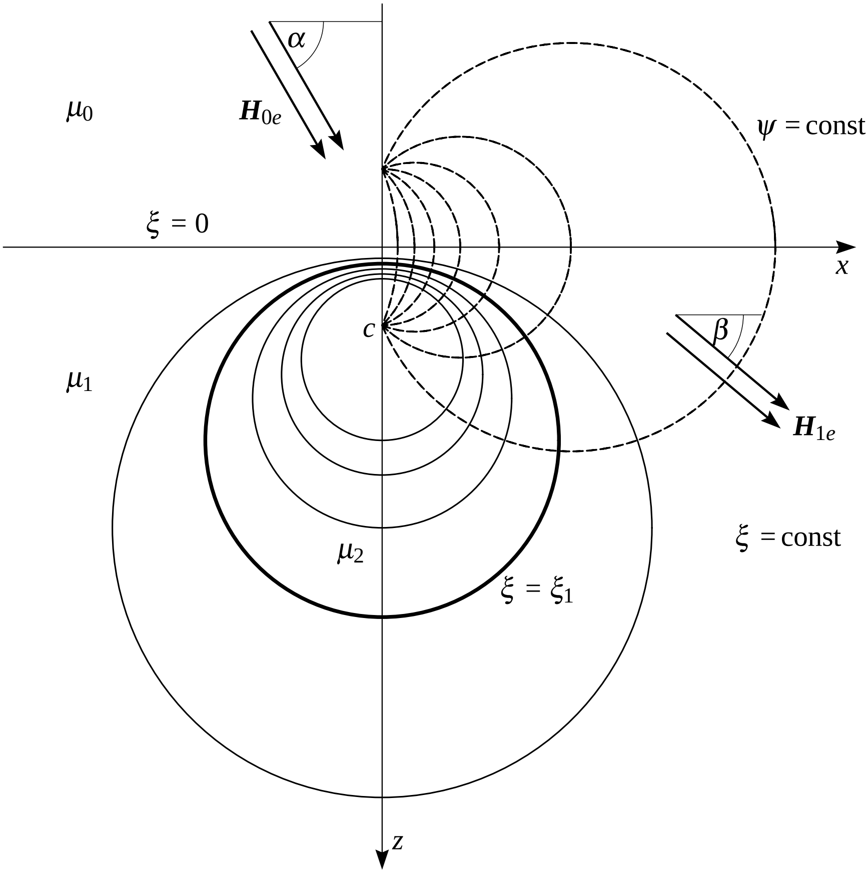

Fig. 1. Scheme of used bipolar coordinate system (ξ, ψ) and vectors of unperturbed magnetic field.

in the y-direction, as shown in Fig. 1. This model enables us to calculate two important cases: a) anomalous magnetic field due to a body with higher permeability $\mu_2 : \mu_2 > \mu_1 > \mu_0$, b) anomalous magnetic field due to a nonmagnetic hollow gallery: $\mu_1 > \mu_0$ and $\mu_2 = \mu_0$. Our experience with solutions of similar geoelectric problems *(Hvoždara, 1975)* or geothermic problem *(Hvoždara and Majcin, 1985)* indicate the possibility of an exact solution of this magnetometric problem by means of bipolar coordinates (ξ, ψ) which are linked with Cartesian coordinates (x, z) by the following transformation relations:

$$x = p\sin\psi/(\operatorname{ch}\xi - \cos\psi), \qquad z = p\operatorname{sh}\xi/(\operatorname{ch}\xi - \cos\psi), \tag{8}$$

where $p = \sqrt{h^2 - a^2}$ is the parameter of the bipolar coordinate system, h is the depth of the cylinder axis, a is its radius. We know that if $\xi \to +\infty$, we have $(x = 0, z = p)$, which corresponds to the positive pole of the bipolar system. The negative pole $\xi \to -\infty$ lies above the surface of the halfspace at the point $(x = 0,\ z = -p)$, as shown in Fig. 1.

The magnetic field in all three considered regions of our problem is split in the form of the sum of normal field and anomalous field, denoted by the asterix:

$$\boldsymbol{H} = \boldsymbol{H}_{ek} + \boldsymbol{H}_k^*\,, \qquad \boldsymbol{H} = -\operatorname{grad} U, \tag{9}$$

$$U_k(x, z) = V_{ek}(x, z) + U_k^*(x, z), \quad k = 0, 1, 2. \tag{10}$$

The potentials of the exciting magnetic field satisfying the boundary conditions (3,4) are already known

$$V_{e0} = -(H_{0x} \cdot x + H_{0z} \cdot z)\,, \quad z < 0, \tag{11}$$

$$V_{e1,2} = -(H_{0x} \cdot x + \mu_{1r}^{-1} H_{0z} \cdot z)\,, \quad z > 0 \quad (k = 1, 2,) \tag{12}$$

and their expression in bipolar coordinates (ξ, ψ) using (8) is easy.

3. Anomalous magnetic potentials

The advantage of introducing the bipolar coordinates (ξ, ψ) lies in the possibility of coincidence of plane boundary of the halfspace $z = 0$ with coordinate line $\xi = 0$ (circular line of infinite radius) and the surface of the circular cylinder with coordinate line $\xi = \xi_1$. The perturbing potentials due to the cylinder obey the Laplace equation:

$$\nabla^2 U^*(\xi, \eta) = 0, \quad \text{i.e.} \quad \frac{\partial^2 U^*}{\partial \xi^2} + \frac{\partial^2 U^*}{\partial \psi^2} = 0, \tag{13}$$

which has particular solution of the form:

$$(U^*)_n = \begin{Bmatrix} e^{n\xi} \\ e^{-n\xi} \end{Bmatrix} \begin{Bmatrix} \cos n\psi \\ \sin n\psi \end{Bmatrix} . \tag{14}$$

Using these particular solutions we must compose perturbing potentials U_0^*, U_1^*, U_2^*. Because in excitation potentials (11), (12) there occur terms with $\cos\psi$ and $\sin\psi$, we will have in perturbing potentials both $\cos n\psi$ and $\sin n\psi$. With respect to the requirement of bounded values of perturbing potentials we can write their general expressions in the form:

$$U_0^*(\xi,\psi) = p\sum_{n=0}^{\infty} e^{n\xi}(C_{0n}\cos n\psi + B_{0n}\sin n\psi), \quad (\xi < 0) \tag{15}$$

$$U_1^*(\xi,\psi) = p\sum_{n=0}^{\infty} e^{n\xi}(C_{1n}\cos n\psi + B_{1n}\sin n\psi) + \\ + e^{-n\xi}(E_{1n}\cos n\psi + F_{1n}\sin n\psi), \tag{16}$$

$$U_2^*(\xi,\psi) = p\sum_{n=0}^{\infty} e^{-n\xi}(E_{2n}\cos n\psi + F_{2n}\sin n\psi), \quad \xi \in \langle \xi_1, +\infty). \tag{17}$$

In (15) there cannot occur terms with $e^{-n\xi}$ since they would be singular for $\xi = -\infty$, which is in the region “0” ($\xi \in (-\infty, 0\rangle$). In (16) we have both terms with $e^{n\xi}$ and $e^{-n\xi}$, because there ξ varies from 0 to ξ_1. In the interior of the cylinder we cannot have terms with $e^{n\xi}$, since they are singular at $\xi = +\infty$. For calculations of coefficients A_{0n}, $B_{0n} \ldots, F_{2n}$ we will use boundary conditions at $\xi = 0, \xi_1$, so we need to know expansions of primary potentials $V_{0e}(x,z) = V_{0e}(\xi,\psi)$ and $V_{1e}(x,z) = V_{1e}(\xi,\psi)$ into particular solutions (14). The basis of their expansion in region $z \geq 0$ are formulae adopted from *Gradstejn and Rhyzhik (1971)*

$$x = \frac{p\sin\psi}{\operatorname{ch}\xi - \cos\psi} = 2p\sum_{n=1}^{\infty} e^{-n\xi}\sin(n\psi), \quad \xi > 0, \tag{18}$$

$$z = \frac{p\operatorname{sh}\xi}{\operatorname{ch}\xi - \cos\psi} = p\left\{1 + 2\sum_{n=1}^{\infty} e^{-n\xi}\cos(n\psi)\right\}, \tag{19}$$

which rapidly converge for $\xi > 0$. Since the primary potential V_{0e} given by (1) continuously transits to V_{1e} at $\xi = 0$ ($z = 0$) and (4) is satisfied too, we can easily find that on the boundary $\xi = 0$ we must treat only the continuity of $U_0^*(\xi, \psi)$ and $U_1^*(\xi, \psi)$, as well as the continuity of their ξ derivatives multiplied by permeabilities μ_0 and μ_1. Then we will have pairs of equations for sine coefficients:

$$B_{0n} = B_{1n} + F_{1n}\,, \quad B_{0n} = \mu_{1r} B_{1n} - \mu_{1r} F_{1n}\,, \quad \text{where } \mu_{1r} = \mu_1/\mu_0\,. \tag{20}$$

Similar pair holds true for cosine coefficients:

$$C_{0n} = C_{1n} + E_{1n}\,, \quad C_{0n} = \mu_{1r} C_{1n} - \mu_{1r} E_{1n}\,. \tag{21}$$

For the application of the boundary conditions at $\xi = \xi_1$ (the surface of the interior cylinder), we use expressions of potentials $U_1(\xi, \psi)$, $U_2(\xi, \psi)$ in the form of:

$$\begin{aligned} U_1(\xi,\psi) = {} & -2p\, H_{0x} \sum_{n=1}^{\infty} e^{-n\xi} \sin(n\psi) - 2p\, H_{1z} \left\{ \frac{1}{2} + \sum_{n=1}^{\infty} e^{-n\xi} \cos(n\psi) \right\} + \\ & + p \sum_{n=0}^{\infty} e^{n\xi} (C_{1n} \cos n\psi + B_{1n} \sin n\psi) + \\ & + e^{-n\xi} (E_{1n} \cos n\psi + F_{1n} \sin n\psi)\,, \end{aligned} \tag{22}$$

$$\begin{aligned} U_2(\xi,\psi) = {} & -2p\, H_{0x} \sum_{n=1}^{\infty} e^{-n\xi} \sin n\psi - 2p\, H_{1z} \left\{ \frac{1}{2} + \sum_{n=1}^{\infty} e^{-n\xi} \cos n\psi \right\} + \\ & + p \sum_{n=0}^{\infty} e^{-n\xi} \left[E_{2n} \cos n\psi + F_{2n} \sin n\psi \right]\,, \end{aligned} \tag{23}$$

where $H_{1z} = H_{0z} \mu_{1r}^{-1}$. We introduce these series into boundary equations at $\xi = \xi_1$ and then, by application of orthogonality Fourier series, we will arrive at the system of linear equations for the coefficients:

$$B_{1n} e^{n\xi_1} + F_{1n} e^{-n\xi_1} = F_{2n} e^{-n\xi_1}, \tag{24}$$

$$2H_{0x} e^{-n\xi_1} + B_{1n} e^{n\xi_1} - F_{1n} e^{-n\xi_1} = 2\,\frac{\mu_2}{\mu_1} H_{0x} e^{-n\xi_1} - \frac{\mu_2}{\mu_1} F_{2n} e^{-n\xi_1}. \tag{25}$$

Similarly, for cosine coefficients we have

$$C_{1n}e^{n\xi_1} + E_{1n}e^{-n\xi_1} = E_{2n}e^{-n\xi_1}, \tag{26}$$

$$2H_{1z}e^{-n\xi_1} + C_{1n}e^{n\xi_1} - E_{1n}e^{-n\xi_1} = 2\,\frac{\mu_2}{\mu_1}H_{1z}e^{-n\xi_1} - \frac{\mu_2}{\mu_1}E_{2n}e^{-n\xi_1}. \tag{27}$$

The continuity for cosine coefficients is kept at $n = 0$ as well, and we see, that the form of equations for sine and cosine coefficients is the same, so we concentrate on the sine coefficients at first. We shall multiply (24) by μ_2/μ_1, on addition to (25), and we eliminate F_{2n}, then we have

$$\left(1 + \frac{\mu_2}{\mu_1}\right) B_{1n}e^{n\xi_1} - \left(1 - \frac{\mu_2}{\mu_1}\right) F_{1n}e^{-n\xi_1} = -2\left(1 - \frac{\mu_2}{\mu_1}\right) H_{0x}e^{-n\xi_1}. \tag{28}$$

From the pair (20) we eliminate B_{0n} which results in

$$(1 - \mu_{1r})B_{1n} + (1 + \mu_{1r})F_{1n} = 0\,. \tag{29}$$

The equations (28) and (29) we arrange to a more suitable form:

$$B_{1n} - F_{1n}\cdot R_{1n} = -2\,H_{0x}R_{1n}\,, \quad q_0\cdot B_{1n} + F_{1n} = 0\,, \tag{30}$$

where $R_{1n} = \dfrac{1 - \mu_2/\mu_1}{1 + \mu_2/\mu_1}e^{-2n\xi_1} = q_2e^{-2n\xi_1}$,

and we introduce reflection coefficients:

$$q_0 = \frac{1 - \mu_{1r}}{1 + \mu_{1r}} = \frac{1 - \mu_1/\mu_0}{1 + \mu_1/\mu_0}\,, \quad q_2 = \frac{1 - \mu_2/\mu_1}{1 + \mu_2/\mu_1}\,. \tag{31}$$

Now we can write necessary coefficients:

$$B_{1n} = \frac{-2H_{0x}R_{1n}}{1 + q_0R_{1n}}\,, \quad F_{1n} = \frac{2H_{0x}\cdot q_0R_{1n}}{1 + q_0R_{1n}}\,. \tag{32}$$

It is clear that if $\mu_2 = \mu_1$, there is $R_{1n} = 0$ and coefficients of perturbing potential $U_1^*(\xi,\psi)$ will be zero.

For coefficients of perturbing potential in the upper halfspace we will find by means of (20):

$$B_{0n} = B_{1n} + F_{1n} = 2\,\frac{R_{1n}(q_0 - 1)}{1 + q_0R_{1n}}H_{0x}\,. \tag{33}$$

For the sake of completeness we give coefficients in the interior of the cylinder as resulted from (24):

$$F_{2n} = F_{1n} + B_{1n} e^{2n\xi_1} = 2q_2 \frac{(q_0 e^{-2n\xi_1} - 1)}{1 + q_0\, q_2 e^{-2n\xi_1}} H_{0x}\,. \tag{34}$$

The form of all coefficients gives explicitly their zero values for $\mu_2 = \mu_1$, since then $q_2 = 0$, $R_{1n} = 0$. In a similar way we repeat the derivation of cosine coefficients, substituting H_{1z} for H_{0x}:

$$C_{1n} = \frac{-2H_{1z} \cdot R_{1n}}{1 + q_0 R_{1n}}\,, \quad E_{1n} = \frac{2H_{1z} \cdot q_0 R_{1n}}{1 + q_0 R_{1n}}\,, \tag{35}$$

$$C_{0n} = 2\, \frac{R_{1n}(q_0 - 1)}{1 + q_0 R_{1n}} H_{1z}\,, \tag{36}$$

$$E_{2n} = 2q_2 \frac{q_0 e^{-2n\xi_1} - 1}{1 + q_0\, q_2 e^{-2n\xi_1}} H_{1z}\,. \tag{37}$$

Let us note, that

$$H_{1z} = H_{0z} \cdot \mu_{1r}^{-1} = H_{0z} \cdot \frac{\mu_0}{\mu_1}\,.$$

Now we will have prepared all necessary expressions for the calculation of potentials of anomalous magnetic field in all regions "0", "1" and "2". Magnetic contrasts are expressed by q_0, q_2; their absolute value is less than 1. In the derived formulae there is an important term $e^{-\xi_1}$ and its powers $e^{-2n\xi_1}$. On the basis of the transformation relations we can show that to the coordinate surface $\xi = \xi_1$ corresponds a circle in the plane (x, z):

$$x^2 + (z - p \coth \xi_1)^2 = (p/ \operatorname{sh} \xi_1)^2\,. \tag{38}$$

This equation is compared with a canonical equation of the circle radius a, and the centre at $z = h$:

$$x^2 + (z - h)^2 = a^2. \tag{39}$$

We see that ξ_1 and p can be determined from equations:

$$p \cdot \coth \xi_1 = h, \quad p/ \operatorname{sh} \xi_1 = a, \tag{40}$$

which gives $p^2 = h^2 - a^2$ and

$$\operatorname{ch} \xi_1 = h/a, \quad \operatorname{sh} \xi_1 = \sqrt{h^2/a^2 - 1} = \frac{\sqrt{h^2 - a^2}}{a},$$

$$\operatorname{ch} \xi_1 = h/a, \quad \operatorname{sh} \xi_1 = p/a, \quad p = \sqrt{h^2 - a^2}, \tag{41}$$

$$e^{-\xi_1} = (h-p)/a, \quad e^{\xi_1} = (h+p)/a\,. \tag{42}$$

From these relations we can easily find that $e^{-\xi_1} < 1$ and powers $e^{-2n\xi_1}$ will rapidly decrease to zero for increasing n.

Since the exciting magnetic field is known, our attention will concentrate onto Cartesian components which follow from perturbing potentials $U_0^*(\xi,\psi)$, $U_1^*(\xi,\psi)$, $U_2^*(\xi,\psi)$.

4. Numerical calculations of anomalous magnetic field $\Delta\boldsymbol{H}$ and ΔT

Calculations of x and z components of anomalous magnetic field in regions "0"–"2" must be performed by the formula:

$$\Delta\boldsymbol{H} = -\operatorname{grad} U^*(x,z). \tag{43}$$

Perturbing potentials are expressed in series with bipolar coordinates, but we can calculate Cartesian components as well (omitting indices 0–2):

$$\Delta H_x = -\frac{\partial U^*(\xi,\psi)}{\partial\xi}\cdot\frac{\partial\xi}{\partial x} - \frac{\partial U^*(\xi,\psi)}{\partial\psi}\frac{\partial\psi}{\partial x}, \tag{44}$$

$$\Delta H_z = -\frac{\partial U^*(\xi,\psi)}{\partial\xi}\cdot\frac{\partial\xi}{\partial z} - \frac{\partial U^*(\xi,\psi)}{\partial\psi}\frac{\partial\psi}{\partial z}. \tag{45}$$

We shall now use transformation relations adopted e.g. from *(Arfken, 1966)*:

$$\xi + \mathrm{i}\,\psi = 2\,\mathrm{Arcoth}\left[(z/p) - \mathrm{i}(x/p)\right], \tag{46}$$

which after some rearrangement gives:

$$e^{2\xi} = \left[(z+p)^2 + x^2\right] / \left[(z-p)^2 + x^2\right], \tag{47}$$

$$\mathrm{tg}\,\psi = 2px / \left[x^2 + z^2 + p^2\right]. \tag{48}$$

Now we can easily find that x and z derivatives of ξ and ψ will be:

$$\frac{\partial\xi}{\partial x} = \frac{x}{(z+p)^2 + x^2} - \frac{x}{(z-p)^2 + x^2}\,, \tag{49}$$

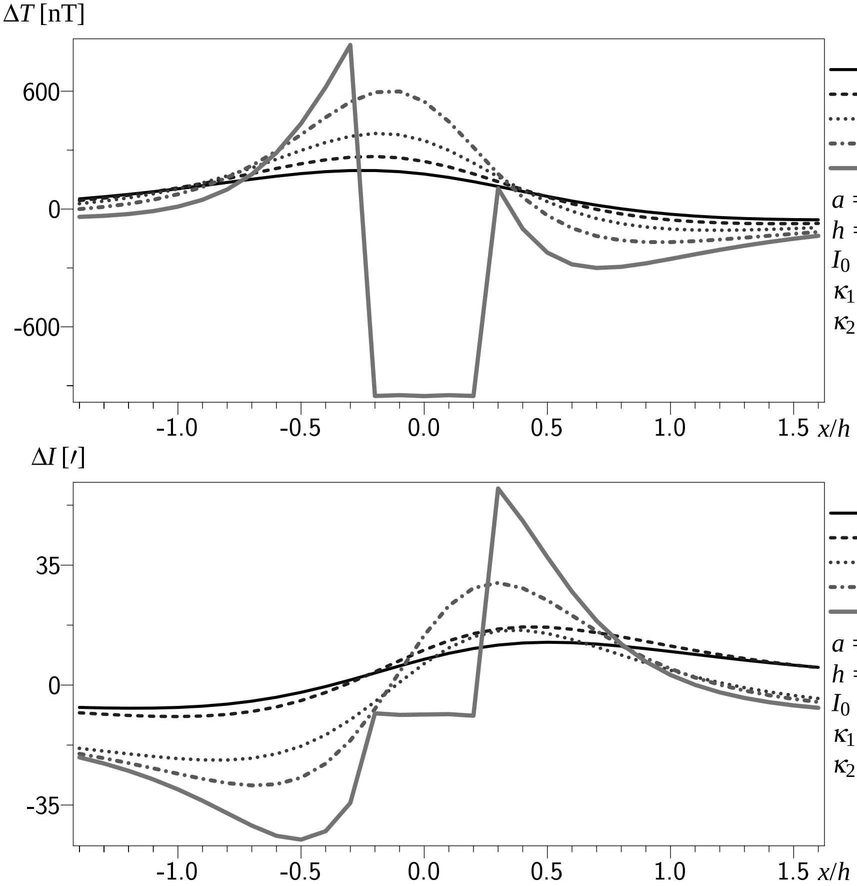

Fig. 2ab. Profile curves ΔT and ΔI calculated for the circular cylinder of magnetic susceptibility $\kappa_2 = 0.05$, radius $a = 100$ m, buried at the depth $h = 150$ m in the magnetic halfspace $\kappa_1 = 0.01$. The inclination of $\boldsymbol{B}_0$ is $I_0 = 75°$.

$$\frac{\partial \xi}{\partial z} = \frac{z+p}{(z+p)^2 + x^2} - \frac{z-p}{(z-p)^2 + x^2}\,, \tag{50}$$

$$\frac{\partial \psi}{\partial x} = \frac{2p(z^2 - p^2 - x^2)}{\left(x^2 + z^2 - p^2\right)^2 + 4p^2x^2}\,, \qquad \frac{\partial \psi}{\partial z} = \frac{-4pxz}{\left(x^2 + z^2 - p^2\right)^2 + 4p^2x^2}\,. \tag{51}$$

These derivatives, when applied to (44) and (45), enable us to calculate ΔH_x and ΔH_z in all regions "0"–"2", and after multiplication by magnetic permeability, also ΔB_x and ΔB_z.

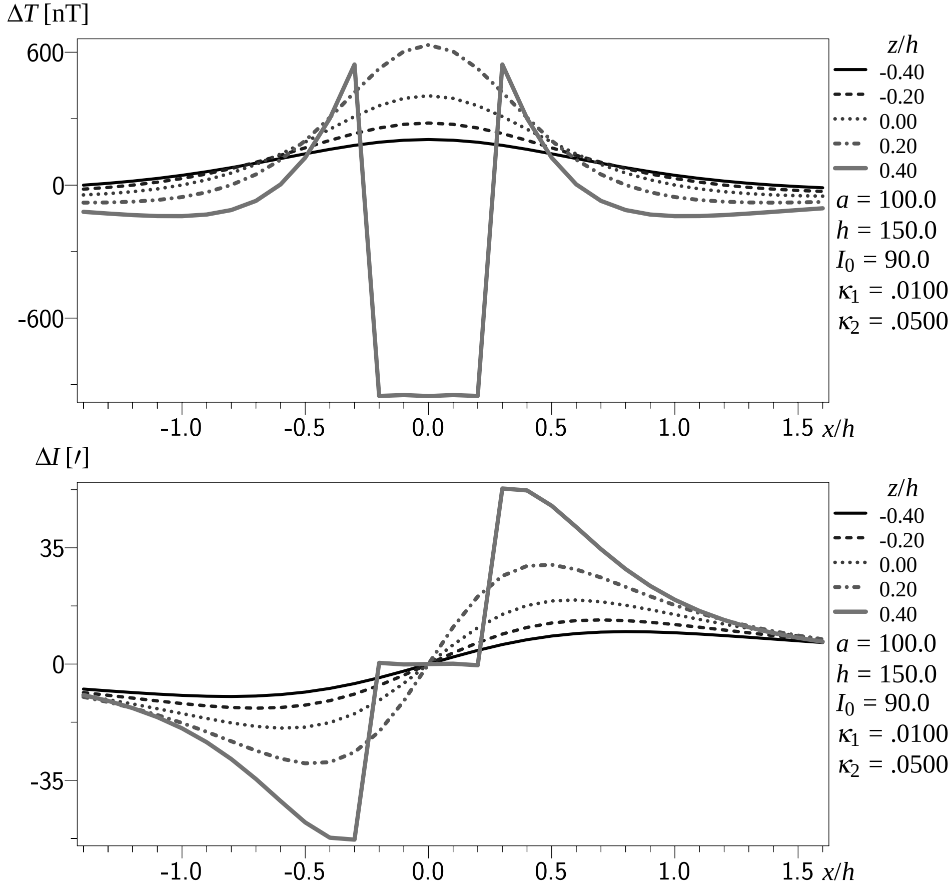

Fig. 3ab. The same as in Figs 2ab, but the inclination of $\boldsymbol{B}$ is 90° (vertical normal field).

Because in magnetometric practice mostly total anomaly field ΔT is used, we will have, by known formula *(Logachev and Zacharov, 1979)*:

$$\Delta T = \Delta B_x \cos\beta + \Delta B_z \sin\beta\,, \tag{52}$$

where β is the inclination angle I for region "0" or "1" and "2" by means of (6):

$$\mathrm{tg}\,\beta = \frac{1}{\mu_r}\,\mathrm{tg}(I)\,. \tag{53}$$

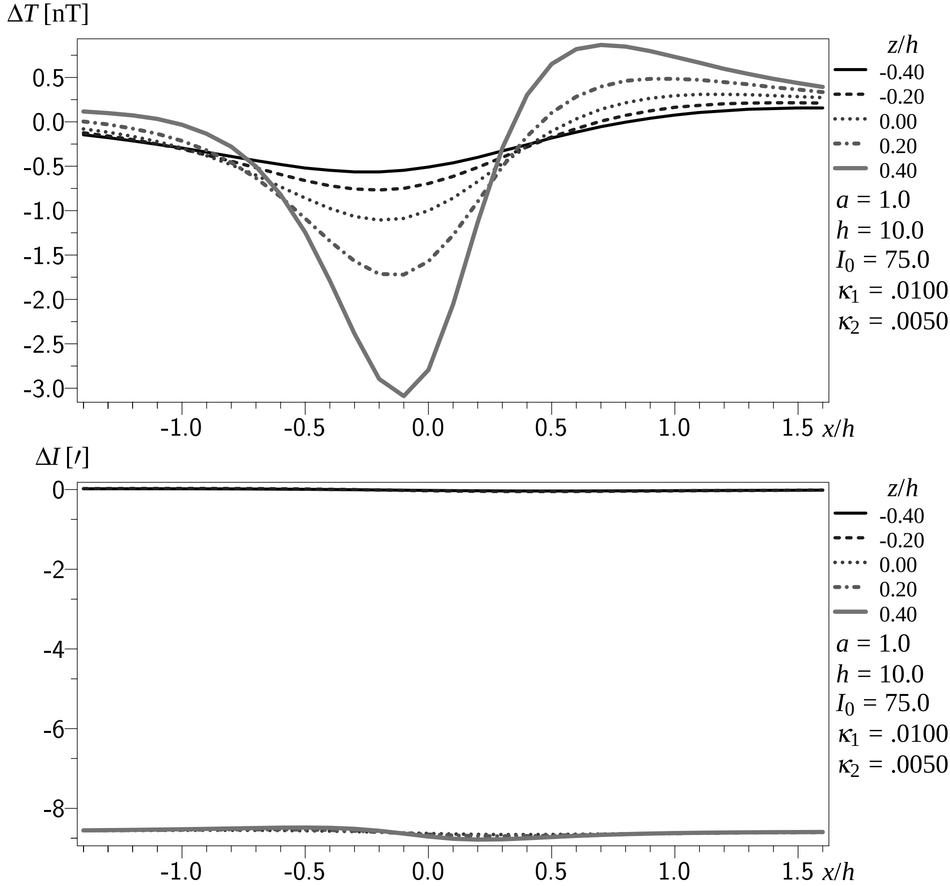

Fig. 4ab. Profile curves of ΔT and ΔI above the shallow circular gallery of radius $a = 1\,\mathrm{m}$, buried in the depth $h = 10\,\mathrm{m}$, with inclination of $\boldsymbol{B}_0$ equal to 75°.

Now we have a complete system of formulae for calculations in our model. In our FORTRAN 77 computer program we introduced truncation of the series for $H^*{}_1$ at indices n which fulfilled relative accuracy better than 10^{-4} to the terms given by $n = 1$.

For numerical calculations we adopted magnetic susceptibility of the halfspace $\kappa_1 = 0.01$ and for the intrusive cylinder body $\kappa_2 = 0.05$. The depth of its axis is assumed as $h = 150\,\mathrm{m}$ and its radius $a = 100\,\mathrm{m}$. The unperturbed magnetic induction $B_0 = 47000\,\mathrm{nT}$, normal inclination

$I_0 = 75°(\equiv \alpha)$. Derived formulae were used for calculations on five levels: $z/h = -0.4,\ -0.2,\ 0.0,\ 0.2,\ 0.4$ and profile curves are plotted in Fig. 2a. We can see that values of ΔT for the first four levels gradually increase with depth level z, they can attain positive values up to 250 nT and then slowly attain smaller negative depression at $x/h > 0.7$. The depth profile $z/h = 0.4$ intersects the interior cylinder, the course of ΔT is more complicated, with jumps on boundaries of the cylinder. Fig. 2b presents values of inclination $\boldsymbol{B}$ for the five depth levels mentioned above. We can see, that changes in inclination can attain $\pm 30'$ (arc minutes). We present an illustrative example of the Figs 3a,b with similar curves calculated for normal vertical inclination ($I_0 = 90°$). In this case we have more symmetric features in the calculated curves. In order to follow magnetic effects of the shallow cylindrical gallery situated near the surface, we adopted $a = 1, h = 10$ m, $\kappa_1 = 0.01, \kappa_2 = 0.001$, $I_0 = 75°$, $B_0 = 47000$ nT. From profile curves ΔT in Fig. 4a we see that the presence of the gallery gives very small anomaly, ΔT about -2 till 0.5 nT. Also the changes of inclination are small, as shown in Fig. 4b. Let us note that the calculations for this analytical model are very fast, so it is possible to perform a lot of model calculations.

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