Improvement of the Euler deconvolution algorithm by means of the introduction of regularized derivatives

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Abstract: The Euler deconvolution method is one of the most used semi-automated methods in potential fields interpretation during last decade. The evaluation of gradients, which enter into the algorithm of the method should be stabilized, because this numerical calculation strongly emphasizes errors and noise in the original data and makes the results of the method instable and defocused. Evaluation of stable derivatives by means of the regularization method demonstrates on a synthetic model study and practical data application the stabilization and focusing of the depth estimates, obtained by means of the Euler deconvolution method. Solutions, obtained for regularized gradients are deeper in comparison with the erroneous shallow ones (obtained without the regularization).

Key words: geophysics, gravimetry, magnetometry, semi-automated interpretation methods, Tikhonov regularization

1. Introduction

Results of interpretation methods in gravimetry and magnetometry build an important part of the geophysical interpretation. As separate methods (e.g. *Blakely, 1995; Mudretsova and Veselov, 1990*) or in the frame of complex/integrated geophysical studies (e.g. *Dérerová and Bielik, 2003; Šefara* and *Bielik, 2004*). The greatest problem during an effective utilization of these results is the well-known inherent instability and ambiguity

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of the inverse problem of potential fields. The only solution (up to present days) is the introduction of the a priori (additional) information about the solution. This information can be of mathematical-physical or geological-geophysical kind. From this point of view, we often speak about so called direct interpretation methods (without the introduction of the geoscientic additional information) and indirect methods (mainly modeling in the $2^1/_2$D, $2^3/_4$D or 3D regime with the acceptation of the results from other geological and geophysical methods, as wells, well-logging, seismics, MT, DC soundings, etc.). The direct methods are often named as deconvolution-, singular points-, depth estimates- or semi-automated methods, etc. In the Western Carpathian region, the indirect methods (based mainly on modeling) have a long-year tradition with very good results, because of the strong school, methodological developments and a high level of a priori geological and geophysical information. But there still exist areas in this region, where the direct methods can bring valuable information about the source distribution (without the direct introduction of the mentioned geological and geophysical data).

Among the direct interpretation methods of potential fields during last years the so called Euler deconvolution method (ED method) (also called Euler homogeneity- or ELDPH-method) became very popular. The reason is probably hidden in its relatively simple realization and application to profile or grided data. It became a part of various professional software packets for processing and interpretation of potential fields (Geosoft, Fugro, Intrepid, etc.). On the other side the basic idea of the method is built on a simple linearization, utilizing gradients of the interpreted field. This combination of properties (linearization and gradients) is responsible for a very high level of instability of the method (this was pointed by many authors and experts from the industry, e.g. *Farrelly, 1997*). There is a huge amount of contributions focused on the improvement of this method (*Fairhead et al., 1994; Stavrev, 1997; Barbosa et al., 1999; Mushayandebvu et al., 2001; Mikhailov et al., 2003; Pašteka, 2004* and many others). One important way to improve the focusing properties of the method is the stabilization of the gradients of the input field, which play an important part in the input of the method. The evaluation of the higher derivatives (gradients) belongs to the so called ill-posed problems of the mathematical physics, because of its very high instability (small perturbations in the input result in great perturbations in the output). There are several ways how to stabilize/smooth the numerically evaluated derivatives of potential fields. Very effective is the concept of the noise separation in the original data by means of Wiener filtering (*Pawlowski and Hansen, 1990*). Another approach is a Fourier-domain low-pass filtering by means of the Gaussian regional filter in the area below the Nyquist wave-number (*Fitzgerald, 2003*, personal communication). The concept of regularized derivatives, introduced by authors of this contribution (*Pašteka and Richter, 2002; Richter and Pašteka, 2003*) can help to stabilize the derivatives calculation. A simple characteristics of the regularized derivatives calculation describes it also as a low-pass filter in the Fourier domain. But the concept of the selection of the optimum parameter of the filter (the regularization parameter) by means of the norm-functions analysis, show some benefits for the user.

The paper presents on a synthetic model and real data application the improvement of the stability and focusing properties of the classical Euler deconvolution algorithm by means of the introduction of the regularized gradients.

2. Euler deconvolution method

The Euler deconvolution method is based on the Euler's theorem for homogeneous functions. The 'potential' of this property of special functions (coming from the basics of the function analysis) was recognized by several geophysicists in the 1950's and 1960's (*Smelie, 1956; Hood, 1965*), but the real introduction into the practice was realized after the publication of fundamental papers from *Thompson (1982*, 2D- profile application) and *Reid et al. (1990*, 3D- grid application).

The basic idea of the method is based on the so called Euler's theorem of homogeneous functions: a function $f(x, y, z)$ is homogeneous of degree n, when it satisfies the following property:

$$f(tx, ty, tz) = t^n f(x, y, z), \tag{1}$$

where t is a real constant. When a homogeneous function has a total differential, then the following equation is valid:

$$x\frac{\partial f}{\partial x}+y\frac{\partial f}{\partial y}+z\frac{\partial f}{\partial z}=nf\left(x,y,z\right). \qquad (2)$$

The mentioned authors (*Smelie, 1956; Hood, 1965; Thompson, 1982*) have recognized that formulae for the direct problem for simple source-types (bodies) in magnetometry and gravimetry are homogeneous functions of form:

$$f\left(x,y,z\right)=\frac{K}{r^N}=\frac{K}{\sqrt{\left(x-x_0\right)^2+\left(y-y_0\right)^2+\left(z-z_0\right)^2}^N}, \qquad (3)$$

where K is a constant (reflecting the physical properties and including basic physical constants), x_0, y_0 and z_0 are the coordinates of the source. Equation (2) can be rewritten:

$$\left(x-x_0\right)\frac{\partial f}{\partial x}+\left(y-y_0\right)\frac{\partial f}{\partial y}+\left(z-z_0\right)\frac{\partial f}{\partial z}=-Nf\left(x,y,z\right). \qquad (4)$$

From this equation it follows that the degree of homogeneity is $n=-N$. N is a very important parameter of the method and it characterizes the type of the source and was named by *Thompson (1982)* as *structural index.* The typical values for different bodies in magnetometry are: 3 (sphere), 2 (horizontal cylinder, pipe), 1 (dike, sill) and 0 (contact). These values in gravimetry are decreased by 1, which is associated with the relationship between gravity and magnetic field (described by the Poisson theorem). This property was generalized and described by *Stavrev (1997).* The role of the value N during the application of the method is very important, it describes the type of the source, whose contribution is recognized in the interpreted data. *Thompson (1982)* defined it as a measure of the rate of change with distance of the potential function – but this property definition holds only for point or line sources (described by rational functions of type K/r^N.

The realization of the method is based on the solution of the equation (4) for unknown x_0, y_0 and z_0. The value of the structural index N is assumed to be known or predicted. The formulation of (4) for several measurement points create a system of linear equations and can be solved in a moving window along a profile (2D modification) or a grid (3D modification). *Thompson (1982)* has found out on practical data (where anomalies are often superposed on a regional trend) that Eq. (4) gives much better results, when it is used in a rewritten format:

$$(x - x_0)\frac{\partial f}{\partial x} + (y - y_0)\frac{\partial f}{\partial y} + (z - z_0)\frac{\partial f}{\partial z} = -N(f - B), \quad (5)$$

where constant B is the so called background term, describing a constant shift (regional field, trend) of the anomalous field. The role of the background term B was generalized by *Pašteka (2004)* – the idea of the interference polynomial from the well-known Werner deconvolution method *(Werner, 1953; Hartman et al., 1971)* was adopted and the right-hand side of Eq. (5) was enlarged by the influence of a polynomial of higher degree:

$$\begin{aligned}(x - x_0)\frac{\partial f}{\partial x} + (y - y_0)\frac{\partial f}{\partial y} + (z - z_0)\frac{\partial f}{\partial z} = \\ = -Nf + A_0 + A_1 x + A_2 y + A_3 xy + A_4 x^2 + A_5 y^2 + ...\end{aligned} \quad (6)$$

where A_0, A_1, A_2, ...are coefficients of the interference polynomial. Introduction of the interference polynomial improves the clustering properties of the method *(Pašteka, 2000; Pašteka, 2004)* and allows to work with non-traditional negative structural index (contact structure in gravimetry) *(Pašteka, 2001, 2005)*. On the other side, its introduction causes a quite intensive growth on the instability of the linear equation system solution *(Pašteka, 2004)*. The instability of the equation system solution is one of the most serious troubles, occurring during Euler deconvolution method application. It can be simply demonstrated on a simple model. We take a case of thin belt fault, which describes a situation, occurring often in sedimentary basins (sub-volcanic sills, disturbed by tectonics, building a small depth shift between two parts of the system – Fig. 1). The correct value of the structural index was derived by *Reid (2003)* and it is equal to 2 for magnetic field (it is very interesting that one separate sill/sheet has $N = 1$). The depth of the upper sheet is 10 m, the lower is in 11 m. In Fig. 1 we can see that the solutions from the classical 2D-Euler deconvolution algorithm (utilizing only the linear background term B) are quite well clustered at the source position of this body-system. The mean depth is – 11.1 m, which is closer to the lower sheet. At present we can not explain this fact, but this error is approximately 11% of the estimated depth, which is in the frame of the usually accepted error interval, when using the semi-automated methods in practice. Important is the fact that the solutions build a well "developed" cluster and there is no false solution. Another situation occurs, when we

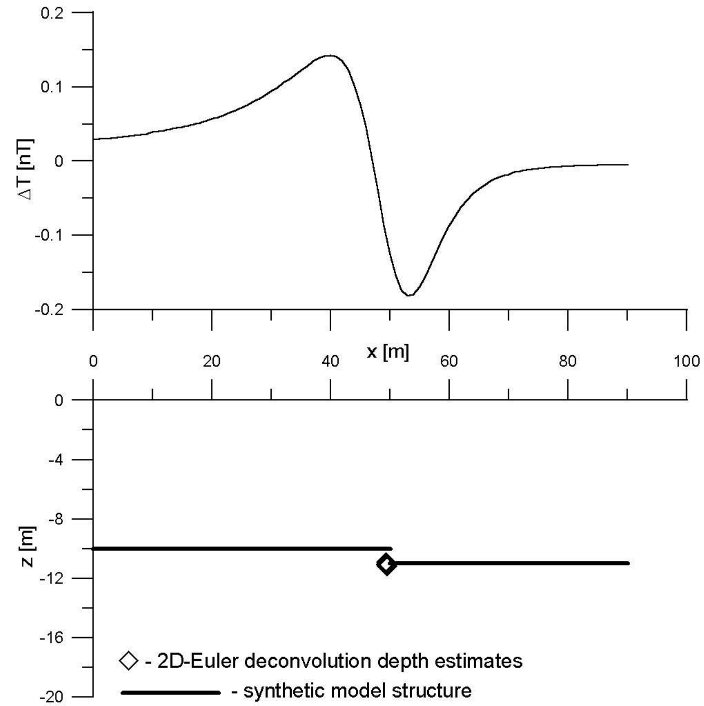


Fig. 1. Results of the 2D-Euler deconvolution algorithm (for $N = 2$) applied on synthetic $\Delta T(x)$ data from the model of a thin belt fault (without additional synthetic normal noise). Clustering of the depth solutions is focused (with a 11% error) in depth of the real source of the anomalous bodies system (window length = 10 m).

apply the same algorithm on $\Delta T(x)$ data with added synthetic normal noise (with the maximum amplitude of 5% of the interpreted anomaly).

The reader can see (Fig. 2) that the obtained solutions are not well clustered in the area of the source (there occur some solutions, but without the knowledge about the source position an interpreter could think that they build only a part of a typical artificial "tail", coming from a shallower source). Important is the occurrence of a relatively great amount of false shallow solutions. In the case of such a simple anomaly from an isolated source there is a straightforward criterion on how to exclude such false solutions – solutions outside the important gradients of the anomaly curve (field) are usually wrong. But when we deal with the interpretation of

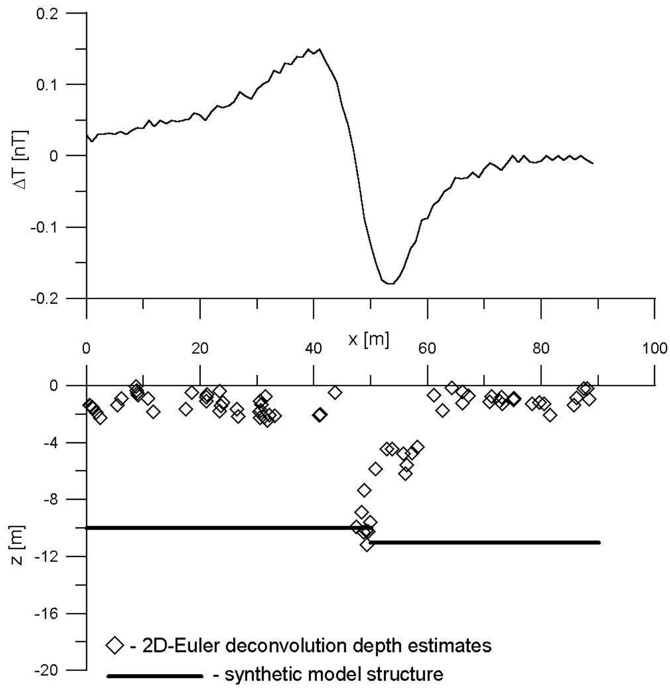


Fig. 2. Results of the 2D-Euler deconvolution algorithm (for $N = 2$) applied on synthetic $\Delta T(x)$ data from the model of a thin belt fault (with an additional 5% synthetic normal noise). Clustering of the depth solutions is not well focused at the depth of the real source of the anomalous bodies system and a large amount of false solutions occur on both sides of the profile (window length = 10 m).

complex anomalies from complicated geological situations, this criterion can not be used.

3. Stabilization of derivatives by means of the Tikhonov regularization

Based on our long year experience with theoretical models and practical applications it follows that the instability of the equation system in the ED algorithm strongly grows after the introduction of disturbed data by synthetic noise (e.g. *Pašteka, 2005*; tests with Gaussian normal noise).

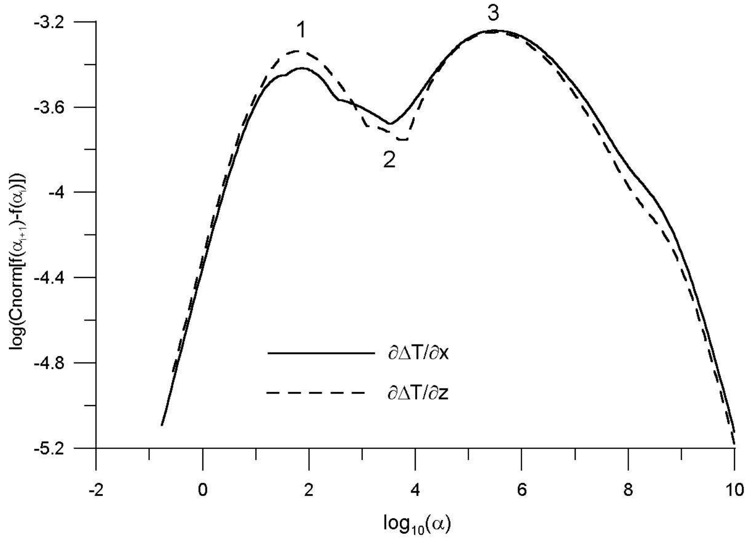


Fig. 3. Norm functions, obtained during the evaluation of regularized x- and z-derivatives in the Fourier domain by means of the Tikhonov's approach. The input $\Delta T(x)$ field was disturbed by an additional 5% synthetic normal noise. The local minimum of the norms (point 2 on the graphs) corresponds to the optimum value of the regularization parameter α. Other local maxima (points 1 and 3) are depicted because of the demonstration of under- and over-regularized solutions.

The influence of the noise and errors in the original input data is strongly emphasized by the derivatives (gradients), which build a part of the input into the interpretation Eq.(4). The numerical evaluation of higher derivatives (in the spatial or spectral domain) is a well know instable operation – small changes in the input signal cause large changes (disturbances, oscillations) in the output signal (in the spectral domain the characteristics of this operations are high-pass filters). This property leads to the breaking of the second of Hadamard's criterions (*Hadamard, 1923*, in *Tikhonov and Arsenin, 1974*), which are set on the so called well-posed problem in mathematical physics. This means that the numerical evaluation of derivatives belongs to the so called ill-posed problems.

Several approaches were developed, how to solve ill-posed problems in mathematical physics – regularization, introduced by Russian authors *Tikhonov and Arsenin (1974)*, is probably most powerful of all approaches.

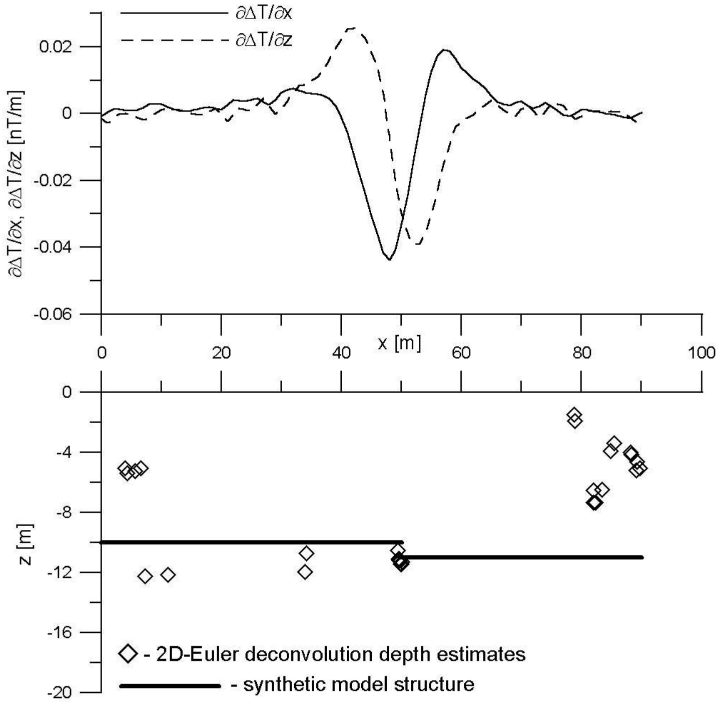


Fig. 4. Results of the 2D-Euler deconvolution algorithm (for $N = 2$) applied on synthetic $\Delta T(x)$ data from the model of a thin belt fault (with an additional 5% synthetic normal noise). The gradients, entering into the method were evaluated by means of the Tikhonov's regularization (for the optimum value of the regularization parameter - taken from the **point 2 in Fig. 3**). Depth solutions are relatively well focused in depth of the real source of the anomalous bodies system. The amount of false solutions was lowered in comparison with the results from non-regularized gradients application (Fig. 2) (window length = 10 m).

The fundamental solution idea of the so called Tikhonov's regularization is based on the solution of the operation as a variational problem. The main functional (to be minimized) of the problem is composed of two partial functionals. The first is the miss-fit functional, describing the classical problem (the inverse transform of the output should be close to the measured/interpreted data). The second (additional) functional describes the stabilizing properties of the solution – it is usually the so called maximum smoothness stabilizing functional (*Zhdanov, 2002*). We can mathemati-

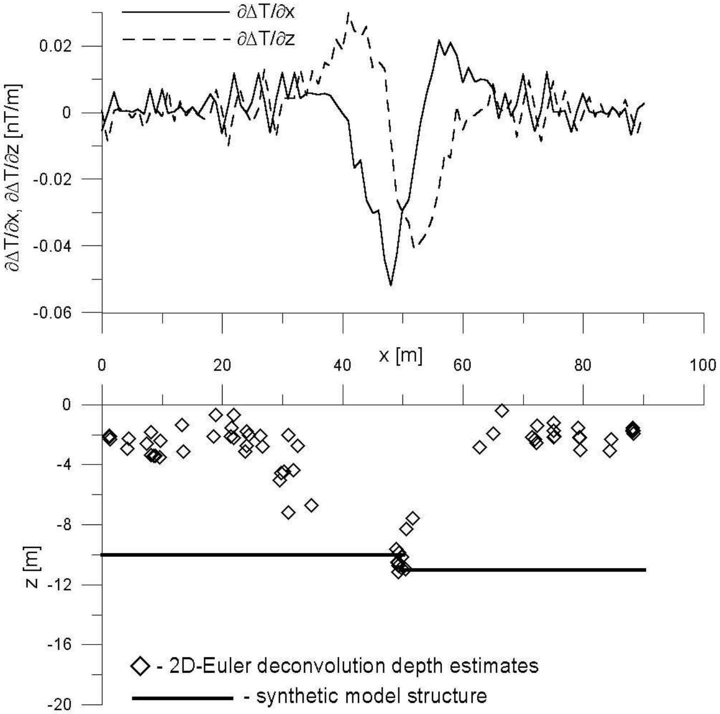


Fig. 5. Results of the 2D-Euler deconvolution algorithm (for $N = 2$) applied on synthetic $\Delta T(x)$ data from the model of a thin belt fault (with an additional 5% synthetic normal noise). The gradients, entering into the method were evaluated by means of the Tikhonov's regularization (for the value of the regularization parameter, taken from the **point 1 in Fig. 3**). Depth solutions are relatively well focused in depth of the real source of the anomalous bodies system, but the amount of false solutions is higher in comparison with the results from (correctly) regularized gradients application (Fig. 4) (window length = 10 m).

cally describe this property by summing (integrating) the squares of the derivatives along the direction of the profile axis (or in two directions, when working with 2D data). Such a sum will reach maximum values for highly distorted (oscillating) field data. The role of both functionals in the solution is "managed" by the so called regularization parameter (α). *Tikhonov et al. (1968)* have shown an elegant solution by means of this approach for the problem of stable analytical continuation downwards. The result of the

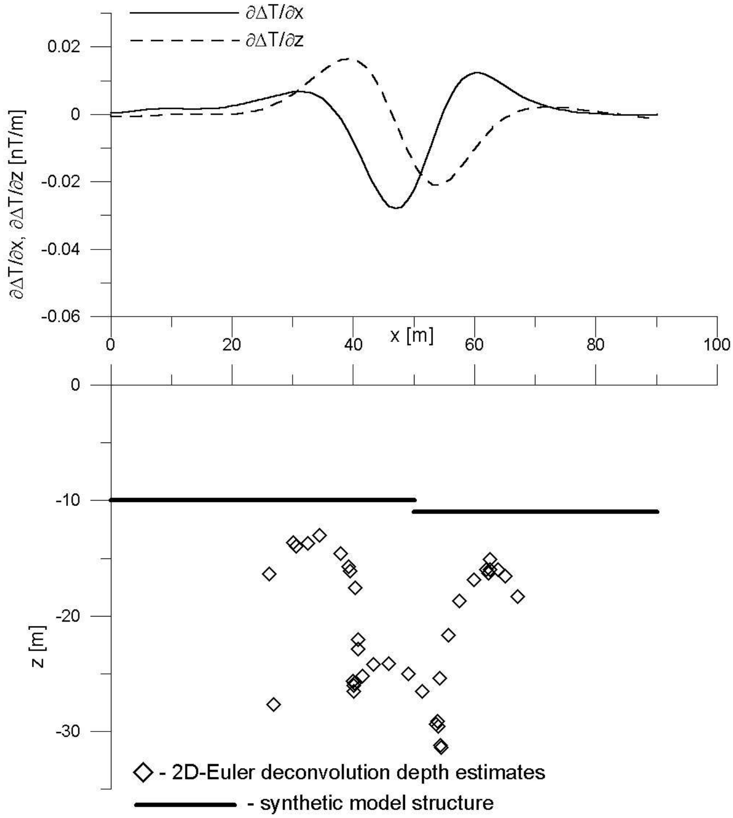


Fig. 6. Results of the 2D-Euler deconvolution algorithm (for $N = 2$) applied on synthetic $\Delta T(x)$ data from the model of a thin belt fault (with an additional 5% synthetic normal noise). The gradients, entering into the method were evaluated by means of the Tikhonov's regularization (for the value of the regularization parameter, taken from the **point 3 in Fig. 3**). ED solutions are too deep and they do not build well "developed" clusters (window length = 10 m).

variational problem in the Fourier domain is a low-pass filter. Its smoothing properties are "managed" by the value of the regularization parameter. The problem of the determination (estimation) of the correct (optimum) value of the regularization parameter is one of the most important tasks during the practical application of this approach. *Tikhonov et al. (1968)* introduced an approach, where a C norm of two following solutions (for

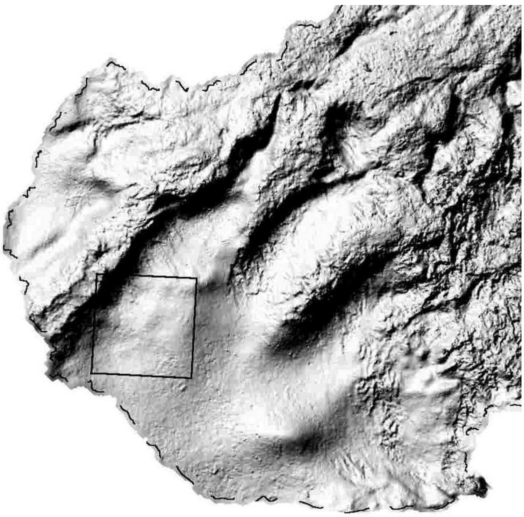


Fig. 7. Shaded relief map of the west-south part of the regional Bouguer gravity anomaly from Slovakia (correction density 2.67 g.cm^{-3}) with the selected area (square) in the west-south part of the Danube Basin region. Data were interpolated into a grid with square step approximately 470 m (64 × 64 points).

various values of the regularization parameter) is evaluated and plotted as a function of the regularized parameter α. This procedure is evaluated for a geometrical sequence of regularization parameter values (typically from 10^{-10} to 10^{+10}). Such a norm-function has usually a specific shape (cf. Fig. 3) – it is a "bulging" function with a local developed minimum. This minimum reflects the area of most stable solutions – small changes of the regularization parameter cause small changes in the evaluated regularized derivatives.

The approach of *Tikhonov et al. (1968)* was transferred to the solution of stable derivatives (*Pašteka and Richter, 2002*) in the Fourier domain, where the Strakhov's (in *Mudretsova and Veselov, 1990*) regularizing low-pass filter was adopted. Second relatively independent approach to the solution of regularized derivatives was presented by Richter (*Richter and Pašteka, 2003*). Comparison of these two independent approaches (*Richter and Pašteka, 2005*) have shown very close results. In this paper we will work with a model situation and practical data application with the Tikhonov's approach in the

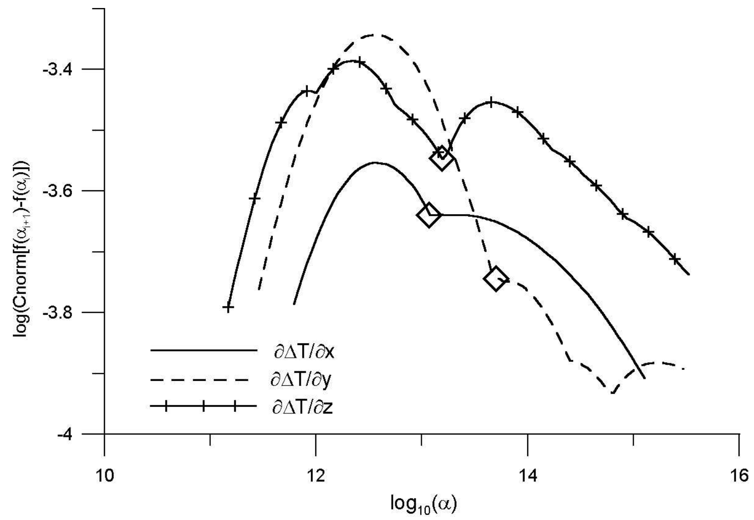
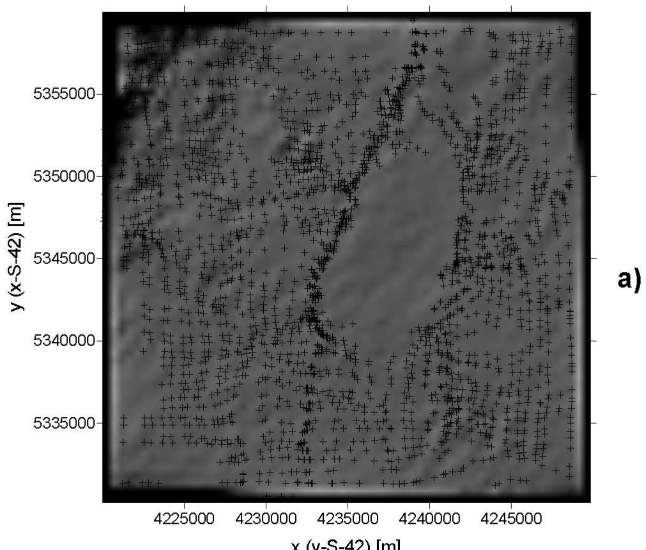


Fig. 8. Norm functions, obtained during the evaluation of regularized x-, y- and z- derivatives in the Fourier domain by means of the Tikhonov's approach, calculated from selected Bouguer anomaly data from Danube lowland. Selected local minima are marked by small squares.

Fourier domain (*Pašteka and Richter, 2002*). In Fig. 3 we can see the C norm-function, obtained for the evaluation of x- and z-derivatives of the $\Delta T(x)$ data with 5% normal noise (displayed in Fig. 2). A well developed local minimum (point 2 in Fig. 3) defines for both norm-functions the *optimum* regularization parameters. ED solutions are displayed in Fig. 4, where we can see that the clustering of the estimates is much better focused in the area of the real source (in comparison with the results from the application of non-regularized gradients – displayed in Fig. 2).

When we will take the values of regularization parameters from two other important extremes of the C norm-function (Fig. 3 – points 1 and 3), we can clearly see that for α from point 1 (lower value than the optimum α) we get the so called *under-regularized* derivatives and the character of the ED soluions (Fig. 5) is very similar to that displayed in Fig. 2. In Fig. 6 we can see the opposite situation – for α from point 3 (higher value than the optimum α) we get the so called *over-regularized* (too smooth) derivatives and the depths of ED solutions are too great (focusing is also very poor).



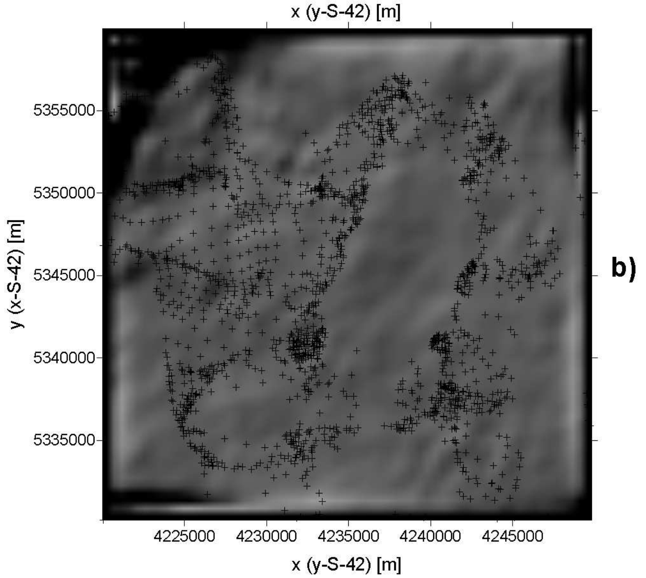


Fig. 9. Results from the 3D-Euler deconvolution algorithm (for $N = 0$) with the non-regularized (a) and regularized derivatives (b) involvement (input field was the selected grid of Bouguer gravity anomalies from the region of west-south Danube lowland). Positions of obtained estimates are plotted with crosses (+). No filtration or clustering of the results were realized. The shaded relief map in the background is the field of the non/regularized z-derivative of Bouguer gravity anomaly (window length $= 10 \times 10$ m).

4. Practical application

For a practical demonstration of the improvement of the Euler deconvolution method an area of a square shape from the regional Bouguer anomaly of Slovak Republic (*Grand et al., 2001*) (correction density 2.67 g.cm^{-3}) was selected (Fig. 7). The shapes of the norm-functions, obtained for this grid during the calculation of all three derivatives (x-, y- and z-derivative) are displayed in Fig. 8. For the final regularized derivatives reconstruction the optimum values from the local minima of the norms (Fig. 8 – these points are marked by small squares) were taken. Some of the local minima are not exact minima (the closest neighbour points do not have greater values in comparison with the "minimum" value), but based on our experiences they can be used for the reconstruction of optimum regularized derivatives. For the norm-functions, obtained during the y- and z-derivative evaluation we got two local minima – this phenomenon is often registered, but at present we have no explanation for it (coming from the influence of two independent sources of errors in the original data?). The optimum value is usually selected from a qualitative analysis of the reconstructed derivatives – usually "average" solutions lying between too noisy (e.g. the first small minimum in the norm-function for the z-derivative) and too smoothed (e.g. the large second minimum in the norm-function for the y-derivative) solutions are selected. We are aware of this ambiguity in our approach and work on the solution of this problem in the future.

Evaluation of the regularized derivatives by means of the presented approach brought desired results (from our point of view). In Fig. 9a we can see the results for the 3D-ED algorithm with non-regularized gradients utilization – there is a large amount on false solutions and a strong influence of the edge effects (artificial sequences of points, parallel to the edges of the map). In Fig. 9b the solutions from regularized gradients involvement show better clustration in the areas of main gradients of the original Bouguer anomaly map. Also the average depth of the solutions, obtained by means of regularized derivations involvement is greater (median: 1986.2 m below the surface) than the non-regularized ones (median: 780.6 m bellow the surface) – here a large influence on the smaller depths have erroneous shallow solutions. The detailed geological interpretations of the obtained solutions (by means of regularized derivatives) would overload the scope of this methodical contribution – it is a topic of a separate publication (interpretation of ED solutions from the regional Bouguer anomaly map from a larger area in West Carpathians region).

5. Conclusions

The introduction of regularized derivatives into the Euler deconvolution algorithm in the frame of the presented study (demonstrated by a simple model and practical data study) brought an improvement of the stability of the received solutions. The solutions are in general better clustered (synthetic model and practical data results) and closer to the real depth (practical data result). The presented results could be, of course, obtained also by a precise manual application of a low-pass filtering procedure on the original data. The involved regularization algorithm helps to select the optimum regularization/smoothing parameter by means of the existence of local minima in the norm-functions. This approach is helpful for interpreters with a lower level of experience and is suitable for automatization (task for future developments). The problem of the selection of the optimum regularization parameter still exists – the occurrence of several local minima of the norm-functions (for practical data) is for us still an opened problem.

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