Some aspects of Bouguer gravity determination – revisited

B. Meurers
Institute of Meteorology and Geophysics, University of Vienna[1]

P. Vajda
Geophysical Institute of the Slovak Academy of Sciences[2]

Abstract: The Bouguer gravity disturbance gets a clear physical meaning only when it is based on ellipsoidal heights. However, applying closed expressions or Taylor series expansions for the normal gravity calculation is not permitted in areas of negative ellipsoidal heights. The paper investigates problems associated with using alternate reference earth models and compares the differences with respect to the classical reference ellipsoid approach. It is shown that the quasi-ellipsoid concept is a suitable way to handle negative ellipsoidal heights. The paper also discusses the consequences of gravity vector transformation into scalar quantities which are commonly used. Strictly speaking the latter are no harmonic functions but can be regarded as such in planar approximation.

Key words: scalar and vector gravity disturbance, normal gravity, reference ellipsoid, quasi-ellipsoid.

1. Introduction

Since modern surveying methods (GPS) make ellipsoidal heights available, several papers have been published recently stimulating the discussion among geodesists and geophysicists on problems like the geophysical indirect effect, gravity anomaly versus disturbance terminology, physical meaning of Bouguer gravity etc. (e.g., *Ivan, 1996; Li and Götze, 2001; Hackney and Featherstone, 2003; Vajda et al., 2004; Hinze et al., 2005; Vajda et al., 2006*). Actually all these papers do not provide much new findings to specialists as the basics are well known. However, it was worth and necessary

[1] Althanstrasse 14, 1090 Vienna, Austria; e-mail: bruno.meurers@univie.ac.at
[2] Dúbravská cesta 9, 845 28 Bratislava, Slovak Republic; e-mail: Peter.Vajda@savba.sk

to clarify the situation as perhaps not everybody was aware that only by using ellipsoidal heights the Bouguer gravity gets a clear physical meaning.

As long as orthometric or normal heights were only available instead of ellipsoidal ones, the problem of the geophysical indirect effect was commonly ignored. This was justified in local and regional studies because on these scales the effect behaves as a long wavelength signal with low amplitude. Besides, orthometric or normal heights allow for applying closed expressions or series expansions for normal gravity calculation almost worldwide as they are positive almost everywhere on and above the earth surface. Contrarily, wide areas exhibit negative ellipsoidal heights not only in certain oceanic regions, but also onshore, e.g., close to some continent-ocean boundaries. Here the usual way of a normal gravity correction is not possible or introduces systematic errors. A possible solution to overcome this problem is to define alternate reference earth figures (e.g., *Vajda et al., 2004*). This paper investigates consequences of such a strategy.

2. On the problem of normal gravity calculation

The 3$^{\text{rd}}$ boundary value problem in physical geodesy requires that Laplace's differential equation (LDE) holds regarding the disturbing gravity potential T at the boundary (geoid) and within the exterior space. The disturbing gravity potential is defined as:

$$T(\boldsymbol{x}) = W(\boldsymbol{x}) - U(\boldsymbol{x}). \tag{1}$$

W is the gravity potential and U denotes the normal potential. The normal potential is uniquely defined by the 4 Stokes' constants (e.g., *Heiskanen and Moritz, 1967*). No assumption regarding the mass density structure inside the reference ellipsoid is required. In geophysics the normal potential is assumed to be caused by an ellipsoidally stratified density distribution inside the reference ellipsoid which matches global seismological findings (e.g., PREM *(Dziewonski and Anderson, 1981)*). Consequently, Poisson's DE holds rather than the LDE, where ellipsoidal heights are negative, and thus the gravitational part of the normal potential is no harmonic function inside the reference ellipsoid.

In physical geodesy this problem is overcome by the non-uniqueness principle of potential theory. Infinitely many mass distributions exist that produce the normal potential. The spherical harmonic expansion of the normal potential converges down to a sphere closely surrounding the focal points of the reference ellipsoid *(Moritz, 1980)*. According to the equivalent source principle a surface density distribution spread over this sphere of convergence exists that generates the normal potential. Hence the gravitational part of the normal potential is a harmonic function far below the surface of the normal ellipsoid.

In geophysics this concept is not applicable as the geophysicist is looking for anomalous density distribution defined as a deviation from the ellipsoidal density stratification of the reference earth model. In addition, the geophysicist needs to calculate the gravity disturbance and has to determine normal gravity at the observation point. This is no problem when the station is located above the ellipsoid, but severe problems arise if the ellipsoidal height is negative. In this case one can no longer use the same closed formulas or Taylor series expansion as for normal gravity outside the ellipsoid. Fig. 1 shows an error estimate based on a PREM like density distribution in spherical approximation.

The problem is solved if the density distribution of the reference earth is known. An alternate way has been proposed, e.g., by *Vajda et al. (2004)*

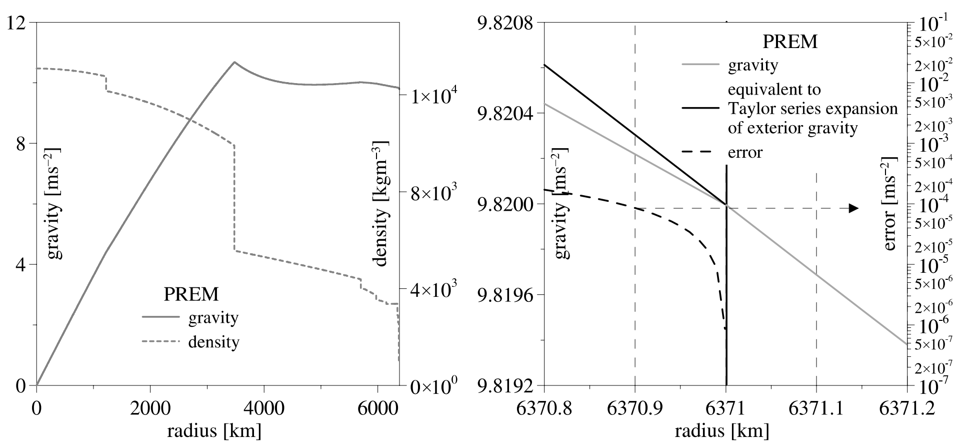

Fig. 1. Gravity inside the earth (left) and close to the earth's surface (right) indicating the error introduced when applying the normal gravity formula even inside the earth.

who suggest to refer to a so-called quasi-ellipsoid. In this article, both concepts are discussed in terms of their physical meaning and differences.

3. Reference ellipsoid concept

All reference earth models have to fulfill following conditions:

1. The mass M_E of the reference earth model is equal to the total mass M of the real earth because the corresponding gravitational potentials have the same zero degree term if expanded into solid spherical harmonics.
2. The surface of the reference ellipsoid coincides with the equipotential surface of the normal gravity field equal to the actual gravity potential at the geoid.

Then the real earth can be regarded as synthetically composed (Fig. 2) of

- ellipsoidal density stratification with total mass M_E (reference ellipsoid)
- topographic surplus mass M_{Ts}
- topographic deficit mass M_{Td}
- mass anomalies inside the reference-ellipsoid: δM_{Ts}, δM_{Td}, $\sum \delta M_E$.

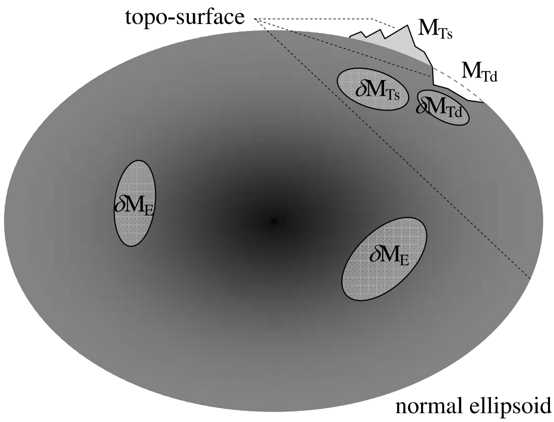

Fig. 2. Synthetic composition of the earth applying the reference ellipsoid concept.

δM_{Ts} and δM_{Td} just compensate the topographic surplus and deficit mass, respectively. The atmospheric mass is either included in the topographic surplus and deficit mass or is considered by the atmospheric correction of normal gravity (e.g., *Wenzel, 1985*).

It should be noted that the actual geoid constrains location and geometry of all compensating masses. That means they cannot be located arbitrarily. Additionally, as the total mass M has to be preserved, the following equation holds:

$$M = \left\{ \begin{array}{l} M_E + \\ + M_{Ts} + \delta M_{Ts} + \\ + M_{Td} + \delta M_{Td} + \\ + \sum \delta M_E \end{array} \right\} = M_E \;\Rightarrow\; \left\{ \begin{array}{l} M_{Ts} + \delta M_{Ts} + \\ + M_{Td} + \delta M_{Td} + \\ + \sum \delta M_E \end{array} \right\} = 0 \tag{2}$$

The gravity $\boldsymbol{g}$ observed at any point $P(\boldsymbol{x})$ outside, on or inside the reference ellipsoid therefore reads as:

$$\boldsymbol{g}(\boldsymbol{x}) = \boldsymbol{\gamma}_E(\boldsymbol{x}; M_E) + \underbrace{\tilde{\boldsymbol{g}}(\boldsymbol{x}; M_{Ts}) + \tilde{\boldsymbol{g}}(\boldsymbol{x}; M_{Td})}_{\substack{\text{topographic} \\ \text{mass correction}}} + \tilde{\boldsymbol{g}}(\boldsymbol{x}; \delta M_{Ts}) + \tilde{\boldsymbol{g}}(\boldsymbol{x}; \delta M_{Td}) + \\ + \tilde{\boldsymbol{g}}(\boldsymbol{x}; \sum \delta M_E) \tag{3}$$

$\boldsymbol{\gamma}_E$ denotes the normal gravity vector and the tildes indicate pure gravitational contributions to gravity. As the Bouguer gravity disturbance $\boldsymbol{\delta g}(\boldsymbol{x})$ is defined by

$$\boldsymbol{\delta g}(\boldsymbol{x}) = \boldsymbol{g}(\boldsymbol{x}) - \boldsymbol{\gamma}_E(\boldsymbol{x}; M_E) - \{\tilde{\boldsymbol{g}}(\boldsymbol{x}; M_{Ts}) + \tilde{\boldsymbol{g}}(\boldsymbol{x}; M_{Td})\} \tag{4}$$

it has a clear physical meaning as Newtonian attraction of all mass anomalies below the topo-surface at the observation point:

$$\boldsymbol{\delta g}(\boldsymbol{x}) = \tilde{\boldsymbol{g}}(\boldsymbol{x}; \delta M_{Ts}) + \tilde{\boldsymbol{g}}(\boldsymbol{x}; \delta M_{Td}) + \tilde{\boldsymbol{g}}(\boldsymbol{x}; \sum \delta M_E) \tag{5}$$

Of course, this holds only if the mass density within topography is exactly known. If the topographic mass correction is performed by assuming constant density, then the Bouguer gravity disturbance is the Newtonian attraction of all mass anomalies below the topo-surface with respect to the mass distribution of the reference earth (if located within the reference ellipsoid) and to the mass of constant density, respectively (if located inside the topographic surplus mass) at the observation point. Inside the reference ellipsoid the components of the Bouguer gravity disturbance vector can be calculated if and only if the internal density structure of the reference ellipsoid is known.

It has to be stressed that the Bouguer gravity disturbance does not exactly correspond to the gradient of the disturbing gravity potential in the sense of physical geodesy because it includes the topographic mass correction term. This has to be considered, when geoidal heights are derived by applying Bruns' theorem on the potential calculated by simple vertical integration of the Bouguer gravity.

Equation (4) shows that the mass correction considers also the deficit mass missing between ellipsoid and topo-surface. This is essential in order to get a harmonic disturbing potential within the space above the topo-surface. Otherwise applying field transformation methods (e.g., upward continuation) would not be strictly permitted. Therefore digital elevation models based on ellipsoidal heights even offshore are required.

4. Quasi-ellipsoid concept

The quasi-ellipsoid is defined as a spheroid located inside the reference ellipsoid. The distance between each surface point and its projection onto the reference ellipsoid along the reference ellipsoidal normal is constant. It is fixed such that each point of the topo-surface is either on or outside the quasi-ellipsoid (e.g., *Vajda et al., 2004*). Following requirements have to be fulfilled:

1. The quasi-ellipsoid exhibits a mass density distribution such that its external gravity field equals exactly to that of the reference ellipsoid. Outside the reference ellipsoid, the potential field is uniquely determined by the normal gravity on the reference ellipsoid. As between the reference ellipsoid and the quasi-ellipsoid the LDE holds, we can make use of the analytical continuation of the external potential into the interior of the reference ellipsoid. Hence, everywhere outside and on the quasi-ellipsoid, the closed expressions or series expansions for the normal gravity can be applied, i.e., the components of the Bouguer gravity disturbance vector can now be calculated without an a priori knowledge of the internal density structure even at points with negative ellipsoidal heights.

2. The total mass of the quasi-ellipsoid M_Q equals to that of the reference ellipsoid M_E (or that of the real earth M) which follows immediately from the first requirement.

The real earth can now be regarded as synthetically composed (Fig. 3) of

- ellipsoidal density stratification with mass M_Q (quasi-ellipsoid)
- topographic surplus mass M_{TQs} filling the space between reference- and quasi-ellipsoid
- topographic surplus mass M_{Ts}
- topographic deficit mass M_{Td}
- mass anomalies inside the quasi-ellipsoid: δM_{TQs}, δM_{Ts}, δM_{Td}, $\sum \delta M_E$.

δM_{TQs}, δM_{Ts} and δM_{Td} again just compensate the topographic surplus (M_{TQs}, M_{Ts}) and deficit mass (M_{Td}), respectively:

$$M = \left\{ \begin{array}{l} M_Q + \\ + M_{TQs} + \delta M_{TQs} \\ + M_{Ts} + \delta M_{Ts} \\ + M_{Td} + \delta M_{Td} + \\ + \sum \delta M_E \end{array} \right\} = M_Q = M_E \ \Rightarrow \left\{ \begin{array}{l} M_{TQs} + \delta M_{TQs} + \\ + M_{Ts} + \delta M_{Ts} + \\ + M_{Td} + \delta M_{Td} + \\ + \sum \delta M_E \end{array} \right\} = 0 \quad (6)$$

Then gravity $\boldsymbol{g}$ observed at any point $P(\boldsymbol{x})$ outside or on the quasi-ellipsoid reads as:

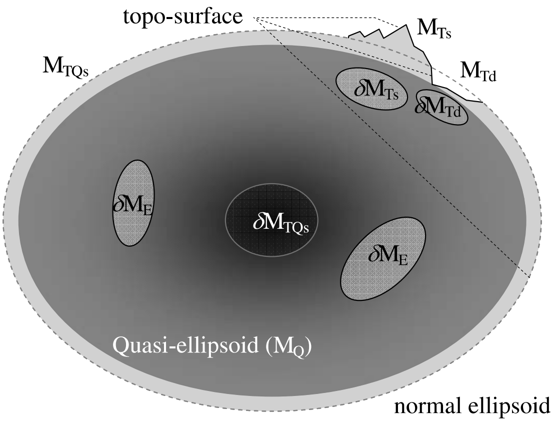

Fig. 3: Synthetic composition of the earth applying the quasi-ellipsoid concept.

$$\boldsymbol{g}(\boldsymbol{x}) = \overbrace{\boldsymbol{\gamma}_Q(\boldsymbol{x}; M_Q)}^{=\boldsymbol{\gamma}_E(\boldsymbol{x})} + \overbrace{\tilde{\boldsymbol{g}}(\boldsymbol{x}; M_{TQs}) + \tilde{\boldsymbol{g}}(\boldsymbol{x}; M_{Ts}) + \tilde{\boldsymbol{g}}(\boldsymbol{x}; M_{Td})}^{\substack{\text{topographic} \\ \text{mass correction}}} + \\ + \tilde{\boldsymbol{g}}(\boldsymbol{x}; \delta M_{Ts}) + \tilde{\boldsymbol{g}}(\boldsymbol{x}; \delta M_{Td}) + \tilde{\boldsymbol{g}}(\boldsymbol{x}; \delta M_{TQs}) + \tilde{\boldsymbol{g}}(\boldsymbol{x}; \textstyle\sum \delta M_E) \tag{7}$$

Again, the Bouguer gravity disturbance $\boldsymbol{\delta g}(\boldsymbol{x})$ has the clear physical meaning as Newtonian attraction of all mass anomalies below the topo-surface at the observation point:

$$\boldsymbol{\delta g}(\boldsymbol{x}) = \tilde{\boldsymbol{g}}(\boldsymbol{x}; \delta M_{Ts}) + \tilde{\boldsymbol{g}}(\boldsymbol{x}; \delta M_{Td}) + \tilde{\boldsymbol{g}}(\boldsymbol{x}; \sum \delta M_E) + \tilde{\boldsymbol{g}}(\boldsymbol{x}; \delta M_{TQs}) \tag{8}$$

However, this expression differs from that of the reference ellipsoid concept (Eq. (5)) just by the last term on the right hand side. Accordingly, applying Gauss' law

$$\iint\limits_{\partial V} \boldsymbol{g} \cdot \boldsymbol{df} = \iiint\limits_{V} \mathrm{div}\boldsymbol{g}\, dv = -4\pi G\, M \tag{9}$$

on Eqs. (5) and (8) yields the total anomalous mass M_S (Eq. (10)) generating the Bouguer gravity disturbance. V is an arbitrary volume including both the reference ellipsoid and the quasi-ellipsoid with volume elements dv, ∂V denotes its boundary with surface elements $\boldsymbol{df}$.

reference ellipsoid concept

$$\begin{array}{llllll} \boldsymbol{\delta g} & = \boldsymbol{g} & - \boldsymbol{\gamma}_E & - & \tilde{\boldsymbol{g}}(M_{Ts}, M_{Td}) \\ \downarrow & \quad\downarrow & \quad\downarrow & & \quad\downarrow \\ M_S & = M & - M & - & (M_{Ts} + M_{Td}) \\ M_S & = & & & -M_{Ts} - M_{Td} \end{array}$$

quasi-ellipsoid concept

$$\begin{array}{llllllll} \boldsymbol{\delta g} & = \boldsymbol{g} & - \boldsymbol{\gamma}_E & - & \tilde{\boldsymbol{g}}(M_{Ts}, M_{Td}) & - & \tilde{\boldsymbol{g}}(M_{TQs}) \\ \downarrow & \quad\downarrow & \quad\downarrow & & \quad\downarrow & & \quad\downarrow \\ M_S & = M & - M & - & (M_{Ts} + M_{Td}) & - & M_{TQs} \\ M_S & = & & & -M_{Ts} - M_{Td} & - & M_{TQs} \end{array} \tag{10}$$

Hence, in the quasi-ellipsoid concept additional anomalous mass exists that balances M_{TQs} and yields to a global and height dependent gravity signal in Eq. (8). The magnitude of this global signal can be estimated by a simple

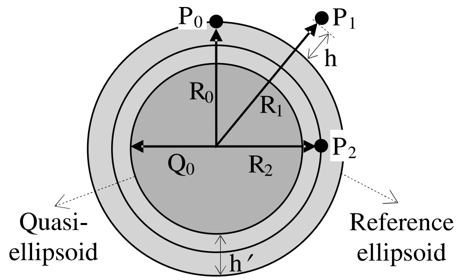

Fig. 4. Earth model in spherical approximation.

model of the Earth (Fig. 4) in spherical approximation (homogeneous sphere without topography).

In the reference ellipsoid concept observed gravity equals to normal gravity at all points outside (P_1), on (P_0) and inside (P_2) the sphere. As no topographic surplus or deficit mass exists, the Bouguer gravity disturbance is zero everywhere. However, in the quasi-ellipsoid concept the observed gravity equals to normal gravity only outside (P_1) and on (P_0) the reference ellipsoid. Additionally, the mass correction δg_T taking the spherical shell between reference ellipsoid and quasi-ellipsoid into account differs for different radii R:

$$g(P_0) = \frac{4\pi G\rho}{3} R_0 = \gamma(P_0) \qquad \delta g_T = \frac{4\pi G\rho}{3} \frac{R_0^3 - Q_0^3}{R_0^2} \qquad \delta g = \frac{4\pi G\rho}{3} \frac{Q_0^3 - R_0^3}{R_0^2}$$

$$g(P_1) = \frac{4\pi G\rho}{3} \frac{R_0^3}{R_1^2} = \gamma(P_1) \qquad \delta g_T = \frac{4\pi G\rho}{3} \frac{R_0^3 - Q_0^3}{R_1^2} \qquad \delta g = \frac{4\pi G\rho}{3} \frac{Q_0^3 - R_0^3}{R_1^2}$$

$$g(P_2) = \frac{4\pi G\rho}{3} R_2 \neq \gamma(P_2) = \frac{4\pi G\rho}{3} \frac{R_0^3}{R_2^2} \qquad \delta g_T = \frac{4\pi G\rho}{3} \frac{R_2^3 - Q_0^3}{R_2^2} \qquad \delta g = \frac{4\pi G\rho}{3} \frac{Q_0^3 - R_0^3}{R_2^2}$$

The Bouguer gravity disturbance is no longer zero but dependent on the radius R:

$$\delta g = \frac{4\pi G\rho}{3} \frac{Q_0^3 - R_0^3}{R^2} = \frac{4\pi G\rho}{3} \frac{(R_0 - h')^3 - R_0^3}{R^2} =$$

$$= \frac{4\pi G\rho}{3} \frac{R_0^3}{R^2} \left(\left(1 - \frac{h'}{R_0} \right)^3 - 1 \right) \approx -4\pi G\rho h' \frac{R_0^2}{R^2} \approx$$

$$\approx -4\pi G\rho h' \left(1 - 2\frac{h}{R_0}\right) \tag{11}$$

where h' denotes the separation of the reference ellipsoid and the quasi ellipsoid, ρ the density and G the gravitational constant. The Bouguer gravity disturbances differ in both concepts by one constant and one height dependent term:

$$\text{height dependent term}: \quad \frac{8\pi G\rho}{R_0} h'h \tag{12}$$

Assuming a crustal density of 2670 kgm^{-3} and $h' = 0.1$ km, the height dependence is estimated to be as small as 70 nms^{-2}/km, and therefore this global signal has no impact on local studies.

5. Vector versus scalar formulation

The previous sections used the vector formulation for gravity in order to be rigorous. In praxis we have to deal with scalar quantities instead. The latter is inevitable to be left with harmonic quantities required for applying interpretation methods like field transformation and for simplifying data processing and modeling. Harmonic quantities can be obtained by either using the vector components or any linear combination of theirs. The norm of a potential gradient vector does not meet Laplace differential equation while its projection to any direction does. For being rigorous we need to know the direction of the observed gravity vector and not only its norm as resulting from gravity measurements. Only in this case we can calculate the projections of all contributing gravitational vectors to the direction of the observed gravity vector. The following recalls which imperfections or errors are introduced by the data processing commonly used in praxis.

Let U be a scalar potential. Then the potential gradient vector is transformed into a scalar quantity W by performing the scalar product with any vector field $\boldsymbol{b}$, and W is written by using the Einstein notation as:

$$W = b_j \frac{\partial U}{\partial x_j} \tag{13}$$

Applying the Laplace operator yields

$$\frac{\partial^2 W}{\partial x_i^2} = \frac{\partial^2 b_j}{\partial x_i^2}\frac{\partial U}{\partial x_j} + 2\frac{\partial b_j}{\partial x_i}\frac{\partial^2 U}{\partial x_i \partial x_j} + b_j \frac{\partial}{\partial x_j}\left(\frac{\partial^2 U}{\partial x_i^2}\right) \tag{14}$$

If U is a harmonic function, i.e., LDE holds, then Eq. (14) simplifies to

$$\frac{\partial^2 W}{\partial x_i^2} = \frac{\partial^2 b_j}{\partial x_i^2}\frac{\partial U}{\partial x_j} + 2\frac{\partial b_j}{\partial x_i}\frac{\partial^2 U}{\partial x_i \partial x_j} \tag{15}$$

Obviously W is a harmonic function if $\boldsymbol{b}$ is any constant vector or if $\boldsymbol{b} = \boldsymbol{x}$. In the scalar formulation of the Bouguer gravity, $\boldsymbol{b}$ is commonly chosen as unit vector parallel to the ellipsoidal surface normal or, in spherical approximation, to the radial direction:

$$\boldsymbol{b} = \boldsymbol{x}/\|\boldsymbol{x}\| \tag{16}$$

It has to be stressed, that, strictly speaking, the scalar Bouguer gravity disturbance is no harmonic function. As for example, if the unit vector in radial direction is used, Eq. (15) results in:

$$\frac{\partial^2 W}{\partial x_i^2} = -\frac{2}{x_k^2}\left(W + x_i \frac{\partial W}{\partial x_i}\right) \tag{17}$$

If Eq. (17) equals zero, then the right-hand side of Eq. (17) contains Euler's homogeneity relation, i.e., W is a homogeneous function of degree -1 and $U = C \ln x_k^2 + D$. However, this is not a harmonic potential function in 3D space.

Conventionally the unit vector $\boldsymbol{n}$ is chosen as pointing downward and parallel to the ellipsoidal normal which passes the observation point $P(\boldsymbol{x})$. Its direction varies with $\boldsymbol{x}$ as long as the observation points are not aligned along the same ellipsoid normal. Note that outside the reference ellipsoid the normal gravity vector is not parallel to the ellipsoid normal because the equipotential surfaces except the reference ellipsoid itself are no ellipsoids. The scalar representation of the Bouguer gravity disturbance follows from Eq. (4):

$$\boldsymbol{n}(\boldsymbol{x}) \cdot \boldsymbol{\delta g}(\boldsymbol{x}) = \boldsymbol{n}(\boldsymbol{x}) \cdot \boldsymbol{g}(\boldsymbol{x}) - \boldsymbol{n}(\boldsymbol{x}) \cdot \boldsymbol{\gamma}_E(\boldsymbol{x}; M_E) - \boldsymbol{n}(\boldsymbol{x}) \cdot \tilde{\boldsymbol{g}}(\boldsymbol{x}; M_T) \tag{18}$$

M_T here denotes the surplus and deficit mass. Using the symbols explained in Fig. 5 yields:

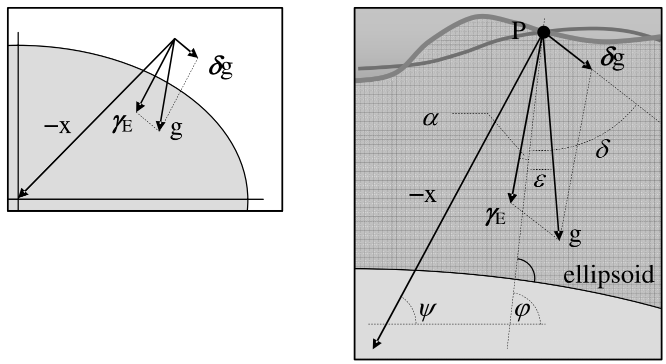

Fig. 5. The problem of vector misalignment.

$$\delta g(\boldsymbol{x}) \cos \delta(\boldsymbol{x}) = g(\boldsymbol{x}) \cos \varepsilon(\boldsymbol{x}) - \gamma_E(\boldsymbol{x}) \cos \alpha(\boldsymbol{x}) - \underbrace{\tilde{\boldsymbol{g}}(\boldsymbol{x}; M_T) \cos \delta_T(\boldsymbol{x})}_{\substack{\text{topographic} \\ \text{mass effect}}} \qquad (19)$$

In praxis, at each observation point a Cartesian coordinate system is locally adjusted so that its vertical axis is parallel to the ellipsoidal normal, and Eq. (19) is evaluated simply by calculating the vertical components. The deflection of the vertical ε and the angle α are disregarded. This is justified because the error is less then 420 nms^{-2} as long as ε does not exceed 60 arcsec. Afterwards the local coordinate systems are rotated such that their vertical axes are aligned. As noted before, strictly speaking the left-hand side of Eq. (19) does not fulfill the LDE, but after alignment of the vertical axes it can be regarded as harmonic function in planar approximation. However, it is important to keep in mind that this approximation introduces errors. They have to be considered when field continuation is used for high accurate 3D interpolation. Moreover, this concept is not applicable for large scale investigations.

6. Conclusions

- The Bouguer gravity disturbance corresponds exactly to the gravity effect of all mass anomalies below the topo-surface with respect to the mass distribution of the reference earth (if located inside the reference ellipsoid or quasi-ellipsoid) and constant density respectively (if located outside) at the observation point. Reference ellipsoid and the quasi-ellipsoid exhibit different density distributions.

- The density distribution of the reference ellipsoid has to be known in order to calculate normal gravity at observation points with negative ellipsoidal heights. Contrarily, the quasi-ellipsoid concept does not require an a priori density information. However, a global and slightly height dependent signal is left in the Bouguer gravity disturbance.
- The mass correction has to include the gravity effect of the deficit mass missing between ellipsoid and topo-surface in the reference ellipsoid concept while in the quasi-ellipsoid concept the mass correction domain is extended down to the surface of the quasi-ellipsoid.
- The Bouguer gravity disturbance vector does not exactly correspond to the gradient of the disturbing gravity potential in the sense of physical geodesy. This has to be considered, when deriving geoidal heights by applying Bruns' theorem on the potential calculated by field transformation (vertical integration) of the Bouguer gravity disturbance.
- The scalar Bouguer gravity disturbance can be regarded as harmonic quantity only in planar approximation. This limits the scale on which field transformation is permitted by applying FFT techniques and introduces additional error sources in upward continuation or other field transformation results.

Acknowledgments. We highly appreciate the constructive critics and valuable suggestions by an anonymous reviewer, which improved the paper essentially.
The authors are grateful to the Science and Technology Assistance Agency under the contract No. APVT-51-002804. This work was also supported by the Slovak Grant Agency VEGA (grants No. 1/3066/06 and 2/6019/06) for the partial support of this work.

References

Dziewonski A. M., Anderson D. L., 1981: Preliminary reference Earth model. Physics of the Earth and Planetary Interiors, **25**, 297–356.

Hackney R. I., Featherstone W. E., 2003: Geodetic versus Geophysical Perspectives of the "Gravity Anomaly". Geoph. J. Int., **154**, 35–43.

Heiskanen W., Moritz H., 1967: Physical Geodesy. Freeman & Co, San Francisco, 364 p.

Hinze W. J., Aiken C., Brozena J., Coakley B., Dater D., Flanagan G., Forsberg R., Hildenbrand Th., Keller G. R., Kellogg J., Kucks R., Li X., Mainville A., Morin R., Pilkington M., Plouff D., Ravat D., Roman D., Urrutia-Fucugauchi J., Véronneau M., Webring M., Winester D., 2005: New standards for reducing gravity data: The North American gravity database. Geophysics, **70**, J25-J32.

Ivan M., 1996: Polyhedral approximations in Physical Geodesy. J. Geodesy, **70**, 755–767.

Li X., Götze H. J., 2001: Ellipsoid, geoid, gravity, geodesy, and geophysics. Geophysics, **66**, 1660–1668.

Moritz H., 1980: Advanced physical geodesy. Herbert Wichmann Verlag Karlsruhe, 500 p.

Vajda P., Vaníček P., Novák P., Meurers B., 2004: On evaluation of Newton integrals in geodetic coordinates: Exact formulation and spherical approximation. Contr. Geophys. Geod., **34**, 289–314.

Vajda P., Vaníček P., Meurers B., 2006: A new physical foundation for anomalous gravity. Stud. Geophys. Geod., **50**, 189–216.

Wenzel F., 1985: Hochauflösende Kugelfunktionsmodelle für das Gravitationspotential der Erde. Wiss. Arb. Fachr. Vermessungswesen Univ. Hannover, Nr. 137.