The overview of precise quasigeoid modelling in Poland

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Abstract: The paper gives an overview of research activity on modelling a precise quasigeoid in Poland, conducted in 2002-2005. It starts with the specification of all available data: terrestrial, marine and airborne gravity data, height data from country-wide GPS/levelling networks, levelling data, deflections of the vertical, tide gauge data, digital terrain models, and lithosphere density data, with their quality assessment. The analysis of performance of different global geopotential models resulted in indicating the most suitable one for a precise quasigeoid modelling in Poland. Quality of terrain corrections calculated for 1 078 046 gravity stations in Poland with the use of the optimum maximum radius of prisms was discussed. The algorithm for calculating mean Faye anomalies was presented and the height data used for computing those anomalies were specified. The GPS/levelling traverse established for quality control of guasigeoid models in Poland was described and the height anomalies of the sites of GPS/levelling control traverse were compared with the corresponding ones computed from the GUGiK 2001 quasigeoid model. Finally the quasigeoid models developed: an astro-gravimetric model, a number of gravimetric and GPS/levelling models, an integrated model and a best-fitted model were described with specifying data used for calculating them and with their quality assessment based on the data from GPS/levelling networks. Brief description of algorithms applied for calculating GPS/levelling quasigeoid models, an integrated quasigeoid model and the best-fitted one was given. An independent assessment of the accuracy of computed quasigeoid models was done with the use of height anomalies at the sites of a GPS/levelling control traverse. The accuracy of quasigeoid models for Poland developed with the use of gravity data is at the level of 2 cm.

Key words: quasigeoid modelling, geopotential models, gravity data, GPS/levelling data, tide gauge data, digital terrain models, density of the upper lithosphere, terrain corrections, mean gravity anomalies

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1. Introduction

The main purpose of geoid determination is the need for a precise geometrical relationship between the Earth surface and the reference ellipsoid that is required for geodetic, geophysical and oceanographic applications. The knowledge of the precise geoid model became substantial in the last decades due to the extended use of precise navigation techniques in surveying, in particular for heighting. Precise modelling of regional geoid became one of the major tasks of numerous research groups and surveying and mapping agencies.

An increase of coverage with deflections of the vertical and with gravity data in the first decades of 20th century resulted in developing regional geoid models (e.g. *Hirvonen, 1934; Tanni, 1948*). The first gravimetric geoid model for the region of Central Europe, including Poland was computed by *Tanni (1949)*. Its accuracy has been estimated as 6–12 m *(Tanni, 1949)*. At the same time the extensive efforts towards data acquiring and regional geoid modelling were taken in Poland. The first regional astro-gravimetric geoid model for Poland of accuracy of about 60 cm was developed at the Institute of Geodesy and Cartography in Warsaw in 1961 *(Bokun, 1961)* from about a hundred of astro-geodetic deflections of the vertical and gravity obtained from the gravity maps. That model was refined in 1970 with the use of more data and more detailed gravity maps. Its accuracy was about 30 cm. The release of gravity data in Poland, as well as progress in computing technology resulted in further advance in regional geoid modelling in Poland. The first gravimetric quasigeoid model for Poland of accuracy of about 10 cm was calculated at the Department of Planetary Geodesy in Warsaw in 1993, using the least squares collocation combined with the integral method *(Lyszkowicz, 1993)*. It was followed by the quasi97b gravimetric quasigeoid model of 5 cm accuracy, developed with the FFT technique using substantially extended gravity data coverage *(Lyszkowicz, 1998)*.

The access to raw gravity data form geological gravity database, development of high-resolution digital terrain models, densification of precise GPS/levelling heights, and re-levelling of Polish vertical control network stimulated the efforts towards undertaking an extensive research on modelling a precise quasigeoid in Poland. The team of researchers from national and foreign research centres, representing different disciplines of Earth sciences, under the leadership of the Institute of Geodesy and Cartography, Warsaw, was conducting in 2002-2005 an advanced research on modelling a centimetre quasigeoid in Poland with the use of geodetic, gravimetric, astronomic, geological and satellite data, in the framework of the project ordered and supported by the Polish Committee for Scientific Research *(Krynski, 2005a)*.

The data available needed to be transformed to unified reference systems in accordance with recent standards, and archived in the respective databases. Transformation concerned, in particular, a large gravity data set (over a million) from Poland, acquired within last 50 years, provided for the project by Polish Geological Institute. The use of the existing data for precise quasigeoid modelling, its methodology and estimation of accuracy of the determined quasigeoid models required an extensive research. It concerned the analysis of digital terrain models, technology of terrain correction computation, methodology of mean gravity anomaly determination, methodology of mean sea level of the Baltic Sea determination and the choice of global geopotential model best suited for quasigeoid modelling in Poland. Where necessary, some astronomic, geodetic and GPS control surveys were performed. To assure the quality control of quasigeoid models the 868 km long control GPS/levelling traverse running from SW part of Poland to NE borders, and consisting of 190 stations on levelling benchmarks, precisely surveyed with GPS, was established. The paper gives a brief overview of the research activity on modelling a precise quasigeoid in Poland in 2002-2005.

2. Input data

All available data, i.e. gravity data, deflections of the vertical, precise GPS/levelling data, tide gauge data, digital terrain models and lithospheric density data have been gathered and extensively qualitatively and quantitatively analysed. The methods of transforming data acquired in different epochs using different techniques, to unified standards, scales and reference systems have been developed. They concerned geodetic datum, gravity systems, gravity units, star catalogues, Earth rotation parameters, time scales, observation epochs and gravity reductions. The data was archived and appropriate databases were developed.

2.1. Gravity data

Gravity data available in the area of interest were not uniform both in terms of quality and coverage (Fig. 1). Terrestrial gravity data acquired within last 50 years consist of 1 089 062 point gravity data ($\sigma = 0.075$ mGal) almost uniformly distributed over Poland as well as point and mean gravity anomalies ($\sigma = 1.0$ mGal) of different spatial resolution from neighbouring countries. Marine gravity data consist of ship-borne Petrobaltic data ($\sigma = 0.5$ mGal) from the southern part of the Baltic Sea, up to 100 km from the coastal line, acquired in 1978-1980 by former USSR research team, data acquired in coastal zone of Poland during the geophysical missions of Zaria and Turlejski vessels in 1971 and 1972 ($\sigma = 2.0$ mGal), as well as data

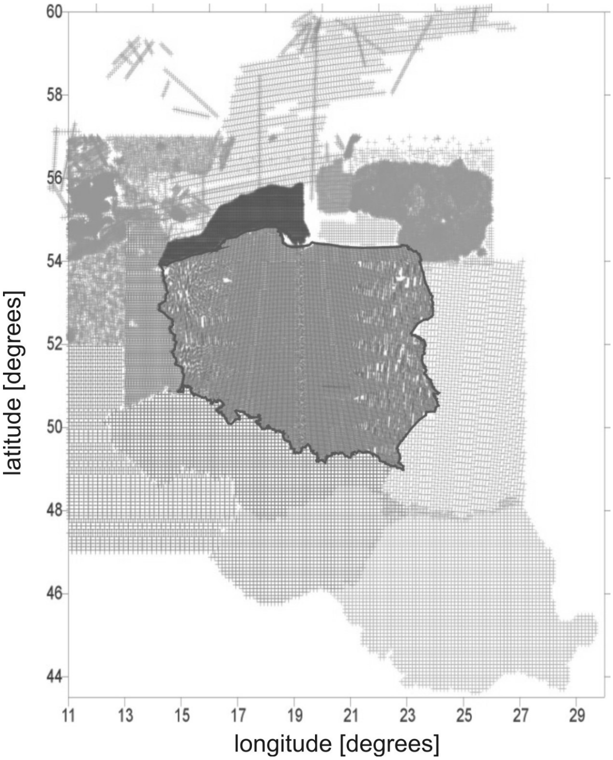

Fig. 1. Distribution of terrestrial and marine gravity data. Data sets including those from different marine gravity surveys in the Baltic Sea are distinguished with different shades and patterns.

from the Swedish coastal area of Southern Baltic Sea, acquired in 1999 by the Norwegian Hakoon Mosby vessel (provided by KMS) ($\sigma = 2.6$ mGal). Airborne gravity data ($\sigma = 1.5$ mGal) acquired in 1999 within the project carried out by the KMS cover the southern part of the Baltic Sea. Also $2' \times 2'$ mean gravity anomalies ($\sigma = 4.0$ mGal) derived at the KMS from the Geosat and ERS-1 altimetry data were used for quasigeoid modelling.

2.2 GPS/levelling data

Precise GPS/levelling heights consist of the POLREF, EUVN and WSSG (Military Satellite Geodetic Network) sites counting all together above 900.

The POLREF network (Fig. 2a) that is a densification of EUREF-POL92 network (11 Polish stations linked in 1993 to ETRS89) consists of 360 sites surveyed in two 4h sessions each, in three campaigns from July 1994 to May 1995 *(Zielinski et al., 1997)*. Stations of the POLREF network were linked to the national vertical control by spirit levelling (Kronstadt86 datum), with standard deviation of normal height equal to 1.0-1.5 cm, standard deviation of ellipsoidal height (GRS80 ellipsoid) 1.0-1.5 cm, and standard deviation of height anomaly equal to 2 cm (optimistic estimate).

The EUVN 1999 network (Fig. 2b) that is a densification of EUVN 1997 network (10 Polish stations linked in 1999 to ETRS89) consists of 52 sites on vertical control benchmarks surveyed in two 24h sessions each, during the campaign in 1999 *(Jaworski, 2000)*. Standard deviations of calculated station coordinates equal $\delta\phi = 0.19$ cm, $\delta\lambda = 0.22$ cm, $\delta\mathrm{h} = 0.28$ cm, and standard deviation of height anomaly equals approximately 2 cm (optimistic estimate).

The WSSG network (Fig. 2c) that is a densification of WPSG network (53 stations of Military Geodetic Control linked in 1993 to ETRS89) consists of 534 sites surveyed in two 3h sessions each, in three campaigns from October 1994 to June 1996. Stations of the WSSG network were linked to the national vertical control by spirit levelling (Kronstadt86 datum), with standard deviation of calculated station coordinates equal $\delta\phi = 2$ cm, $\delta\lambda = 2$ cm, $\delta\mathrm{h} = 3$ cm, standard deviation of ellipsoidal height (GRS80 ellipsoid) 1.0-1.5 cm, and standard deviation of height anomaly equal to 3.5 cm (optimistic estimate).

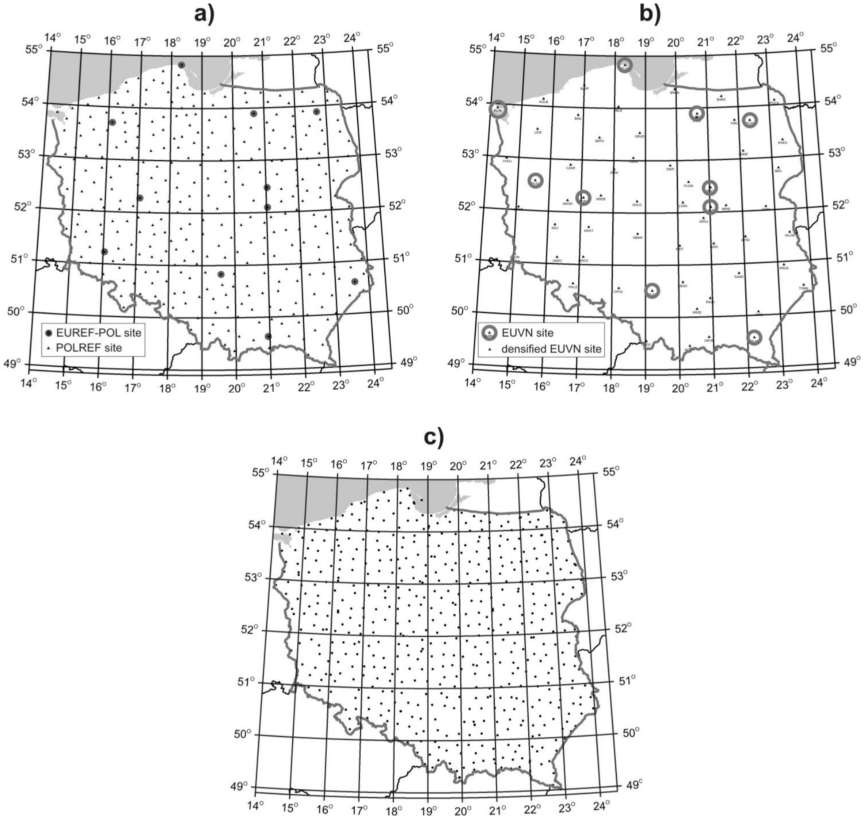

Fig. 2. Sites of the POLREF (a), EUVN (b), and WSSG (c) networks.

2.3. Levelling data

The re-levelling of the 1st order vertical control in Poland (Fig. 3) with 382 lines of total length of 17 516 km has started in May 1999 and was completed in June 2002. Based on the differences between back and fore levelling of a section (about 16 000 sections surveyed) the mean square error of levelling equals to ±0.278 mm/km$^{1/2}$. Random and systematic errors of levelling estimated using Lallemand formula are ±0.264 mm/km$^{1/2}$ and ±0.080 mm/km, respectively *(Gajderowicz, 2005).* Normal heights determined in preliminary adjustment of the network in the mode of free network adjustment fit very well to those from the previous levelling campaign in

Fig. 3. Network of levelling control surveyed in 1999-2002.

1974–1982. Estimated standard error of observation of unit weight equals to ±0.088 mm/km and coincides with the one obtained in the previous campaign.

2.4. Deflections of the vertical

The set of the available deflections of the vertical in Poland (Fig. 4) consists of 165 astro-geodetic deflections acquired in 1952-1975 ($\sigma = 0.5''$) and

Fig. 4. Distribution of astro-geodetic and astro-gravimetric deflections of the vertical.

370 astro-gravimetric deflections of the vertical determined in the 1960s ($\sigma = 0.7''$) *(Rogowski et al., 2005)*. Additional new astronomical observations at 29 points were performed with circumzenithal provided by the Slovak University of Technology, Bratislava, from May 2003 until July 2004 with standard deviation $\sigma = 0.3'' - 0.5''$.

2.5. Tide gauge data

Data from 25 tide gauges from Danish, Finnish, German, Polish and Swedish coast line, that took part in the Baltic Sea Level Project has been acquired from the database of the Permanent Service of Mean Sea Level (monthly and annual means). They were supplemented with data from Gdansk, Hel, Kolobrzeg tide gauges, provided by the division of the Institute of Meteorology Water Management, Gdynia, and from Kronstadt tide gauge *(Bogdanow et al., 1994)* (Fig. 5).

Fig. 5. Baltic Sea tide gauges.

2.6. Digital terrain models

Three DTMs: DTED2, SRTM3 and SRTM30 were used in the project on precise quasigeoid modelling (Fig. 6).

The DTED (Digital Terrain Elevation Data) digital terrain model elaborated in Poland according to the NATO-STANAG 3809 standard (NGA, 1996) covers completely the territory of Poland (between 49° – 55°N and 14° – 24°E) and is available at two resolution levels: level 1 of $3'' \times 3''$ resolution, and level 2 (DTED2) of $1'' \times 1''$ ($49° - 50°$N) and $1'' \times 2''$ ($50° - 55°$N) resolution. The horizontal datum of the model is WGS84 while the elevations are referred to MSL. Estimated vertical accuracy of the DTED2 varies from 2 m to 7 m, while its horizontal accuracy equals to 15 m *(Krynski et al., 2005c).*

The SRTM (Shuttle Radar Topography Mission) data, acquired during the 11-day mission in February 2000, covers land areas over nearly 80%

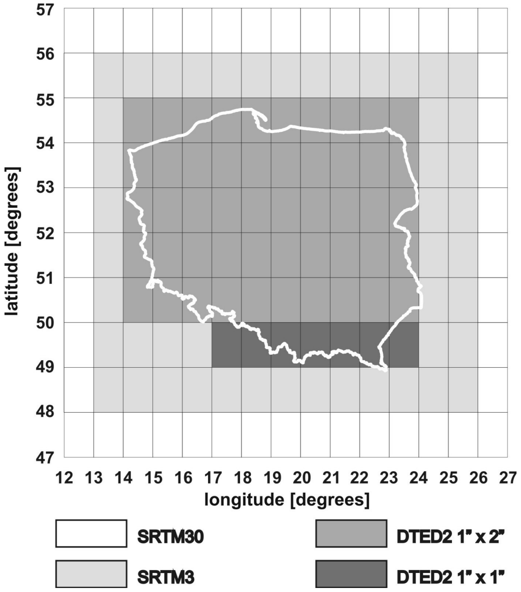

Fig. 6. Coverage of Poland and neighbouring regions with the three DTMs: DTED2, SRTM3 and SRTM30.

of Earth's land surfaces between 54° south latitude and 60° north latitude *(Bamler, 1999)*. As a product of radar interferometry survey the SRTM provides elevations that are not always referred to actual ground level. Both natural and human-made land coverage affect the model *(Showstack, 2003)*. The SRTM3 model of $3'' \times 3''$ resolution, released to public in 2004, is a preliminary one as not yet fully consistent with map accuracy standards *(JPL, 2004)*. The horizontal datum of the model is WGS84 while the elevations are referred to EGM96 geoid. The absolute vertical accuracy is specified as 16 m and the absolute horizontal accuracy is 20 m *(Bamler, 1999; JPL, 2004)*. The SRTM30 model of $30'' \times 30''$ resolution is a generalization of the SRTM3 model.

2.7. Lithosphere density data

Since 1945 an extensive research on the distribution of density of the upper lithosphere in Poland is conducted by the Polish Institute of Geology.

The actual database consists of 1363 sites (1/230 km^2) where the density of the lithosphere above the sea level was determined (Fig. 7a) *(Krolikowski and Zoltowski, 2004)*. Density at 419 sites were determined as the result of laboratory measurements, at 901 sites were evaluated, and at 43 sites were based on geophysical well logging data. Evaluated densities of the upper lithosphere in Poland vary within the range 1.71-2.76 g/cm^3 and their standard deviation equals to 0.15 g/cm^3. Mean density in Poland equals to 2.17 g/cm^3. Map of density distribution in Poland is given in Fig. 7b *(Polechonska and Krolikowski, 2005)*.

3. Suitability of global geopotential models

The choice of the global geopotential model (GGM) used in the remove-restore technique for the determination of a regional quasigeoid from gravity data may affect the solution, in particular when the accuracy is supposed to reach a centimetre level. GGM plays also an important role in validating height anomalies at GPS/levelling sites that are used for the estimation of the external accuracy of quasigeoid models. Six different GGMs (Table 1) were investigated *(Krynski and Lyszkowicz, 2005a, 2005b)*.

Three kinds of numerical tests with the use of terrestrial gravity data and GPS/levelling height anomalies were conducted. The first one concerned the comparison of height anomalies at GPS/levelling POLREF sites with corresponding values computed from various GGMs (Table 2 and Fig. 8). Similar statistics was obtained when using the heights of the EUVN sites *(Krynski and Lyszkowicz, 2005a, 2005b)*.

Height anomalies computed from the EGM96 exhibit bias larger by almost 20 cm than the other obtained from the remaining GGMs, based on CHAMP and GRACE mission data (Table 2 and Fig. 8).

In the second test, the terrestrial gravity anomalies were compared with corresponding gravity anomalies computed from GGMs (Table 3 and Fig. 9).

Finally, the quasigeoid models obtained from gravity data with the use of different GGMs were verified against corresponding height anomalies at the sites of GPS/levelling networks in Poland. High-resolution GGM02S/EGM 96 GGM that was derived as a combination of GRACE-based data with the EGM96 model fits best the height anomalies at POLREF and EUVN GPS/

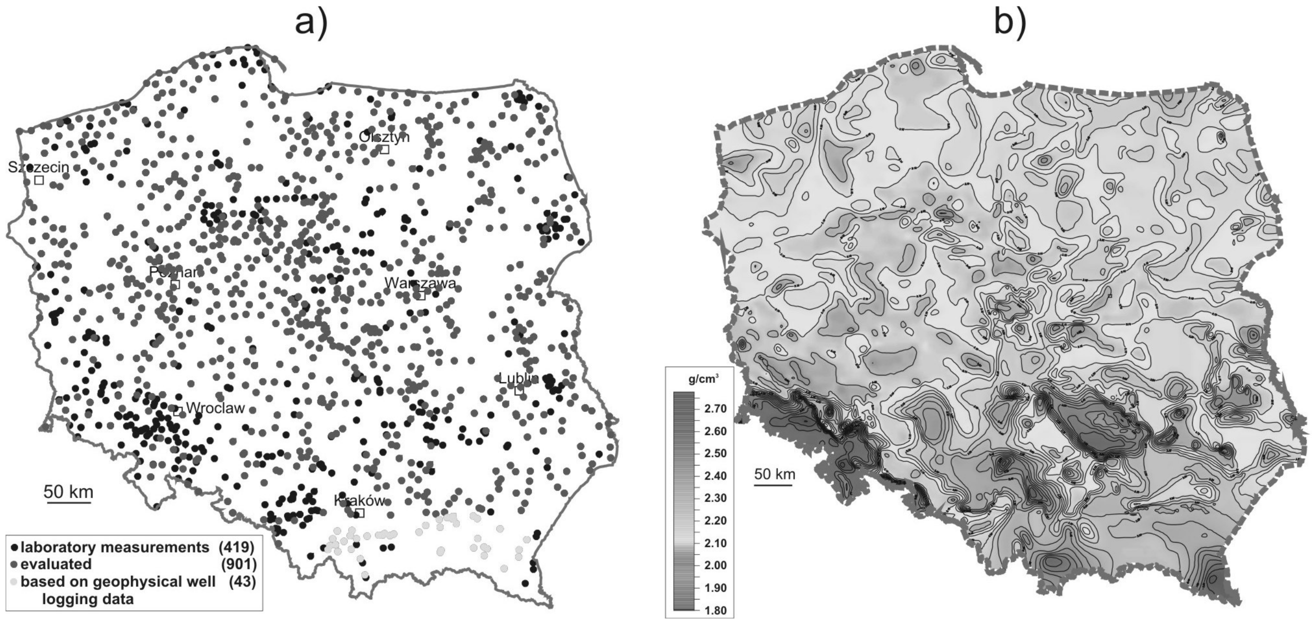

Fig. 7. Sites where the density of the lithosphere above the sea level was determined (a), and the map of density distribution (b).

Table 1. Investigated global geopotential models

Model	Degree	Type
EGM96	360	combined
EIGEN-CH03S	140	satellite only
GGM01S	120	satellite only
GGM02S (140)	160	satellite only
GGM02C	200	combined
GGM02S/EGM96	360	combined

Table 2. Statistics of the differences between height anomalies computed from GGMs and the corresponding values derived from GPS/levelling at the POLREF sites ($\zeta_{GPS/lev} - \zeta_{GGM}$)

Model	Mean	Std dev.	Min	Max
EGM96	-0.53	0.19	-1.03	0.08
EIGEN-CH03S	-0.33	0.76	-2.22	1.06
GGM01S	-0.36	0.46	-1.70	1.05
GGM02S (140)	-0.34	0.47	-1.53	1.23
GGM02C	-0.35	0.26	-1.09	0.49
GGM02S/EGM96	-0.37	0.13	-0.79	0.05

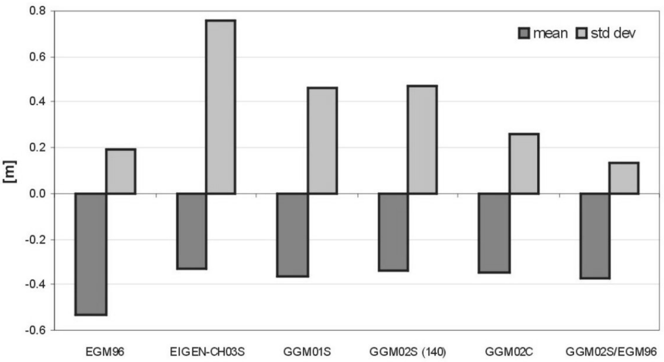

Fig. 8. The means and standard deviations of the differences between height anomalies computed from GGMs and the corresponding values derived from GPS/levelling at the POLREF sites ($\zeta_{GPS/lev} - \zeta_{GGM}$).

levelling sites. It also fits best the terrestrial gravity data in Poland, although in this case the fit of the EGM96 is almost equally good *(Krynski and Lyszkowicz, 2005a).*

Table 3. Statistics of the differences between gravity anomalies computed from GGMs and the respective values derived from terrestrial and marine gravity survey ($\Delta g_{grav}-\Delta g_{GGM}$) [mGal]

Model	Mean	St. dev.	Min.	Max.
EGM96	-0.18	9.39	-112.01	137.34
EIGEN-CH03S	0.00	17.30	-111.42	182.54
GGM01S	0.26	15.37	-109.94	166.07
GGM02S (140)	-0.14	14.81	111.89	157.57
GGM02C	-0.20	12.44	-115.57	153.86
GGM02S/EGM96	-0.30	9.31	-115.56	135.44

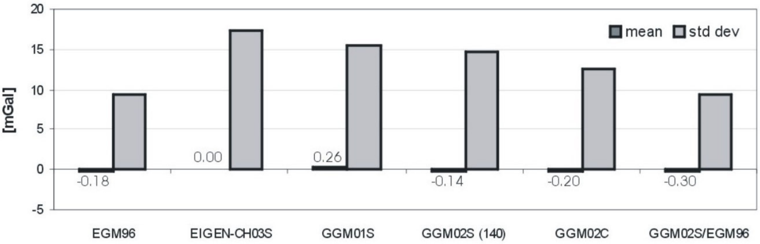

Fig. 9. The means and standard deviations of the differences between gravity anomalies computed from GGMs and the respective values from terrestrial and marine gravity survey ($\Delta g_{grav}- \Delta g_{GGM}$).

4. Terrain corrections

Quasigeoid modelling with a centimetre accuracy requires taking into account irregularities of topography in the vicinity of a gravity station, i.e. the terrain correction to surveyed gravity. The accuracy of the determination of the terrain correction affects the quality of the quasigeoid model. It depends mainly on the resolution and accuracy of terrain data that usually is provided in the form of a digital terrain model DTM.

In the gravity database for Poland, established in 1974-1992 by the Polish Geological Institute for geological and geophysical prospective, the terrain corrections were computed only at the stations where the slopes exceeded 6° within the radius of 100 m from the station. The terrain corrections were calculated within the radius of 22.5 km. In the inner zone – up to 100 m – the approach of the circular diagram of Lukavtchenko based on measured terrain slope angles was used *(Krolikowski, 2004)*. In the intermediate zone ranging from 100 m to 1500 m, the Kane's approach *(Kane, 1962)* based on mean heights for 200 m × 200 m sectors, determined from the maps at the scale of 1:25 000 was applied. In the outer zone, ranging from 1.5 km to 22.5 km, the terrain correction was calculated using the Bott's method *(Bott, 1959)* based on mean heights for 1 km × 1 km sectors.

Precise quasigeoid modelling requires an extensive investigation on the procedure of the terrain correction computation. Elevation data provided by the DTED2, SRTM3 and SRTM30 as well as heights from the gravity database was considered. The prism method of determination of terrain corrections was applied in the majority of numerical tests. The size of the area covered by elevation data used for computing the terrain corrections, e.g. the radius d of integration of prisms, depends on the required accuracy. It dramatically decreases when lowering the accuracy demand towards the computed terrain corrections. Numerical experiments with computing the terrain corrections using the prism method based on high resolution DTMs enabled to precisely determine the optimum maximum radius d of integration of prisms for a required accuracy of the solution (Table 4). Practical method for determining the optimum radius $d$of the integration cap considering roughness of topography $\sigma\Delta h$ relative to a gravity station (it might be the standard deviation of differences between the elevations of topography within the radius d and the elevation of a gravity station) as well as required accuracy of terrain corrections was developed *(Grzyb et al., 2006)*.

Table 4. The optimum maximum radius d [km] for different roughness of topography $\sigma\Delta h$

Accuracy of the terrain correction [mGal]	The optimum maximum radius d [km]		
	$\sigma\Delta h = 15$ m (flat)	$\sigma\Delta h = 50$ m (hilly)	$\sigma\Delta h = 300$ m (mountainous)
0.1	14.9	205	280
0.2	0.22	118	265
0.3	0.06	33	249

The results of the investigations were used for determining the strategy of computation of the terrain corrections to point gravity data in the gravity database for Poland. Considering the roughness of topography in Poland, the optimum radius $d = 200$ km was chosen. The "2005" terrain correction set has been calculated for 1 078 046 gravity stations (Fig. 10a). Maximum

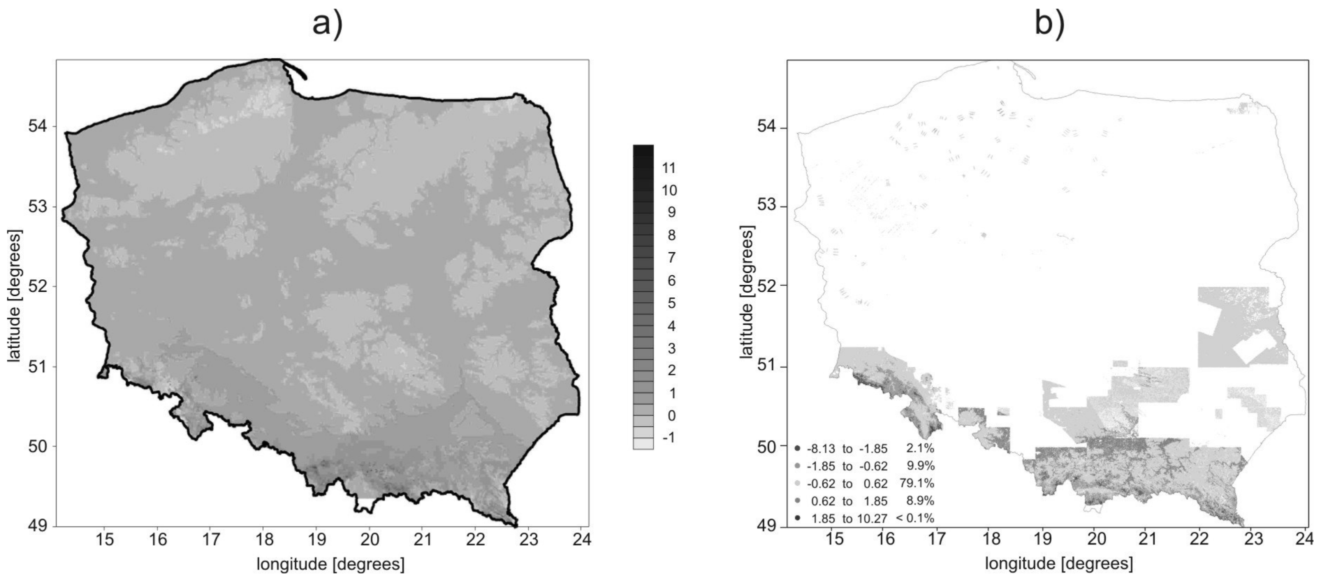

Fig. 10. The map of "2005" terrain corrections calculated at all gravity stations from the gravity database for Poland [mGal] (a), and the differences between the "1992" and "2005" terrain corrections ("2005" – "1992") [mGal] (b).

terrain correction in Poland – excluding the region of Tatra Mountains, where no terrain corrections were computed because of a lack of gravity stations there – reaches 22.3 mGal. Only 10% of all calculated terrain corrections exceed 0.5 mGal, and 3% are larger than 1 mGal. The "1992" terrain corrections existing in the gravity database were compared with "2005" terrain corrections (Fig. 10b) *(Grzyb et al., 2006)*. The statistics of differences between corresponding terrain corrections are given in Table 5.

Table 5. Statistics of differences between "1992" terrain corrections descended from the gravity database and the "2005" terrain corrections ("2005" – "1992") [mGal]

Number of stations	Min	Max	Mean	Std dev.
288 507	-8.135	10.260	-0.050	0.616

Height data available in Poland seem to be sufficient in terms of resolution for computing the terrain corrections in most areas (over 80%) with an accuracy required for a centimetre quasigeoid modelling. There is no need to use the DTM as dense as the DTED2 model for computing the terrain corrections with accuracy better than 0.1 mGal. In that case a DTM of resolution 100 m × 100 m seems quite sufficient. The use of the DTED2 model in hilly regions ensures the determination of terrain corrections with accuracy of 0.1 mGal. To reach, however such accuracy in the determination of terrain corrections in mountainous regions a DTM of higher resolution is needed *(Grzyb et al., 2006)*.

5. Mean gravity anomalies

The gravimetric quasigeoid model is determined with the use of free-air anomalies or, when available, Faye anomalies that include the terrain corrections. Point gravity data set is usually transformed into the equivalent set of mean gravity anomalies that are used for computing of quasigeoid heights with Molodensky's integral formulae in the discrete form. Both free-air and Faye anomalies are strongly correlated with elevations. Mean anomalies additionally depend on the number and distribution of point gravity data. In Poland there is 1-6 point gravity data per km^2 *(Krolikowski, 2004)*. The process of computation of mean anomalies involves Bouguer anomalies or isostatic anomalies that are smoother and less correlated with topography. Their use results in the reduction of the interpolation error.

$1' \times 1'$ mean Faye anomalies for Poland were generated using Bouguer anomalies *(Krynski et al., 2005b).* They were calculated starting from point gravity data according to the following algorithm:

1. calculation of point free-air anomalies Δg_P^F

   $$\Delta g_P^F = g_P + \delta g_P^F - \gamma_{P_0} \quad \text{where} \quad \delta g_P^F = 0.3086 \times h_P \tag{1}$$

2. calculation of point Faye anomalies Δg_P^{Faye}

   $$\Delta g_P^{Faye} = \Delta g_P^F + c_P \tag{2}$$

3. calculation of point Bouguer anomalies Δg_P^B

   $$\Delta g_P^B = \Delta g_P^{Faye} - 2\pi G\rho\, h_P \tag{3}$$

4. interpolation of point refined Bouguer anomalies (Δg_i^{int}) and heights (h_i^{int}) on the grid of higher resolution than the resulting grid of mean Faye anomalies

5. calculation of mean Bouguer anomalies $\overline{\Delta g}^B$ in $1' \times 1'$ blocks

   $$\overline{\Delta g}^B = \frac{1}{\sigma}\sum_i \Delta g_i^{int}\Delta\sigma \tag{4}$$

6. calculation of mean heights $\overline{h}$ in $1' \times 1'$ blocks

   $$\overline{h} = \frac{1}{\sigma}\sum_i h_i^{int}\Delta\sigma \tag{5}$$

7. calculation of mean Faye anomalies $\overline{\Delta g}^{Faye}$ in $1' \times 1'$ blocks

   $$\overline{\Delta g}^{Faye} = \overline{\Delta g}^B + 2\pi G\rho\overline{h} \tag{6}$$

where P – surface point, P_0 – projection of P on the ellipsoid, γ – normal gravity, c – terrain correction, G – gravitational constant, ρ – density of the upper lithosphere, σ – area of the block.

The concept of calculating the mean anomalies is illustrated in Fig. 11. $1' \times 1'$ mean Bouguer anomalies and Faye anomalies for Poland were calculated using heights h_P of the gravity points from the gravity database, $\rho = 2.67$ g/cm^3, and computing mean heights $\overline{h}$ in $1' \times 1'$ blocks from the SRTM3 model of $3'' \times 3''$ resolution *(Krynski et al., 2005b)*.

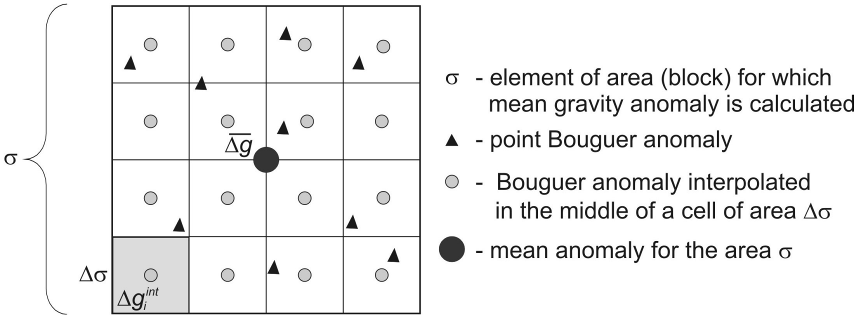

Fig. 11. The concept of calculating mean gravity anomalies.

6. The GPS/levelling control traverse

For the verification of the developed quasigeoid models, as well as for the estimation of their accuracy and evaluation of interpolation algorithms used for application of the GPS/levelling quasigeoid, a 868 km long control GPS/levelling traverse across Poland has been established (Fig. 12) *(Krynski et al., 2004)*.

The traverse of 868 km surveyed in 2003 and 2004 in 5 campaigns consists of 190 stations (in average 1 station per 4.6 km) with precisely determined ellipsoidal and normal heights. The stations are located at the benchmarks of the 1st or 2nd order vertical control, or in their close vicinity. The observation strategy and processing methodology ensure the accuracy of quasigeoid heights at the traverse points on a centimetre level. The 49 stations of the traverse, considered as the 1st order control, were surveyed in one or

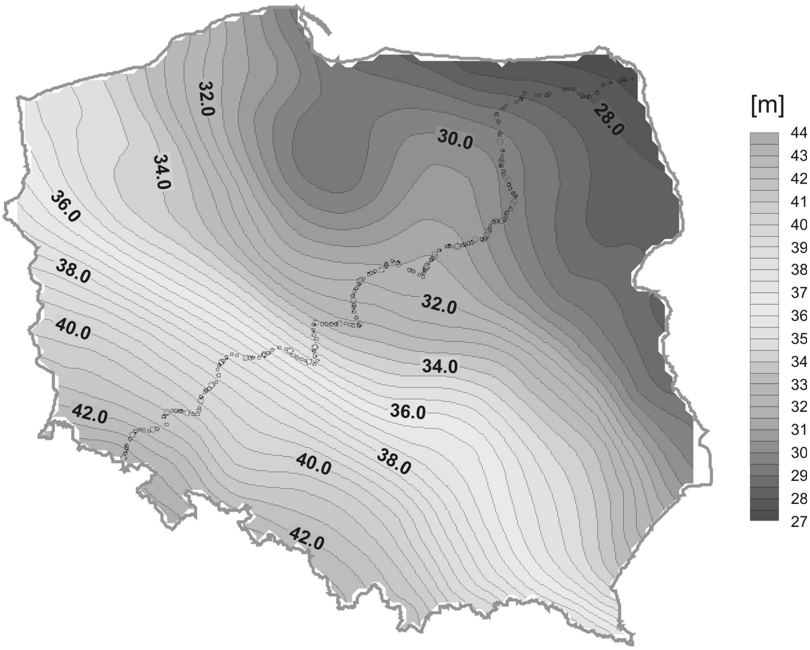

Fig. 12. The control GPS/levelling traverse.

two 24h sessions. The remaining 141 stations, considered as densification points, were surveyed in 4h sessions. The coordinates of 49 1st order control stations were determined using the EPN strategy with the Bernese v.4.2 program. The accuracy of the determined coordinates is at the level of single millimetre for the majority of stations. The coordinates of densification points were calculated using the Pinnacle program with the 1st order control stations as reference *(Krynski et al., 2005a; Cisak and Figurski, 2005).* Differences between height anomalies of the GPS/levelling control traverse and the corresponding ones computed from the GUGiK 2001 quasigeoid model are shown in Fig. 13.

7. Quasigeoid models developed

A variety of available data made possible to develop a number of different quasigeoid models: astro-gravimetric quasigeoid model, numerous

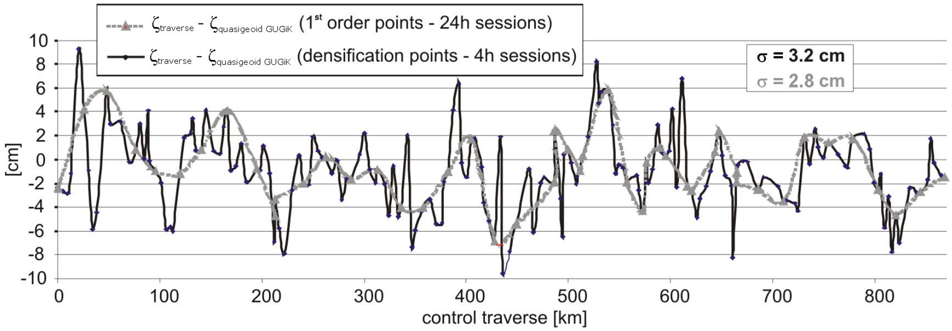

Fig. 13. Deviations of height anomalies of GPS/levelling control traverse from the GUGiK 2001 quasigeoid.

gravimetric quasigeoid models, GPS/levelling quasigeoid models as well as the integrated (combined) quasigeoid model and the best-fitted quasigeoid model.

7.1. Astro-gravimetric quasigeoid model

The astro-gravimetric quasigeoid model was developed using the principle of astronomical levelling. The network consisting of 384 astro-gravimetric deflections of the vertical has been fitted to the network of 197 astro-geodetic deflections of the vertical. The statistics of standard deviation of the adjusted quasigeoid undulations are given in Table 6.

The map of the adjusted quasigeoid heights as well as fitting correction (after removing the bias) of the astro-gravimetric quasigeoid to the quasi97 gravimetric quasigeoid model are shown in Fig. 14.

Table 6. Statistics of standard deviation of adjusted quasigeoid undulations [m]

σ_{max}	σ_{av}	σ_{min}
0.225	0.065	0.000

7.2. Gravimetric quasigeoid models

Gravimetric quasigeiod models were calculated using the remove-restore strategy, generally adopted for the determination of the regional quasigeoid.

Fig. 14. The map of the adjusted quasigeoid undulation (a), and fitting the astro-gravimetric quasigeoid to the quasi97 gravimetric quasigeoid model (b).

Height anomaly ζ is expressed as a sum of three components that represent global, regional and local effects, respectively. The first component is calculated using the coefficients of the global geopotential model, the second one – using mean Faye anomalies Δg_F, and the third one – using topography (heights H) *(Forsberg, 2005).* The integral representing the second component was evaluated in the frequency domain by the multi-band fast Fourier Transform. Proper zero padding (100%) was applied to the gridded data to eliminate the effects of circular convolution *(Krynski and Lyszkowicz, 2006).*

The consecutive models of gravimetric quasigeoid (Table 7) have been calculated with the use of various GGMs (Table 1) and six gravity data sets getting improved in the process of conducting the project:

1. non-unified gravity data covering the area of $47° < \phi < 57°$ and $11° < \lambda < 26°$, used for computing quasi97b quasigeoid model; data from Poland consist of $1' \times 1'$ mean free-air anomalies calculated by the Polish Institute of Geology;
2. gravity data (1) (excluding data from Bornholm), with the rough upgrade of data from Poland; $1' \times 1'$ mean free-air anomalies from (1) were corrected to the POGK-99 gravity system and from Borowa Gora datum to the ETRS89;
3. gravity data (1) including data from Bornholm
4. gravity data (1) with more rigorous upgrade of data from Poland; $1' \times 1'$ mean Faye anomalies were calculated from point gravity data expressed in the POGK-99 gravity system and ETRS89 using heights from the DTED2; they include new terrain corrections;
5. gravity data (3) with additional data from the east of Poland;
6. gravity data (1) with additional data from the east of Poland, and with the rigorous upgrade of data from Poland; $1' \times 1'$ mean Faye anomalies were calculated from point gravity data expressed in the POGK-99 gravity system and ETRS89 using heights from the gravity database and the DTED2; they include new terrain corrections.

Height anomalies precisely determined at the sites of the POLREF network by GPS/levelling were used for estimating the external accuracy of the computed gravimetric quasigeoid models. On the other hand, gravimetric quasigeoid models were used for evaluating the quality of the height anomalies at the POLREF sites. The statistics of fitting the quasigeoid models to the POLREF sites after eliminating the outliers and removing the bias are given in Table 8. The means after eliminating outliers and standard deviations of height anomalies are shown in Fig. 15. Number of outliers detected and eliminated in statistical analysis of investigated quasigeoid models is shown in Fig. 16.

Table 7. Input data used for developed gravimetric quasigeoid models

No	Quasigeoid model	Data set	GGM
1	quasi97b	1	EGM96
2	quasi04a	2	EGM96
3	quasi04b	2	GGM02S
4	quasi04c	2	GGM02S/EGM96
5	quasi04d	3	GGM02C
6	quasi05a	4	EGM96
7	quasi05b	5	EGM96
8	quasi05c	5	GGM02S/EGM96
9	quasi06a	6	GGM02S/EGM96

Table 8. Statistics of fitting quasigeoid models to the POLREF sites ($\zeta_{GPS/lev} - \zeta_{grav}$) after removing the bias (the mean = 0) [cm]

Statistics	quasi97b	quasi04a	quasi04b	quasi04c	quasi04d	quasi05a	quasi05b	quasi05c	quasi06a
Mean	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Std dev.	3.4	3.2	4.1	3.9	3.6	3.6	3.6	3.7	3.8
Min	-9.2	-8.4	-12.0	-10.2	-10.5	-9.2	-9.2	-10.1	-10.3
Max	12.4	10.1	10.8	11.4	10.9	9.0	9.0	9.7	10.1

Table 8 and Fig. 15 show that for all recently developed quasigeoid models, the standard deviation of their fit to the POLREF data remains at the same level; it slightly varies within the range from 3.2 cm to 4.1 cm, and does not indicate any improvement due to refining the gravity data. Variation of standard deviation is related to the varying outliers (Fig. 16) eliminated before the statistics is evaluated. The bias, however, gets significantly reduced from −30 cm to −13 cm after using refined gravity data from

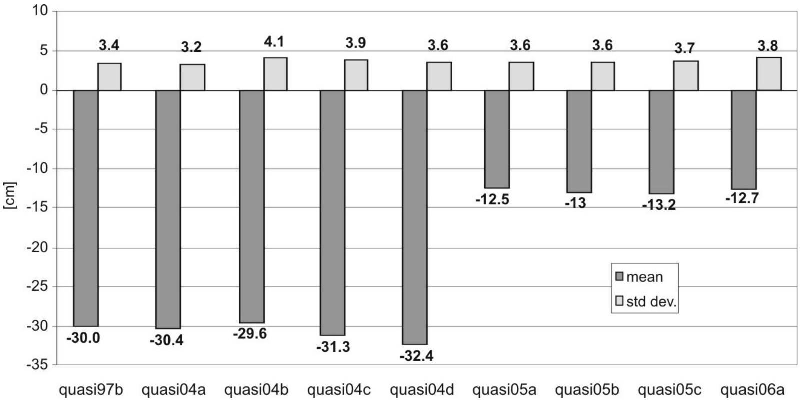

Fig. 15. The means and standard deviations of fitting quasigeoid models to the POLREF sites ($\zeta_{GPS/lev} - \zeta_{grav}$).

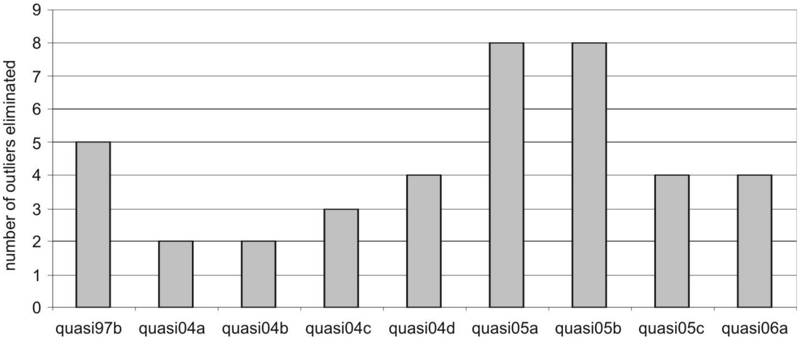

Fig. 16. Histogram of the eliminated outliers.

Poland. The results obtained indicate that the quality of height anomalies of the POLREF sites is no longer sufficient to evaluate the quality of recent precise gravimetric quasigeoid models for Poland *(Krynski and Lyszkowicz, 2006)*.

7.3. GPS/levelling quasigeoid models

Two purely numerical quasigeoid models spanned on quasigeoid heights of the POLREF sites have been developed *(Osada et al., 2003; Krynski et* *al., 2005d).* The first one applies the "kriging" interpolation technique that is based on least squares collocation with 4th order polynomial trend and a signal s

$$\zeta(x, y) = a + bx + cy + dx^2 + exy + fy^2 + ... + s(x, y). \tag{7}$$

The coefficients $a, b, c, d, e, f, \ldots..$ and the signal $s_i = s_i(x_i, y_i)$ at the POLREF sites, are obtained by solving the system of equations

$$\zeta_i(x, y) + v_i = a + bx_i + cy_i + dx_i^2 + ex_iy_i + fy_i^2 + ... + s_i(x_i, y_i) \tag{8}$$

The second approach of modelling involves the "minimum curvature" interpolation technique with parameters determined using spline functions a_k = spline (x, y, ζ, σ), where variance coefficients $\sigma = m_p$ must ensure $\nu = \zeta - \zeta_{calc}$ in the range of mean square errors m_p of quasigeoid heights at the POLREF sites. The quasigeoid surface of a minimum curvature is given as follows *(Osada et al., 2003)*:

$$\begin{aligned}&\zeta(x, y, X, Y, a) = a_{k+1} + a_{k+2}x + a_{k+3}y...... + \\ &+\frac{1}{2}\sum_{i=1}^{k}\left[(X_i - x)^2 + (Y_i - y)^2\right]\ln\left[(X_i - x)^2 + (Y_i - y)^2\right]a_i.\end{aligned} \tag{9}$$

Both GPS/levelling quasigeoid models provided very similar results. Statistics of the fit of GPS/levelling quasigeoid model to the EUVN and WSSG sites are given in Table 9 *(Krynski, 2006).*

Table 9. Statistics of the fit of GPS/levelling quasigeoid model ("kriging") to the EUVN and WSSG sites [cm]

	Mean	Std dev.	Min	Max
$\zeta_{\text{model}} - \zeta_{\text{EUVN}}$	-3.4	4.8	-21.3	10.3
$\zeta_{\text{model}} - \zeta_{\text{WSSG}}$	-3.1	5.4	-16.7	21.6

Another GPS/levelling quasigeoid model spanned on quasigeoid heights at the POLREF sites has been developed with support of gravity data represented by mean $1' \times 1'$ gravity anomalies Δg *(Osada et al., 2003; Krynski et al., 2005d).* The model is based on the equation

$$\zeta(x, y) = \zeta_{\Delta g}(x, y) + a + bx + cy + dx^2 + exy + fy^2 + ... + s(x, y) \tag{10}$$

where height anomalies $\zeta_{\Delta g}$ computed using planar approximation of Stokes' formula

$$\zeta_{\Delta g}(X,Y) = \frac{1}{2\pi\gamma}\int\limits_{-\infty}^{+\infty} \frac{\Delta g}{\sqrt{(x-X)^2+(y-Y)^2}} dxdy \tag{11}$$

provide an important information on the quasigeoid between the POLREF sites. The 4th order polynomial was used in computations (10).

The coefficients $a, b, c, d, e, f, \ldots$ and the signal $s_i = s_i(x_i, y_i)$ at the POLREF sites, are obtained by solving the system of equations

$$\begin{aligned}\zeta_i(x,y) + v_i = \zeta_{\Delta g}(x_i,y_i) + a + bx_i + cy_i + dx_i^2 + ex_iy_i + fy_i^2 + ... + \\ + s_i(x_i,y_i)\end{aligned}$$

Statistics of the fit of GPS/levelling quasigeoid model with support of gravity data to the EUVN and WSSG sites are given in Table 10 *(Krynski, 2006).*

Table 10. Statistics of the fit of GPS/levelling gravity quasigeoid model with support of gravity data to the EUVN and WSSG sites [cm]

	Mean	Std dev.	Min	Max
$\zeta_{\text{model}} - \zeta_{\text{EUVN}}$	-3.1	2.9	-12.4	2.1
$\zeta_{\text{model}} - \zeta_{\text{WSSG}}$	-3.2	5.5	-25.9	43.8

All the developed GPS/levelling quasigeoid models spanned on quasigeoid heights at the POLREF sites exhibit almost the same bias of 3 cm. Adding gravity data substantially improves a GPS/levelling quasigeoid model with respect to the heights of the EUVN sites. Evaluation of the effect of adding gravity data on a GPS/levelling quasigeoid model, with respect to the heights of the WSSG sites, is not representative due to the variation of the eliminated outliers.

7.4. Integrated quasigeoid model

An integrated GPS/levelling/gravity/topography quasigeoid model has been developed *(Osada et al., 2005).* It is based on Molodensky's approximate solution *(Heiskanen and Moritz, 1967)*

$$\zeta = \frac{1}{4\pi\gamma R}\iint\limits_{\sigma}(\Delta g + G_1)S(\psi)d\sigma \tag{12}$$

in which free-air gravity anomaly Δg is represented as follows

$$\Delta g = (g_P^{GGM} - \gamma_Q) + (g_P - g_P^{GGM}) = \Delta g^{GGM} + \delta g \tag{13}$$

where Δg^{GGM} is the major component of the free-air anomaly Δg computed with the use of gravity g_P^{GGM} determined from a global geopotential model GGM (e.g. EGM96) at point P, and δg is a small term similar to the gravity disturbance.

After substituting (13) to (12), the height anomaly becomes the sum of three components, i.e. $\zeta_{GGM} + \zeta_{\delta g} + \zeta_{G_1}$. The GGM component of the height anomaly can practically be computed from the definition *(Heiskanen and Moritz, 1967)*

$$\zeta_{GGM} = \frac{W_P^{GGM} - U_P}{\gamma_Q} \tag{14}$$

where W_P^{GGM} is the gravity potential at P calculated from the GGM, and U_P is the normal gravity potential at P obtained from GRS80 *(Moritz, 1984)*.

Numerical calculation of the remaining components of the height anomaly, i.e. $\zeta_{\delta g}$ and ζ_{G_1} can be performed using the plane approximation of the sphere of radius R, with the Stokes' kernel expressed as $S(\psi) \approx 2/\psi = 2R/r$, where r is the planar distance between the running point and the computed point. The gravimetric component of the height anomaly is thus given as *(Shimbirev, 1975)*

$$\zeta_{\,\delta g} = \frac{1}{2\pi\gamma}\iint\limits_{\sigma}\frac{\delta g}{r}d\sigma \tag{15}$$

while its terrain component as *(Heiskanen and Moritz, 1967)*

$$\zeta_{G_1} = \frac{1}{2\pi\gamma}\iint\limits_{\sigma}\frac{G_1}{r}d\sigma \tag{16}$$

where

$$G_1 = \frac{1}{2\pi} \iint\limits_{\sigma} \frac{H - H_P}{r^3} \Delta g d\sigma \tag{17}$$

with $d\sigma$ - the surface integration element.

The quasigeoid model $\zeta_{GGM} + \zeta_{\delta g} + \zeta_{G_1}$ is next fitted using least squares collocation to height anomalies $\zeta_{GPS/lev}$ obtained at GPS/levelling sites, with simultaneous determination of model parameters

$$\zeta = \zeta_{GGM} + \zeta_{\delta g} + \zeta_{G_1} + t + s \tag{18}$$

where t is the trend expressed by 4^{th} order polynomial function, and s – random signal with the expected value equal to zero and with the isotropic covariance function. The method has been proven efficient and robust for outliers detection *(Osada et al., 2005)*.
Mathematical model reads as follows:

$$\mathbf{v} = \mathbf{AX} + \mathbf{s} - \mathbf{L}, \tag{19}$$

where $\mathbf{v}$ is the vector of observation errors, $\mathbf{A}$ is the coefficient matrix of trend $t[(1, x_i, y_i, \ldots)i = 1, 2, \ldots, n]$, $\mathbf{X}$ is the vector of trend parameters, $\mathbf{s}$ is the vector of signals, and $\mathbf{L}$ is the vector of residuals $l = \zeta_{GPS/lev} - (\zeta_{GM} + \zeta\ {}_{\delta g} + \zeta_{G_1})$.

The integrated GPS/levelling/gravity/topography quasigeoid model was evaluated at the POLREF, EUVN and WSSG sites. Statistics of the fit of quasigeoid model to the POLREF EUVN and WSSG sites are given in Table 11. The fit of the integrated GPS/levelling/gravity/topography quasigeoid model to the POLREF EUVN and WSSG sites corresponds to a priori variances used for height anomalies at those sites.

Table 11. Statistics of the fit of the integrated GPS/levelling/gravity/topography quasigeoid model to the POLREF EUVN and WSSG sites [cm]

	Mean	Std dev.	Min	Max
ζ_{model} - ζ_{POLREF}	0.5	0.7	-1.3	3.0
ζ_{model} - ζ_{EUVN}	-0.5	0.6	-2.3	1.2
ζ_{model} - ζ_{WSSG}	-1.5	4.0	-24.3	38.9

7.5. Best-fitted quasigeoid model

Gravimetric quasigeoid model fitted to GPS/levelling at the POLREF sites has been developed *(Krynski and Lyszkowicz, 2006)*. The difference between the GPS/levelling height anomaly $\zeta_i^{GPS/lev}$ and gravimetric quasigeoid height ζ_i^{grav} is expressed as follows

$$\zeta_i^{GPS/lev} - \zeta_i^{grav} = t_i + s_i + n_i, \tag{20}$$

where the trend component was modelled by 3-parameter datum shift in the form

$$t_i = \cos\varphi_i \cos\lambda_i \Delta X + \cos\varphi_i \sin\lambda_i \Delta Y + \sin\varphi_i \Delta Z \tag{21}$$

with φ_i, λ_i - geodetic latitude and longitude, and ΔX, ΔY, $\Delta Z-$ datum shift components.

After computing the trend parameters, an empirical covariance function of the de-trended residuals $\zeta_i^{GPS/lev} - \zeta_i^{grav} - t_i$ was calculated and modelled by least squares collocation *(Krynski and Lyszkowicz, 2006)*. Hirvonen's covariance function *(Heiskanen and Moritz, 1967)* was applied. Based upon the computed set of 330 height anomaly differences, datum shift components ΔX, ΔY, ΔZ were determined using quasi05c gravimetric quasigeoid model and the data from the POLREF sites. It yields $\Delta X = -30.1$ cm, $\Delta Y = -27.5$ cm, and $\Delta Z = 12.8$ cm. Statistics of the fit of obtained "best-fitted" quasigeoid model to the POLREF, EUVN and WSSG sites are given in Table 12, and the fit of that model to the POLREF sites is shown in Fig. 17. Mean difference between the heights of "best-fitted" quasigeoid and the respective height anomalies of the POLREF sites equals to 1 cm.

Table 12. Statistics of the fit of the "best fitted" gravimetric quasigeoid model to the POLREF EUVN and WSSG sites [cm]

	Mean	Std dev.	Min	Max
$\zeta_{\mathrm{model}} - \zeta_{\mathrm{POLREF}}$	0.0	1.0	-5.3	4.2
$\zeta_{\mathrm{model}} - \zeta_{\mathrm{EUVN}}$	-2.3	4.0	12.4	8.6
$\zeta_{\mathrm{model}} - \zeta_{\mathrm{WSSG}}$	-6.0	10.5	-99.6	79.8

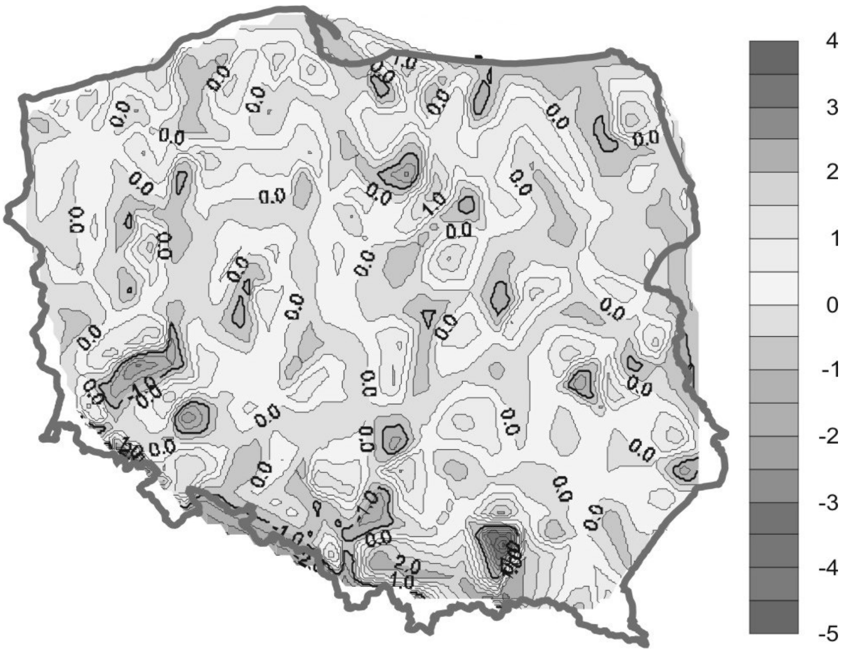

Fig. 17. The fit of "best-fitted" quasigeoid model to the POLREF sites [cm].

8. Quality of developed quasigeoid models

The integrated, best-fitted and gravimetric quasigeoid models developed were mutually compared and the residual height anomalies were analysed *(Krynski, 2006)*. The accuracy of the models was also tested with the use of height anomalies of the POLREF sites, as well as, in some cases, of the EUVN and WSSG sites. An independent assessment of the accuracy of the computed quasigeoid models was done with the use of height anomalies at the sites of the precise GPS/levelling control traverse. The fit of the representative quasigeoid models to the sites of the GPS/levelling control traverse is shown in Fig. 18. Quasigeoid models quasi05a and quasi06a run similarly along the control traverse to quasi05b and quasi05c. Quasigeoid models quasi97b, quasi04a, quasi04b, quasi04c and quasi04d form a consistent group of a similar pattern to the models of 05 and 06 but with the

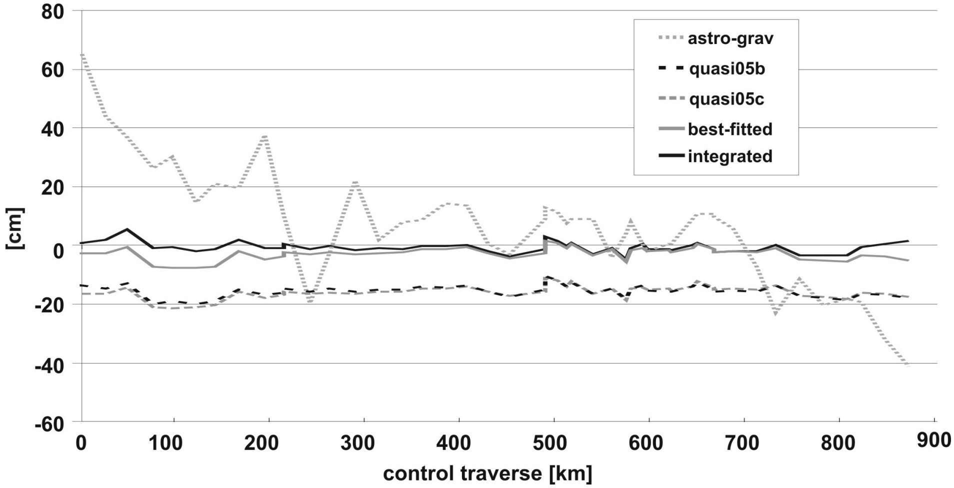

Fig. 18. The fit of the quasigeoid models to the sites of the GPS/levelling control traverse.

larger bias of about −30 cm. Statistics of the fit of those models to the sites of the GPS/levelling control traverse and the inclination of the models with respect to the control traverse in the N-E direction are given in Table 13.

The quality of the astro-gravimetric quasigeoid is substantially lower than the quality of the remaining quasigeod models. It might be improved by some densification of astronomic deflections of the vertical and generating a new set of high resolution gravimetric deflections of the vertical using available unified gravity data.

Table 13. Statistics of the fit of representative quasigeoid models to the sites of the GPS/levelling control traverse and the inclination of the models with respect to the control traverse in N-E direction [cm]

Model	Mean	Std dev.	Min	Max	N-E incl.
astro-gravimetric	8	21	-40	65	-0.15"
gravimetric (quasi05b)	-15	2.1	-20	-11	~0
gravimetric (quasi05c)	-16	2.3	-21	-11	~0
gravimetric-best-fitted	-2.7	2.0	-7.4	2.0	0.004"
integrated („2005”)	-0.7	1.8	-4.7	5.4	-0.004"

Height anomalies of the GPS/levelling control traverse sites are more precisely determined than the ones of the POLREF network and therefore more suitable for evaluation of accuracy of gravimetric quasigeoid models. Standard deviations of fitting the quasigeoid models to the control traverse sites are at the level of 2 cm, while the respective ones referred to the fit to the POLREF sites are hardly below 4 cm (see Table 8).

The quality of the best-fitted gravimetric quasigeoid, as well as the integrated quasigeoid in Poland, seems to be at the moment mainly limited by the quality of the GPS/levelling height anomalies.

9. Conclusions

- The existing gravity data from Poland fulfill in terms of accuracy, distribution and resolution the requirements for modelling the quasigeoid at a centimetre level of accuracy.
- Recently developed quasigeoid models in Poland exhibit very high quality. Accuracy of the developed quasigeoid models evaluated by comparing quasigeoid heights with the respective ones on the sites of the GPS/levelling control traverse are in terms of the standard deviations as follows: 21 cm for the astro-gravimetric quasigeoid, 2.2 cm for the gravimetric quasigeoid, 2.0 cm for the gravimetric best-fitted quasigeoid, and 1.8 cm for the integrated quasigeoid.
- Precise quasigeoid models of high accuracy and resolution require the use of good quality gravity data. The accuracy of quasigeoid models developed with the use of gravity data is at the level of 2 cm.
- Poor quality of the astro-gravimetric quasigeoid model results mainly from poor quality of old astro-gravimetric deflections of the vertical derived in 1960s from small scale gravity maps. Substantial improvement of the astro-gravimetric quasigeoid model can be expected when using a newly generated set of high resolution gravimetric deflections of the vertical with the use of recently available unified gravity data.
- Quality of heights of the POLREF sites that results from surveying strategy and technology does not correspond to the accuracy of below 2 cm claimed in the documentation. The POLREF network is not sufficient

for quality control of precise quasigeoid models in Poland. It is also not sufficiently accurate as the basis for best-fitted quasigeoid models, that could provide height accuracy required for levelling with GPS in Poland.

- A GPS/levelling control traverse established across Poland proved to be a powerful tool for reliable quality assessment and accuracy estimate of precise quasigeoid models in Poland.
- The best-fitted gravimetric quasigeoid model developed should replace the GUGiK 2001 quasigeoid model that has recently been used in surveying practice in Poland.
- The results obtained in the project can efficiently be applied to further improvement of quasigeoid models for Poland. They indicate the methods and tools that ought to be used to improve quasigeoid models. They also make it possible to develop more quasigeoid models.
- The system of developed databases is a unique source of complex information for geodynamics research in Poland. It is also a valuable starting point for further research on precise quasigeoid modelling in Poland. The system of databases can widely be used for research on the figure of the Earth, study the structure of the upper crust, geological cartography, as well as for solving hydrological and environmental problems.

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