Terrain correction in extremely disturbed terrain

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A b s t r a c t: It has been for several decades that computers have enabled us to perform the terrain correction computation much faster and accurate then before. In the meantime several programs for terrain correction computation have grown up among geophysicists and geodesists. All these programs have something in common – they all use a digital elevation model (DEM) as an input. On the other hand there are some differences, either in form of the integration kernel (planar, spherical), or in the integration scheme and integration method. While browsing over the flat region, all of the programs give almost identical results, assuming all use the same DEM. However, when terrain grows higher and gets more broken, the differences in results become significant. Now, it is not an easy task to decide which program produces better results. One way how to do it would be to produce an etalon computed analytically from idealized terrain. Such an approach, in fact, has several disadvantages: it is rather complicated and it cannot be done in real, or at least real-like, terrain. A different way how to compare the quality of terrain corrections in the mountains is presented in this contribution. The main idea is very simple: terrain corrections that produce smoother refined Bouguer gravity anomalies

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are better. This approach, of course, can only be used when the computation points are sufficiently close each other and when the real gravity at these points is known from direct measurement.

During summer 2004 the unique joint measurements of gravity and 3D position had been collected in High Tatra Mountains. The amount of 153 points had been measured using Scintrex CG-3 gravity meter and Trimble 5700 GPS receivers (3D position). At some points the additional measurements of 3D position in close surrounding area had been performed. These measurements are very convenient for our investigation.

In this contribution we compare three different programs for terrain correction computation. At first we briefly explain the basic differences between the programs. As a next step we compute the terrain corrections by each program using the same DEM obtaining the 3 sets of terrain corrections. The integration radius is 166.7 km $\approx$ 5390 arcsec. and density is standard 2670 kg$\cdot$m^{-3}. Consequently we compute the 3 sets of refined Bouguer gravity anomalies corresponding to 3 sets of terrain corrections. Finally we compare 3 different sets of refined Bouguer anomalies and evaluate the smoothness of each set using statistical testing. At the end we give conclusions and recommendations based on obtained results.

Key words: terrain correction, refined Bouguer gravity anomaly, smoothness, statistical testing

1. Introduction

Let us define the terrain correction and introduce the principle of terrain correction computation in general.

The first term we introduce is topographical masses. Simply, topographical masses are masses between the basic equipotential surface – geoid and physical surface of the Earth. In geophysics and also in geodesy it is often important to remove the attraction of topographical masses. This can be done applying so called topographic reduction $\delta g_T = -A_T$. Computation of this reduction is often divided into reduction of a Bouguer plate (or reduction of a spherical Bouguer shell) $\delta g_B = -A_B$ and terrain correction $\delta g_t = -A_t$ *(Heiskanen and Moritz, 1967)*. The terrain correction by definition removes gravitation attraction of part of the topographical masses rising above the Bouguer plate (or shell) and missing within the Bouguer plate (or shell), see Fig. 1. In other words, terrain correction is affected by masses bounded with equipotential surface passing through the point of computation from one side, and with physical surface of the Earth from the other side. Therefore, to compute the terrain correction, some kind of elevation model is necessary. The best case is to have a digital elevation model (DEM).

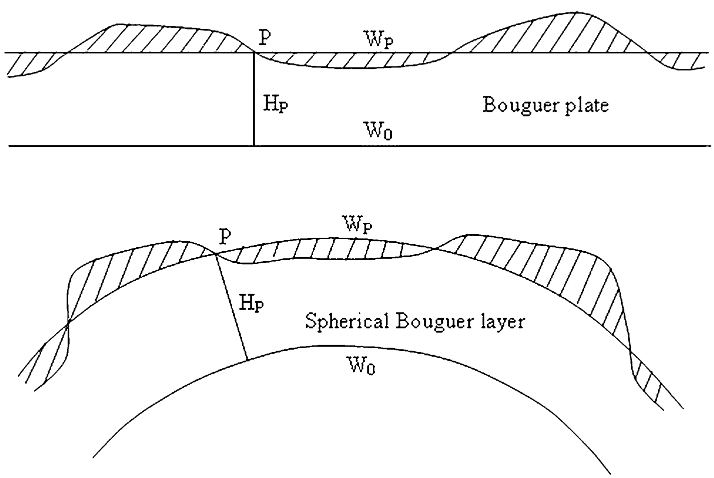

Fig. 1. Principle of planar and spherical approach to terrain correction computation.

Let us describe three different approaches to terrain correction computation represented by three independent software products developed in one non-academic and two academic institutions.

2. Approach A – developed and used in Geocomplex Corporation

The basic partitioning of the area around the computation point respects the Hayford zones, following suggestions of *(Pick et al., 1960)*, see Tab. 1 and Fig. 2.

Terrain for T_1 computation is approximated using vertical prisms with triangular bases with the edges length from 10 to 20 meters. The Boundary of T_1 is square shaped, therefore the gravitational effect of the body shown in Fig. 3 must be subtracted in order to assure the compatibility with T_2. T_1 is computed using a planar model, see Fig. 1.

Tab. 1. Partitioning of surrounding area for terrain correction computation (approaches A and B)

Label	Limits (m)	Hayford
T_1	0-250	A-C_2
T_2	250-5240	D_1-H
T_{31}	5240-28800	I-L
T_{32}	28800-166735	M-O_2

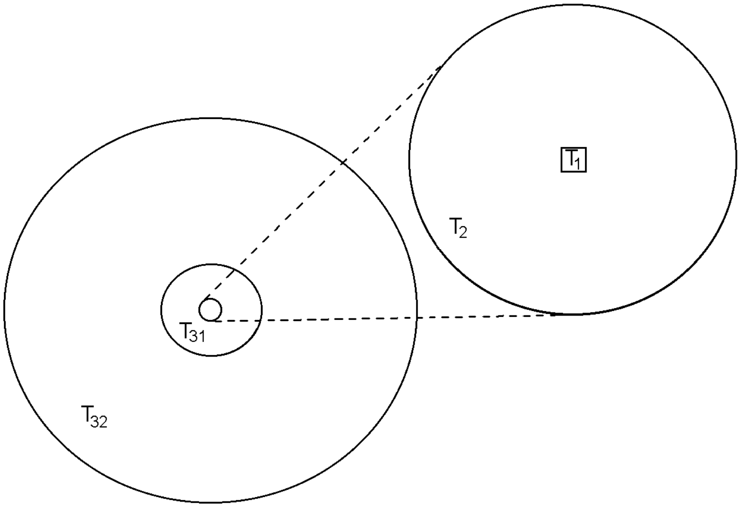

Fig. 2. Partitioning of the area for approach A.

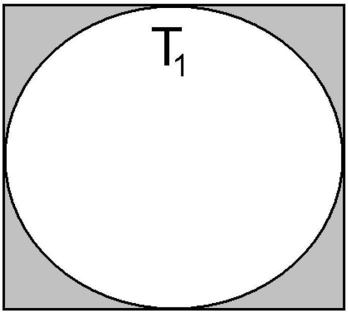

Fig. 3. Transition from square-shaped to circle-shaped area for T_1 part of approach A.

T_2, T_{31} and T_{32} is computed as a sum of gravitational effects of segments of a spherical layer with thickness obtained as a difference between the height of the point of computation and height of the terrain in the middle of the segment obtained from DEM by interpolation. This approach operates in S-JTSK coordinate system.

3. Approach B – developed in Comenius University and used in project "Atlas of geophysical maps and profiles"

The basic partitioning of the area around the computation point is the same as in approach A, see Tab. 1 and Fig. 4. T_1 part is computed using vertical prisms with triangular bases and T_2 using rectangular vertical prisms. Both, T_1 and T_2 use a planar model, see Fig. 1. T_{31} and T_{32} parts are computed using a pseudo-spherical model, i.e. with the vertically immersed rectangular prisms.

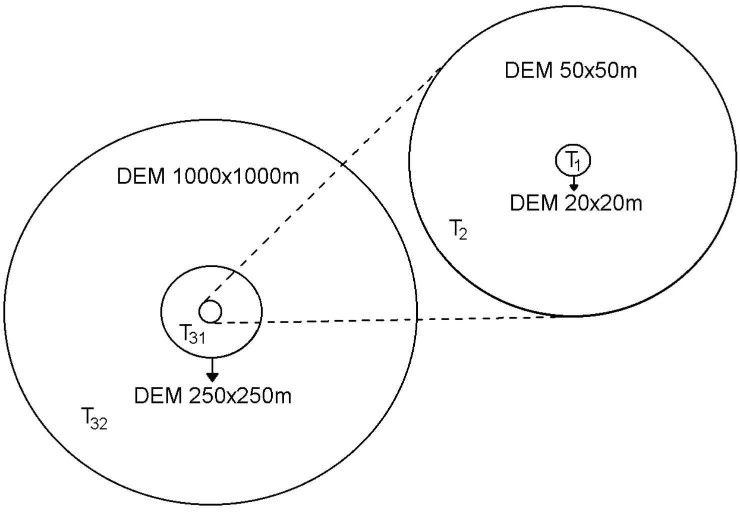

Fig. 4. Partitioning of the area for approach B and DEM grid spacing.

The elevation grid using for calculation of T_1 is 20 × 20 m, for T_2 is 50 × 50 m, for T_{31} is 250 × 250 m and for T_{32} is 1000 × 1000 m. This approach also operates in S-JTSK coordinate system and it was used for re-computation of terrain corrections in geological research project Atlas of Geophysical Maps and Profiles *(Kubeš et al., 2001)*. For more detailes about this approach see *(Grand et al., 2004)*.

4. Approach C – developed in University of New Brunswick, improved in Slovak University of Technology and used mostly within geodetic community

Program used for terrain correction computation is part of the software package for precise geoid determination SHGEO (Stokes-Helmert's GEOid software) *(Tenzer and Janák, 2002)*. It operates in geographical coordinates and uses a spherical model, see Fig. 1, with the analytical spherical integration kernel derived by *(Martinec, 1998)*. The basic partitioning of the area around the computation point is shown in Tab. 2 and Fig. 5.

Particular parts T_{i1}, T_{i2}, T_{m1}, T_{m2} are computed as a sum of gravitational effects of segments of spherical layer, see Fig. 6, with the thickness computed as a height difference obtained from DEM using biquadratic interpolation.

5. Some ideas, how to check the quality of terrain corrections

Quality of terrain correction depends on several factors. First of all it depends on quality and minuteness of used DEM. Next, the roughness of terrain and density variation, also play an important role. Last but not least, the computation procedure (partitioning of the area, interpolation method used for heights estimation, integration algorithms etc.), affects the results as well. An important question is how we can test the real quality of computed terrain correction. Three ideas are presented here, but only the second one has been applied.

The first idea is to use an idealized terrain, where terrain correction can be computed exactly without any approximation. Advantage of this method is that the true error of terrain correction is obtained. Disadvantage is that it is very complicated to create such synthetic terrain, that would look like

Tab. 2. Partitioning of surrounding area for terrain correction computation (approach C)

Label	Limits	Cell size
T_{i1}	Spherical trapezoid 1′×1′	0.1″×0.1″
T_{i2}	Spherical trapezoid 5′×5′	1″×1″
T_{m1}	Spherical trapezoid 55′×55′	30″×30″
T_{m2}	Up to chosen spherical radius (e.g. 5390″≈166.7km)	5′×5′

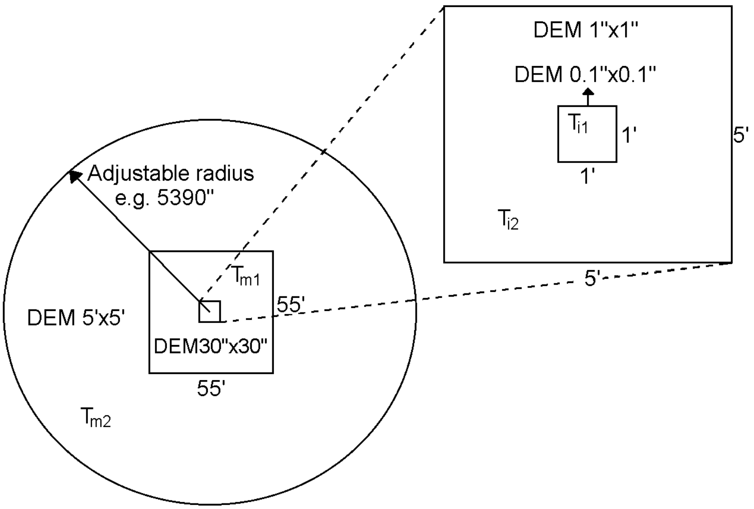

Fig. 5. Partitioning of the area for approach C and DEM grid spacing.

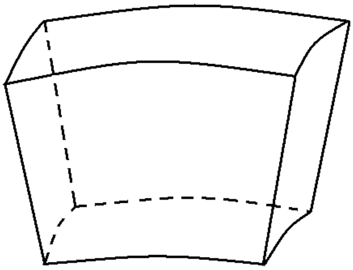

Fig. 6. Segment of spherical layer bounded by meridian and prime-vertical planes.

real and that would be analytically computable. Especially it holds when we want to simulate high mountains terrain.

The second idea is to compare several sets of terrain corrections indirectly, in terms of refined Bouguer gravity anomalies. According to empirical experience, the terrain corrections that produce smoother field of refined Bouguer gravity anomalies are more accurate and therefore better. Degree of smoothness can be measured in various ways. We chose to do it in terms of standard deviation of particular sets of refined Bouguer gravity anomalies and also in terms of standard deviations of their residuals with respect to quadratic polynomial surface.

The third idea is based on fact that spherical refined Bouguer gravity anomaly multiplied by geocentric distance is a harmonic function *(Vaníček et al., 2004)* and harmonicity can be tested numerically performing the second partial derivatives, e.g. using regularization approach *(Pašteka and Richter, 2002)*. The terrain corrections producing the most ideal harmonic field of refined Bouguer gravity anomalies should be the best.

6. Experiment with smoothness of refined Bouguer gravity anomalies

Data used for our experiment were collected during 2004 summer season in High Tatra mountains. It was a unique joint measurement of GEOCOMPLEX Corporation and Department of Theoretical Geodesy of Slovak University of Technology. Gravity was measured by Scintrex CG3 gravity meter and position was measured by Trimble 5700 GPS receivers. Normal heights were derived by subtracting the quasigeoid undulation from ellipsoidal heights obtained from GPS. The quasigeoid model GMSQ03C *(Mojzeš et al., 2004)* was used.

Some basic statistics of measured points is given in Tab. 3 and distribution of measured points is shown in Fig. 7.

In order to make a reasonable comparison, the following conditions were kept during terrain correction computation: the same DEM ($1'' \times 1''$), the same integration radius of 166.7 km $\approx 5390''$ and the same reduction density of 2670 kg $\cdot$m^{-3}. The three sets of terrain corrections were computed using approach A, B and C, described above. The basic statistics can be found in Tab. 4.

Tab. 3. Statistics of measured points

Number of points	152
Min. height	919.5 m
Max. height	2631.5 m
Mean height	1922.53 m

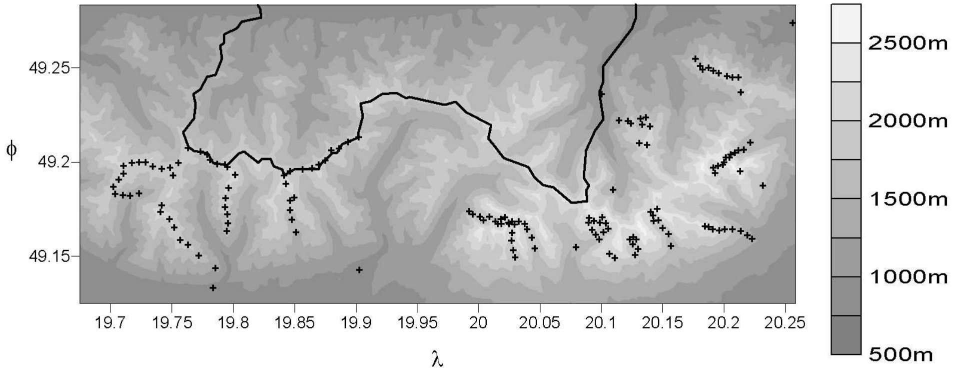

Fig. 7. Distribution of the measured points and topography in our experimental area.

Tab. 4. Statistics of terrain correction sets

Approach	Min.	Max.	Mean	Range	Standard deviation
	mGal				
A	3.69	80.97	25.04	77.28	10.97
B	3.66	79.66	24.74	76.01	10.56
C	3.64	78.16	24.40	74.52	10.02

Consequently, the refined Bouguer anomalies Δg_B were computed according to following equation (*Pick, 2000, Eq. (1.73)*), using particular sets of terrain corrections.

$$\Delta g_B = g - \gamma_0 - \frac{\partial \gamma}{\partial H} H + \delta g_B - B + \delta g_t \tag{1}$$

In Eq. (1): g is measured gravity, γ_0 is normal gravity at the reference ellipsoid, H is normal height, $\delta g_B = -2\pi G \rho H$ is reduction of the Bouguer plate, B is so called Bullard's term computed according to *(Pick, 2000, Eq. (8.6))* and finally δg_t is terrain correction computed according to particular approach A, B and C, respectively. The basic statistics of refined Bouguer gravity anomalies can be found in Tab. 5.

Tab. 5. Statistics of corresponding refined Bouguer anomaly sets

Approach	Min.	Max.	Mean	Range	Standard deviation
	mGal				
A	-60.2	-46.5	-52.5	13.7	3.28
B	-61.3	-46.8	-52.8	14.5	3.39
C	-59.8	-44.5	-52.1	15.3	3.29

As the next step we compute the quadratic polynomial surfaces, one for each refined Bouguer gravity anomaly set, given by following general form

$$z(x, y) = A + Bx + Cy + Dx^2 + Exy + Fy^2, \tag{2}$$

best fitting the refined Bouguer gravity anomalies in sense of L_2 norm. In Eq. 2: x, y, z are general coordinates and A, B, C, D, E, F are corresponding coefficients. The polynomial surfaces for particular approaches are shown in Fig. 8.

Residuals obtained as differences between the value of refined Bouguer gravity anomaly and corresponding value of polynomial surface at the measurement point were tested and basic statistics is given in Tab. 6.

7. Conclusions

Based on our numerical experiments we can conclude that all tested programs for terrain correction computation give similar results. Analysis of statistical values in Tab. 5 and Tab. 6 revealed some minor differences:

- Approach C produces the refined Bouguer anomalies with wider range then approaches A and B. The same holds for residuals above the best fitting quadratic polynomial surface.
- Approach B produces the refined Bouguer anomalies with slightly larger standard deviation. The same holds for residuals above the best fitting quadratic polynomial surface.
- Approach A seems to give the best results in terms of our statistical testing. Most likely, the reason is that, in this approach, for the prisms touching the point of computation, the slope of the upper base was estimated according to direct observation in the field.

- Based on previous conclusion we can say that any local information about the shape of the very near area can improve the accuracy of terrain correction computed in an extremely disturbed terrain.

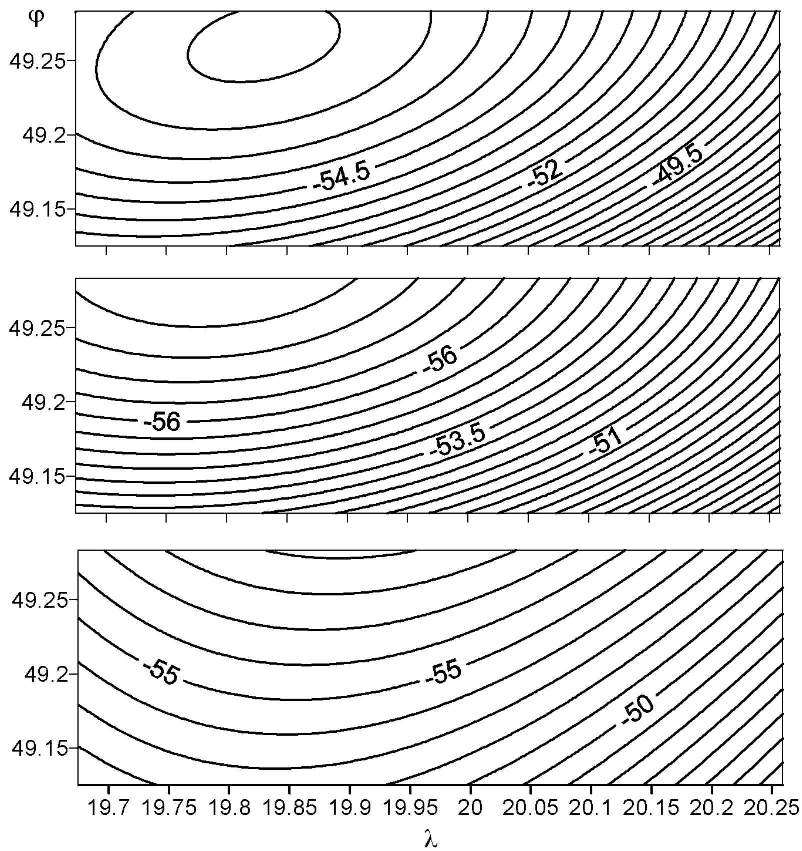

Fig. 8. The best fitting quadratic polynomial surface for a particular approach. From the top: approach A, B and C.

Tab. 6. Statistics of residuals above the best fitting quadratic polynomial surface

Approach	Min.	Max.	Mean	Range	Standard deviation
	mGal				
A	-5.0	3.3	0.0	8.3	2.03
B	-5.2	3.7	0.0	8.9	2.10
C	-5.4	4.9	0.0	10.3	2.08

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