Diurnal and semi-diurnal coordinate variations observed in European permanent GPS network: deterministic and stochastic constituents

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Abstract: Regular analysis of network of permanent Global Positioning System (GPS) monitoring stations covering almost the whole European continent is performed at Slovak University of Technology in Bratislava. It allows detecting small short-periodic oscillations of station coordinates. We analyze one-year interval of station time series comprising of coordinates evaluated from 4-hour observing samples. The variations of horizontal coordinates and ellipsoidal height with amplitudes from 0.5 to 2.0 mm are detected at majority of analyzed sites. They can be assigned to unmodelled solid Earth and ocean tides, polar motion effects and satellite orbits biases. After elimination of these deterministic signals still residual diurnal oscillations of stochastic character can be observed in the coordinate series. They can be observed at majority of analyzed stations by means of ARMA process modeling. We suppose that these signals have origin in perturbing effects resulting from atmosphere refraction variations, site monumentation movements, multipath effects and other relevant phenomena with no strict periodic character but with diurnal variability.

Key words: permanent GPS network, coordinate time series, tidal variations of site coordinates, linear stochastic process

1. Introduction

The European Reference Frame (EUREF) Permanent Network (EPN) started its activities in year 1995. Now, EPN comprises more than 160 permanent stations situated in 30 European countries. The whole network is

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divided to smaller sub-networks, which are processed at 16 local analysis centers. In 2002 the Department of Theoretical Geodesy at Slovak University of Technology was accepted as one of the EPN Local Analysis Center (LAC SUT).

At the LAC SUT the sub-network comprising 39 EUREF permanent GPS stations is regularly processed. For the regular processing the Bernese GPS Software 4.2 (*Hugentobler et al., 2001*) is used. The analysis procedure follows the rules applied to EUREF permanent network analysis (*Bruyninx et al., 2003*). The theoretical tidal effects affecting the site coordinates are consistently modeled according to IERS Conventions (*McCarthy, 1996*). Outputs from the EPN processing are the daily station coordinates obtained form 24-hours observing intervals and their covariance matrix, which are merged in weekly solutions.

The additional alternative data processing performed at SUT results in coordinates obtained from observations from shorter intervals, covering separate 4-hours observing spans. The aim of such processing mode is to obtain information about sub-daily variations of coordinates of observing stations. Such information is usually absorbed in coordinates obtained from daily observing intervals.

All the network solutions are referenced to International Terrestrial Reference Frame 2000 (ITRF 2000) through one site strongly constrained. The Borowiec (BOR1) in Poland is used at LAC SUT as the fiducial station. This method of network referencing means, that all the observed network station variations are relatively to the BOR1 station.

2. Time series from EPN subnetwork analyzed at SUT Bratislava

In this paper we analyze the 4-hour interval solutions of EPN subnetwork covering the period from Oct. 1, 2003 to Sept. 29, 2004. We will present here results obtained for selected subset of analyzed EPN stations, namely Ondrejov, Czech Republic (GOPE), Helgoland, Germany (HELG), Kootwijk, Netherlands (KOSG), Ohrid, Macedonia (ORID), Lviv, Ukraine (SULP), Stavanger, Norway (STAS) and Brno, Czech Republic (TUBO). The choice of these stations is motivated by the fact, that their series are continuous and these stations are situated in various regions of the continent.

The resulting station time series are formed by geocentric coordinates X, Y, Z. These time series were examined for outliers and missing parts were interpolated. Length of the time series is one year and is comprised of 2190 observations – 4-hour interval coordinates. In the next step, the geocentric coordinates X, Y, Z are transformed to local system *n, e, v (Hefty and Husár, 2003)* according to

$$\mathbf{N} = \begin{bmatrix} n \\ e \\ v \end{bmatrix} = \begin{bmatrix} -\sin\bar{B}\cos\bar{L} & -\sin\bar{B}\sin\bar{L} & \cos\bar{B} \\ -\sin\bar{L} & \cos\bar{L} & 0 \\ \cos\bar{B}\cos\bar{L} & \cos\bar{B}\sin\bar{L} & \sin\bar{B} \end{bmatrix} \begin{bmatrix} X-\bar{X} \\ Y-\bar{Y} \\ Z-\bar{Z} \end{bmatrix} = \mathbf{R}\left(\mathbf{X}-\bar{\mathbf{X}}\right), \quad (1)$$

where $\mathbf{R}$ is transformation matrix, $\bar{X}, \bar{Y}, \bar{Z}$ are reference geocentric coordinates and $\bar{B}, \bar{L}$ are reference ellipsoidal coordinates. The axis of local system n, e, v are shown in Fig. 1.

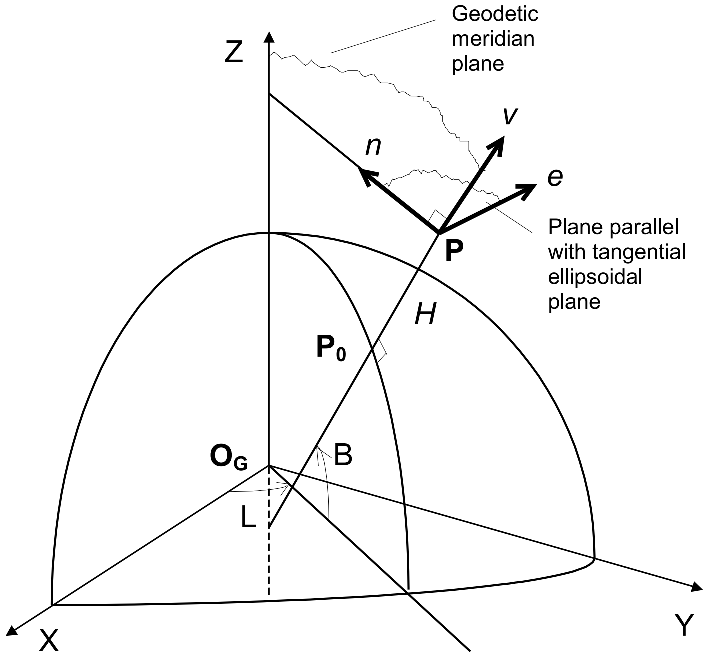

Fig. 1. Geocentric system X, Y, Z and local system n, e, v.

3. Deterministic constituents in station coordinates series

Deterministic constituents of $\{n_t\}$, $\{e_t\}$, $\{v_t\}$ time series are expressed usually by means of additive decomposition. The additive model has the form

$$Y_t = T_t + S_t + C_t + E_t, \tag{2}$$

where Y is one of the n, e, or v constituents, T is trend, S is seasonal component, C is periodic component and E is residual component. The subscript t is used for expressing the time dependence of observations and the modeled components.

The trend component is expressed by linear regression in the form

$$T_t = \alpha + \beta\, t, \tag{3}$$

where α, β are regression coefficients.

Significant periods of n, e, v series are determined after elimination of linear trend by spectral analysis method, which uses for Fourier transformation the estimates of autocovariance function. Spectral density function has then the form

$$f(\omega) = \frac{1}{2\pi} \sum_{k=-\infty}^{\infty} \gamma(k)\, e^{-ik\omega}, \tag{4}$$

where $\gamma(k)$ is autocovariance function estimate and ω is frequency. Example of spectra of coordinate time series for station GOPE is given in Fig. 2.

The most significant terms are with seasonal frequencies (annual and semi-annual), and tidal frequencies: diurnal (O_1, P_1, S_1, K_1) and semi-diurnal (M_2, S_2, K_2). The periods of observed tidal waves are given in Tab. 1.

Seasonal components model has the form

$$S_t = \sum_{i=1}^{2} \left(a_i \sin\left(\frac{2\pi\, t}{P_i}\right) + b_i \cos\left(\frac{2\pi\, t}{P_i}\right)\right), \tag{5}$$

and the periodic tidal components are

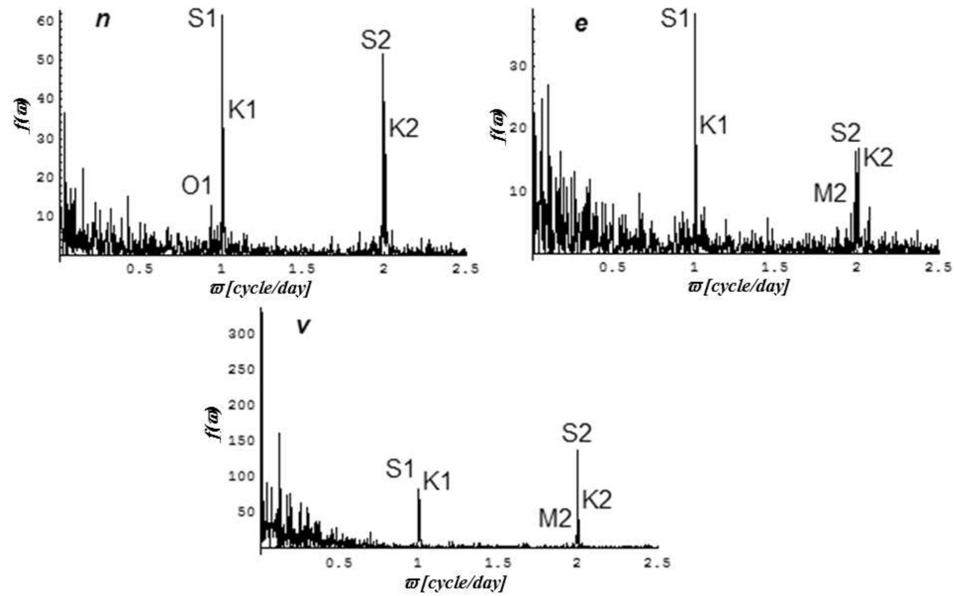

Fig. 2. Spectra of coordinate time series of station GOPE.

$$C_t = \sum_{i=3}^{9} \left(a_i \sin\left(\frac{2\pi\, t}{P_i}\right) + b_i \cos\left(\frac{2\pi\, t}{P_i}\right)\right), \tag{6}$$

where a_i, b_i are regression coefficients, P_i are annual, semi-annual, diurnal and semi-diurnal periods. Each harmonic component has amplitude A and phase ϕ

$$A_i = \sqrt{a_i^2 + b_i^2} \quad , \quad \tan\phi_i = \frac{a_i}{b_i}. \tag{7}$$

Tab. 1. Periods of observed tidal waves

Tidal waves band	Wave	Period P_i [hours]
Semi-diurnal	M_2	12.420 601
	S_2	12.000 000
	K_2	11.967 234
Diurnal	O_1	25.819 320
	P_1	24.132 120
	S_1	24.000 000
	K_1	23.934 469

Parameters of (1, 3, 5, 6), namely α, β, a_i, b_i are estimated for each of the $\{n_t\}$, $\{e_t\}$, $\{v_t\}$ time series for all 7 examined stations. Amplitudes A_i of seasonal, diurnal and semi-diurnal terms of coordinate time series are displayed in Fig. 3.

The periodic variations of horizontal coordinates and ellipsoidal height with amplitudes from 0.5 to 2.0 mm are observed at majority of analyzed sites. They can be assigned mainly to unmodelled solid Earth and ocean tides, polar motion effects and satellite orbits biases. Some of the terms like O_1 and P_1 have generally amplitude less than 0.5 mm for majority of stations. The amplitudes of K_1 and K_2 are very different at the various analyzed sites and reach up to 3 mm.

4. Stochastic constituents

After elimination of deterministic signals (the time series $\{E_t\}$) still residual diurnal oscillations of stochastic character can be observed in the station coordinate series. This effect is evident on the correlogram of the residual station coordinates time series. An example of the autocovariance function behavior is shown in Fig. 4. The peaks in time intervals corresponding to 24 hours are clearly visible (each 6^{th} value with 4-hours sampling).

To describe the stochastic constituents we will use the Box-Jenkins methodology (*Box and Jenkins, 1970*). We expect that coordinates from single 4-hours observations are not mutually independent but they are correlated in time. Considering this, the value E_t of a time series free from systematic influences might be expressed by equation

$$E_t = \varphi_1 E_{t-1} + \varphi_2 E_{t-2} + \cdots + \varphi_p E_{t-p} + \widetilde{N}_t, \tag{8}$$

where $\varphi_1, \ldots, \varphi_p$ are auto-regressive (AR) coefficients (deterministic part) and $\widetilde{N}_t$ is stochastic part or Moving-average (MA) part. MA part is expressed by

$$\widetilde{N}_t = N_t + \Theta_1 N_{t-1} + \cdots + \Theta_q N_{t-q}, \tag{9}$$

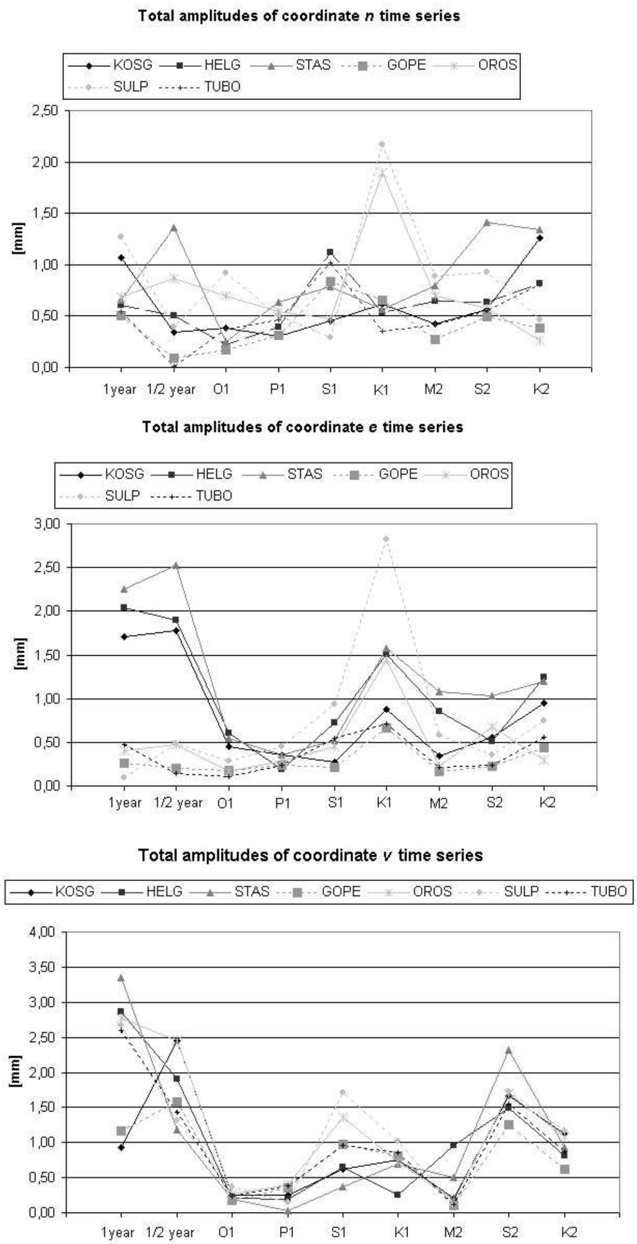

Fig. 3. Amplitudes of periodic terms in local coordinates time series.

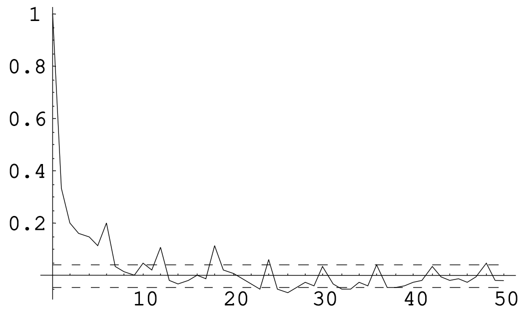

Fig. 4. Behavior of autocovariance function of residual coordinates n, station KOSG.

where $\Theta_1, \ldots, \Theta_q$ are MA coefficients. The series $\{N_t\}$ is white noise with zero mean value and constant variance $D\{N_t\} = \sigma^2$.

The set of AR and MA coefficients defines the ARMA (p, q) process, p and q being orders of AR and MA constituents, respectively. Firstly, following the Box-Jenkins methodology, orders p and q are estimated. They are determined by Akaike's and Bayesian information criterions. Then the AR and MA coefficients are estimated. The model suitability is verified through standard diagnostic-checking tests, which detect randomness of residuals (*Box and Jenkins, 1970*). The values of AR and MA coefficients of the analyzed series are plotted on Fig. 5.

All the series exhibit similar behavior with significant values of 6^{th} both the AR and MA coefficients. As the time series are sampled in 4-hours intervals, this phenomenon corresponds to 24-hour periodicity. The exactly 24-hour periodic terms were eliminated by introducing the S_1 terms. The significant 6^{th} AR and MA terms are due to variations with variable period oscillating around 24-hours. We suppose that these signals have origin in perturbing effect resulting from atmosphere refraction variations, site monumentation movements, multipath effects and other relevant phenomena with no strict periodic character but with diurnal variability.

5. Conclusions

Processing of permanent GPS observations with sub-daily resolution showed the presence of diurnal and semi-diurnal signals in station coordi-

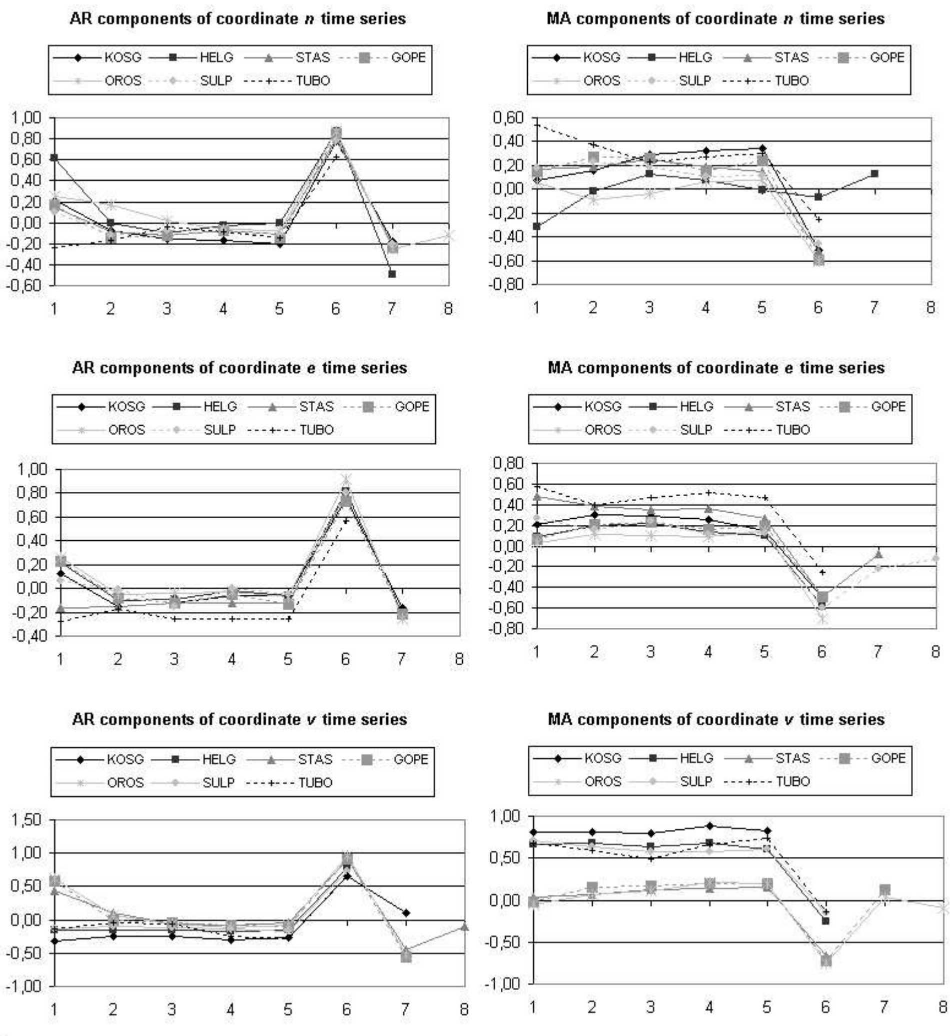

Fig. 5. AR and MA coefficients of residual $\{E_t\}$ time series.

nate series. We performed the decomposition of the series into deterministic and stochastic part. In the deterministic part seasonal terms and frequencies corresponding to tidal waves O_1, P_1, S_1, K_1, M_2, S_2 and K_2 dominate. The amplitudes of terms with tidal frequencies are of magnitude from 0.5 to 2.0 mm and are station dependent. Stochastic linear modeling leads to diurnal repeatability with no strict periodic character.

Our analysis pointed out that intra-diurnal variability of GPS determined coordinates is a phenomenon with amplitude comparable to seasonal effects. The geodynamical interpretation of these variations needs additional information like meteorological observations, ground water storage measurements, etc. We have to keep in mind also other periodic effects influencing the GPS permanent positioning like satellite orbits modeling, troposphere and ionosphere variations which can be responsible for the diurnal variations.

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