On the derivation of local/regional geodetic system

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Abstract: The systemic and technological aspects in derivation of the national/regional integrated *geodetic system* are set out. Here, instead of consistency of three independently derived *reference systems*, the procedures are suggested which integrate networks, technologies, data and models connecting elements from different *reference systems*. The proposed procedures should be applied to the following areas 1) the design of layers of the complex *basic geodetic network*, 2) the measurement technologies in building of *network*, 3) the consistency of datums, 4) the modelling of inhomogeneous data, 5) the comparison and concordance of results, obtained by different methods. In order to analyse these procedures a term "*integrating process*" is introduced. The different elements, stages and varieties of these *processes* are set out. Two technological schemes for determination of the consistent parameters of *geodetic system* by means of *integrating processes* are proposed. The procedures for derivation of an integrated *system* with corresponding to it geocentric 4D *integrating geodetic datum* are outlined. It is suggested the materialization of the *system* to be realized by integral *basic geodetic network*. The *network's* structure and the possibilities of integrating technologies for determination of *quasigeoidal deviation parameters* are discussed. The integrating model for adjustment of 1D heterogeneous (leveling and GPS) height data with evaluation of the *basic quasigeoid* and the national/regional *vertical datum* is proposed.

Key words: geodetic system, integrating model, basic geodetic network, basic quasigeoid, vertical datum

1. Introduction

The recent progress in geodesy is characterized with an extensive theoretical researche and realization of a number of national/regional projects

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on the integration of elements from three *Reference Systems* - *Horizontal* (RHS), *Vertical* (RVS) and *Reference Gravity System* (RGS). The regional realization of 3D-network was EUREF 89. The integration of vertical networks in 3D reference frames is put on the agenda. Such procedure gave rise to derivation of *European reference vertical systems*, to realizations of *height systems* - UELN 95, EUVN 97, EVS 2000, EVRF 2000, but not to the integration of RHS and RVS in the consistent *Geodetic System* (GS). The realization of EUVN 97 was performed by adding of ellipsoidal height from RHS to the RVS without integration of both *systems*.

The need of integral *Basic Geodetic Network* (BGN) is provoked by the necessity of updating of traditional networks and by the global process of unification the national datums into geocentric *International Terrestrial Reference System* (ITRS). The integral $\mathrm{BGN}^{\mathrm{S}}$ as a union of at least three layers from the basic kinds of networks began to be realized during the past twenty years. The combination of horizontal and vertical networks was an urgent matter of theoretical research investigations in the 80's but now it is the practical topic of the day. The cosmic technologies give the geometric base in the building of BGN. The geoid is connecting physical unit.

Here the principles for integral derivation of national/regional GS different from the concordance of *Reference Systems* (RS^{S}) applied up to now will be formed. It is suggested to perform this by BGN, materializing the integrated GS. This *system* should be characterized by a number of integrating procedures of elements from vertical and 3D space and by a geocentric 4D *integrating geodetic datum*. The basic *integrating processes* in different stages are analysed. The technological schemes for these *processes* are proposed. The established schemes connect the elements from different RS^{S}, the results from modelling of quasigeoid and the deduced *Reference Surfacefor Vertical Datum* ($\mathrm{RS_fVD}$). The integrating model for combined processing of 1D heterogeneous height data with optimal evaluation of the *basic quasigeoid* are worked out, as well as the geopotential differences regarding to the *Conventional Equipotential Reference Surface* ($\mathrm{C_vERS_f}$).

2. Systemic aspects in derivation of the integrated geodetic system

The proposed approach for building the integrated GS is being differed from traditional one in:

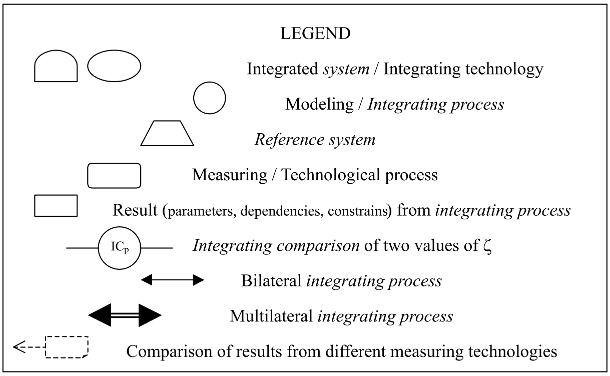
- falling off of the derivation of three independent RS^S,
- dealing with elements from space $\Im$, which represents an embedding of 1D vertical space V in 3D-space, (V is the gravity space, which describes the vertical dimension and is represented by the geopotential W) by the establishment of a 4D *integrating geodetic datum* of GS under the minimum datum constraints,
- applying of the *integrating processes* between elements from $\Im$ in the different stages of the building of GS,
- materializing of GS by the integral BGN with the space, vertical (levelling, gravimetric) and other measurements in each BGN-station,
- treating of BGN-stations as collocated stations for the lower-class networks,
- derivation of the *quasigeoidal deviation parameters* in BGN-stations and other parameters as a result of *integrating processes*,
- elimination of the systematic effects, datum inconsistencies, heterogeneity of the data and discrepancies of elements from different R^S before their integration,
- integration of space (VLBI-, GPS-, LPS-, SLR-, EDM-measurements) with vertical observations in *synthetic observations* having components from different RS^S (see *Kotsakis and Sideris, 1999*),
- combination of *hybrid networks* (with data from one and the same RS) in BGN,
- considering in the evaluation models the time dependent influences on the RS^S and observations.

Here we discuss the derivation of integrated local/regional but not global GS for lack of reliable global vertical information - global VD, geoid, *world height system*, i.e. there never have been vertical measurements on a global scale with the accuracy and time resolution now possible.

The existing GS is a combination of fragments from three RS^S and the consistent processes between them. A contemporary integrated GS (see Fig. 1) is a synthesis of space-fixed and Earth-fixed systems and parameters as a function of time connecting both systems. These systems are the *Conventional Inertial (celestial) RS* (CIRS) and the *Conventional Terrestrial*

RS (CTRS). The latter *system* has to contain the three completely built RS^{S} with the consistent transitions from one to another. If GS is integrated system of the three RS^{S}, it opens up a chance for definition of vertical measurements by the geopotential differences but not by kind of physical heights.

The following legend describes the graphic symbols used in the figures.



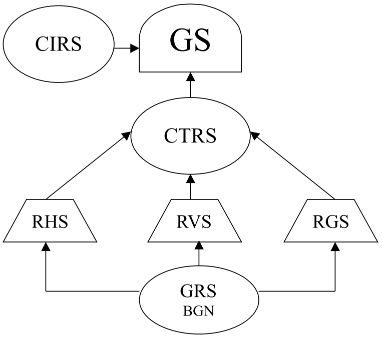


Fig. 1. *Reference systems* and *basic geodetic network* building the integrated *geodetic system*. The physical principles are attached to the connecting arrows. The arrows indicate the direction leading toward the building of *geodetic system*.

Physical height is not a rectilinear geometrical distance but defined by means of $\mathrm{RS_f}$VD, the geopotential difference and a curved trajectory. In principle, height is independent of geometry of RHS and unrelated to CTRS.

In order to overcome this the RS_fVD, as element of RVS and RGS, have to be embedded conventionally in CTRS, achieving into the 4D *integrating geodetic datum*. The *integrating datum* includes 1) CTRS-based *horizontal, vertical* and *gravity datums*, 2) transformed relations between datums and their epochs. All these datums have some variations at various levels both in the spatial and time domain. An *integrating datum* is 4D because it contains (except 3D space) the vertical (geopotential) dimension, RS_fVD at fixed epoch, vertical direction (trajectory) all of them from 1D space V. Where a local (national) *integrating geodetic datum* is established within CTRS then it is possible to unite such datums in regional ones in the presence of fixed connections between them.

The *Vertical Datum* (VD), geoid, *height system*, motion of the geocenter, geopotential and tidal variation, crustal deformations and Earth rotation dynamics are included in the up-to-date extending of CTRS realized by IERS. In this way the space and temporal variations characterize GS and corresponding *integrating datum*.

3. Integrating processes in building of integrated geodetic system

The realization of integrated GS has to contain *processes* integrating the geometrical geocentric coordinates to the geopotential numbers, the directions of the vertical and the shape of RS_fVD. Upon the implementation of combined processing of elements from physical and geometrical space in an GS, it should be analysed the unified, consistent and incorporate procedures connecting the elements from space $\Im$. In order to analyse the technological, datum, transformed, constrained and comparison problems for these elements, we introduce the term *Integrating Process* (IP).

Here the concept of IP includes:

- the derivation of integrated GS,
- the integral BGN from two or more networking layers,
- the integrating technology of two or more types of geodetic measurements or instruments, e.g. the satellite altimetry integrates elements from RHS, RVS and such from the tidal, atmospheric and timed system, i.e. the altimetry is sensitive to the *vertical* and 3D-space and to the geophysical and physical systems,

- the integrating model whose results - the parameters that are being determined, are elements from RS which differs from the *system* that contains the used data,
- the methods for combined processing of the data from space $\Im$,
- the result from combination of the integrating technology or measuring process and integrating model,
- the optimization and transformation model connecting the elements from $\Im$,
- the *integrating connection* - the dependence between elements from space $\Im$,
- the *Integrating Comparison* ($\mathrm{IC_p}$) between elements established in the previous $\mathrm{IP^S}$, i.e. this is IP from the second generation,
- the *Integrating Constraint* ($\mathrm{IC_s}$) for the data or unknown parameters in adjustment and calculating model - this is physical or geometrical dependence that the elements should be satisfied (if these parameters are also the observation values, then they are involved in the adjustment as *synthetic observations*).

The CTRS performes a basic role in derivation of integrated GS. In other words the geodetic data and corresponding elements from $\Im$ have to be referred to the *global geocentric conventional terrestrial coordinate systems.* This is being realized in a large measure by the GPS-technology, which underlines almost all $\mathrm{IP^S}$ corresponding to the integral BGN. The *Geodetic Reference System* (GRS) performs the integrating role too, providing equal constants and parameters for all $\mathrm{RS^S}$. This role is described in details in (*Stoyanov and Ivanov, 2000*).

The modelling of the geoid, which is the classical example for IP - an integrating model, connects data and elements from RHS, RVS and RGS. The other simple and effective integrating model is the geometric method for geoidal determination. It integrates the GPS-technology, typical of RHS, with the geometric levelling, typical of RVS. The result is undulation of the geoid that is traditionally determined in RGS.

An example of a geometric $\mathrm{IC_s}$ is the most frequently applied *constraint*:

$$h - H - \zeta = 0, \tag{1}$$

where h is the ellipsoidal height, H – the normal height and ζ – the height anomaly.

The examples of physical $\mathrm{IC}_{\mathrm{s}}^{\mathrm{S}}$ are different *boundary conditions* in mixed *Geodetic Boundary Value Problems* (GBVP), connecting the geopotential – a physical measure for height (achieved most frequently by levelling and gravimetry) above the $\mathrm{C_vERS_f}$, with gravimetric data of the type of different gravity anomalies. A similar *constraint* is the dependence of the Bruns theorem which is a unique $\mathrm{IC_s}$, linking geometry and physics.

There exist $\mathrm{IP^S}$ which are basic in:

- design and optimising of integral BGN,
- unification of structural basis and standards (*Stoyanov, 1996*), measuring technologies and combined processing techniques,
- consisting of *datums* and derivation of *integrating geodetic datum*,
- building of consistent transformation models among elements and epochs of different $\mathrm{RS^S}$ and extracting the parameters from them (*Stoyanov, 1996*),
- working out of integrating models for combination and comparison of inhomogeneous networking observations from space $\Im$ by means of overcoming their differences in datum and stochastic features with the help of appropriate *synthetic – stochastic* and *functional, models*,
- connection and consisting of $\mathrm{RS^S}$ within the framework of the national and international (European and world) GS.

The modelling processes arouse precondition for the another $\mathrm{IP^S}$. Here the *Modelling of Quazigeoid* (MQGd) through GBVP is a powerful IP. The contemporary interpretation of GBVP is based on inhomogeneous geodetic measurements on the physical surface of the Earth (e.g. gravimetric, coordinate) and satellite (e.g. GPS, altimetric, gradiometric), as well as the additional information about the *global geopotential models*, the *digital topographic models*, etc. The solutions of $\mathrm{GBVP^S}$ determinate directly 1) the physical Earth surface or the quazigeoid surface in particular – a *problem with a free boundary*, 2) the geopotential W or the geopotential number $C = W_0 - W$ above C_vERS_{f} $W_0 = \mathrm{const}$ - *problem with a fixed boundary.*

H. Moritz (*Moritz, 2001*) names aptly the last problem as GPS-GBVP. It appears as a physical more common IP, which result W (with a quality of height in particular) calculated by gravimetric data can be compared with the normal height determined by levelling and gravimetry. GPS-GBVP is an illustration of the mutual *integrating connection* between elements of 3D-space and V by modelling and imposition of specific physical *constraint.* In essence this physical approach for determination of heights by gravimetric data is an IP with physical nature.

Between the technological and modelling processes for elements from different $\mathrm{RS^S}$ can be (see Fig. 2) established 1) integration of technologies, 2) realization of bilateral and multilateral $\mathrm{IP^S}$ and $\mathrm{IC_p^S}$, 3) derivation of gravity potential W_0 on the mean $\mathrm{C_v}ERS_\mathrm{f}$, 4) *integrating connection* in the frame of one VD or between different $\mathrm{VD^S}$ by fundamental relation

$$\Delta H_{\mathrm{ZP}}^{\mathrm{ij}} = \Delta h - \Delta\zeta^{\mathrm{ij}} - (\Delta H_{\mathrm{B}}^{\mathrm{j}} - \Delta H_{\mathrm{A}}^{\mathrm{i}}). \tag{2}$$

The symbols in (2) and Fig. 2 are: $\Delta h(\Delta h^{\mathrm{pq}})$ - the difference in ellipsoidal heights h of point B and A which are refereed to one and the same *Horizontal Datum* (HD) (to two different $\mathrm{HD^S}$, $\mathrm{HD^q}$ and $\mathrm{HD^p}$ respectively, derived by using of two different measuring techniques or valid for two countries); $\Delta H(\Delta H^{\mathrm{ij}})$ - the difference between normal height H_B and H_A (H_B^j and H_A^i) of point B and A which are refereed to one and the same VD (to two different $\mathrm{VD^S}$, $\mathrm{VD^j}$ and $\mathrm{VD^i}$ respectively); δg - gravity anomaly; $\Delta\delta g^{\mathrm{ij}}$ - bias in the gravity anomalies caused by the VD-inconsistencies; ΔW - the difference between the gravity potential in point B and A; $\Delta H_\mathrm{A}^\mathrm{i}$ - the altitude difference of point A which is relative to normal height $H_{\mathrm{ZP}}^{\mathrm{i}}$ of *Zero Point* (ZP) for reading of heights in relation to $\mathrm{VD^i}$; $H_{\tilde{O}}^{\mathrm{i}}$ - the normal height of *Vertical Origin* (VO) point, denoted by $\tilde{O}$, in relation to $\mathrm{VD^i}$; $\Delta H_{\mathrm{ZP}}^{\mathrm{ij}}$ - the difference between normal height $H_{\mathrm{ZP}}^{\mathrm{j}}$ and $H_{\mathrm{ZP}}^{\mathrm{i}}$; $\Delta H_{\mathrm{SST}}^{\mathrm{ij}}$ - the same as ΔH^{ij} in the case of A, $B \in$ SST (*quasistationary Sea Surface Topography*); $\Delta\zeta^{\mathrm{ij}}$ - the difference in height anomaly ζ of point B and A, which are refereed to $\mathrm{VD^j}$ and $\mathrm{VD^i}$ respectively; i, $j(p,q)$ - the number of VD (or HD) using in different measuring/technological processes or different countries; ΔW_0^{ij} - the difference between potential W_0^{j} and W_0^{i}, which are refereed to $\mathrm{VD^j}$ and $\mathrm{VD^i}$ respectively; H_m and H_n - the normal height of mareograph station m, respectively n, which are refereed to their common VD; H_{SST} - the normal height of SST; H_{MSL} - the normal height of *Mean Sea Level* (MSL).

The relation (2) is basic for IP realizing a connection between two $\mathrm{VD^S}$ by means of trilateral IP between levelling, space technologies and GBVP. The analogous results are being obtained from the trilateral $\mathrm{IP^S}$ between

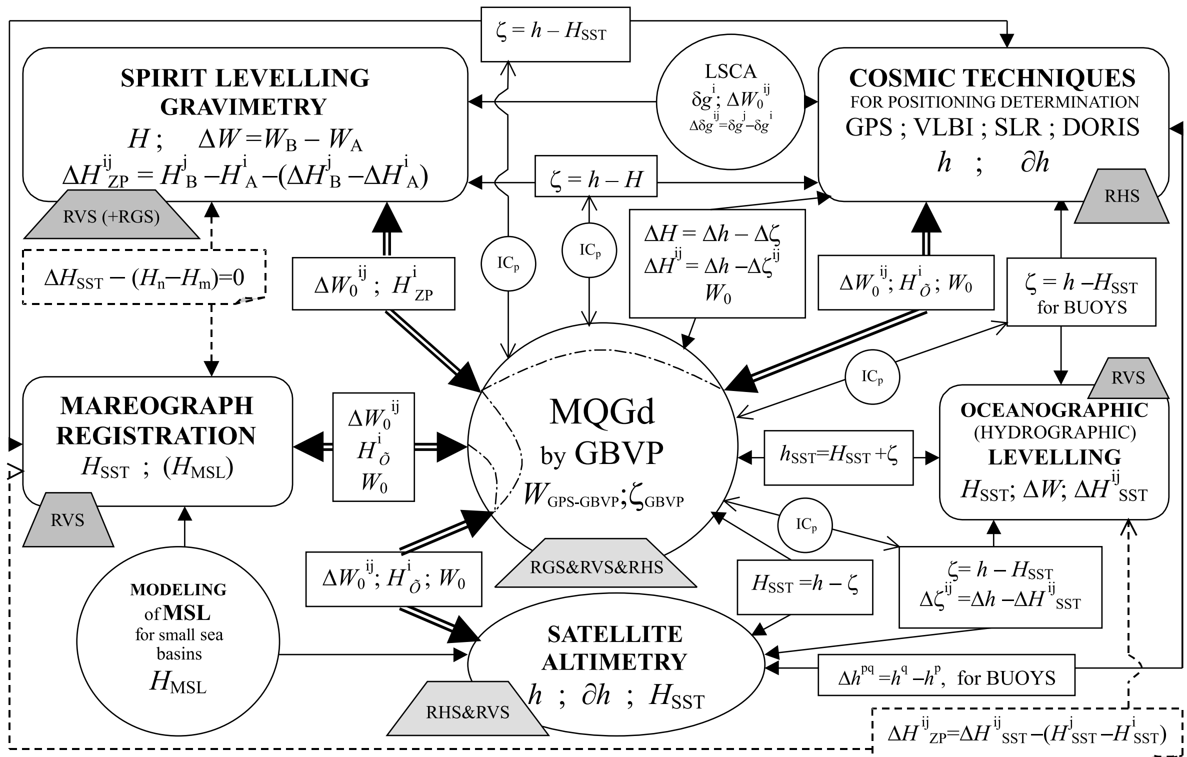


Fig. 2. *Integration processes* between the elements of different RS^S establishing *connections* between: 1) GBVP and VD, 2) different VD^S, 3) different technologies in case of deriving the geopotential differences ΔW, the bias $\Delta\delta g$ in the gravity anomalies, ellipsoid, normal and quasigeoid heights, their variations ∂ and differences Δ about VD^i and HD^p respectively.

levelling, satellite altimetry and GBVP, as well as between satellite altimetry, mareograph recordings and GBVP.

The solution of the *problem with a fixed boundary* gives the base for $\mathrm{IC}_\mathrm{p}^\mathrm{S}$ between geopotential differences $\Delta W_{\mathrm{GPS-GBVP}}$ and their corresponding once from levelling, ocean levelling or ΔW_{SST}, determined by the geopotentials on the SST in the mareograph stations, forming the VD. These $\mathrm{IC}_\mathrm{p}^\mathrm{S}$ are not depicted on Fig. 2.

The height anomalies ζ determined in the bilateral IP^S (see Fig. 2) are appropriate to compare (fitting) with the values ζ_{GBVP} determined by the GBVP *with a free boundary.* This *comparison* is an optimal IP in essence. The analogous *comparison* of the heights H_{SST} can be carried out. It is between H_{SST} - values determined by the bilateral IP and the indirectly established ones by corresponding technologies. If the values of ζ and H_{SST} are compared, then the transformation models and *connections* can be established and the reference and correction surfaces and parameters derived.

The precise GPS-derived coordinates of levelling benchmarks in two vertical datum zones in combination with gravimetry are the extra information in a *Least-Squares Collocation Adjustment* (LSCA) of gravity anomalies δg. The bias $\Delta\delta g^{\mathrm{ij}}$ and the value ΔW_0^{ij} can be determined by means of this modelling process.

The primary parameter W_0 determining the Earth's dimension has long-term variations. Also W_0 defines directly the *geoid*, *world height system* and fundamental constant in realization of the *space-time* RS - the time scale difference between *Coordinate Geocentric Time* (TCG) and observable *terrestrial time*. It should be noticed that the ITRS-scale is consisted with the TCG. On the other hand, the determination of W_0 remains a problem of GBVP. The GBVP with two different boundary conditions on ocean and lend and two additional conditions has as a result two constants (W_0 and another one for land) and pieces of a broken $\mathrm{RS_f}$ VD (see *Sacerdote and Sanso, 2001*). That is why we consider the determination of W_0 and derivation of integrated GS is possible in regional scale in the actual reality.

The realization of IP^S and $\mathrm{IC}_\mathrm{p}^\mathrm{S}$ between different $\mathrm{RS_f}$ VD on solid and ocean Earth which are derived from inhomogeneous measurements and modelling, as well as the *comparisons* of surfaces derived from different measurement technologies, are the basic problems in integration and addition of these surfaces. The illustration of these IP^S gives the technology scheme on Fig. 3.

4. Structure of the integral basic geodetic network

In the BGN-stations is desirable to include the following complex data:

- the astronomical coordinates $\{\Lambda, \Phi\}$ and the astronomical azimuths A,
- the mareograph and tidal registrations,
- the parameters of crustal dynamics determined by permanent geodetic and geophysics trackers,
- the additional (meteorological, geophysical) parameters,
- the 3D geocentric coordinates at a particular epoch and their velocities,
- the gravity values and its variations,
- the normal heights (the gravity potentials or geopotential numbers respectively) of fundamental (secular) benchmarks and their vertical velocities derived from a kinematic adjustment of all measurements.

The obligatory observations - new and old in every BGN-point, are the last three.

5. On the integrating technologies in the realization of basic geodetic network

Till now in the geodetic practice the three main types of national/regional networks from corresponding $\mathrm{RS^S}$ have referred to three different datums. The space technologies achieve the necessary information for geocentric linking of these networks. Through this technology the horizontal and vertical observations can be integrated and 4D *integrating geodetic datum* can be derived. Here the separation of sea level from vertical changes of solid Earth surface has to be taken into account imperatively. The derivation of the *equipotential* and *reference surfaces* and their described parameters is of primary significance in realization of BGN. An additional *constraint* would be the spectral harmonization of the existent gravimetric quasigeoid and the same one from GPS/levelling.

The local/regional GPS-solution (used in Section 6 as a background) of

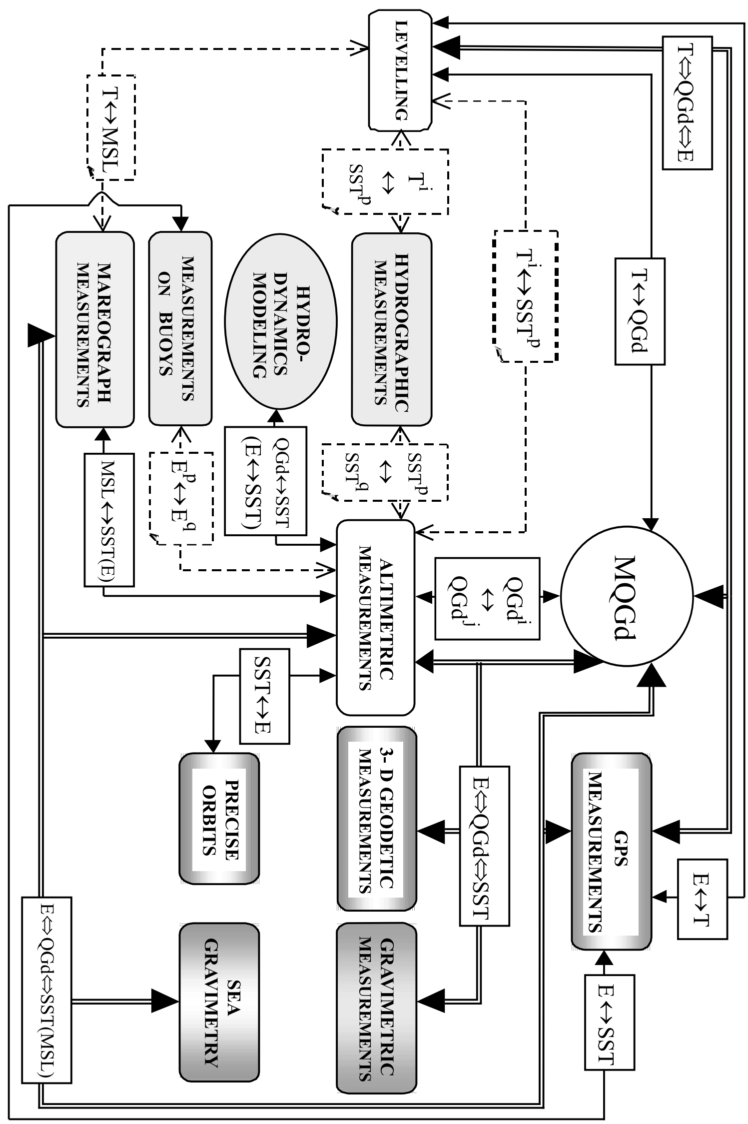


Fig. 3. Technological scheme of *integrating processes* and *comparisons* between heterogeneous measurements and modellings for establishment of connection between the surfaces of ellipsoid (E), topography (T), SST, MSL and quasigeoid (QGd) in relation to VD^i and HD^p.

BGN-station positions has respective datum (origin and scale but not orientation), that could be far from that of ITRS i.e. *integrating horizontal geodetic datum*. This datum discrepancy is due to network configuration effect. An optimal procedure for embedding the GPS-solution in the global ITRS has to be performed. This could be achieved by 1) constraining coordinates of a subset of BGN-stations to their ITRS-values, 2) reducing the regional GPS-solution to the geocentric datum of ITRS by means of transformation model.

It is desirable that the specific measuring integrating technologies for determining the component of deflection of vertical $\{\eta, \xi\}$ are applied in every point of proposed BGN. This is not substitution of astronomical observations, but is the duplicated deduce of $\{\eta, \xi\}$ with alternative technological procedures. Thus the possibility arises to perform an IC_p between "technological" and astrogeodetic deflections of vertical in astronomical BGN-points. By means of including of classical angular-distance and levelling information in integrating technologies the succession of part of the data from existent networks is reached.

Here in capacity of the basic combined measuring technologies in the building of integral BGN are suggested 1) GPS/levelling, 2) GPS/zenith camera/levelling, 3) GPS/Local Positioning System/levelling. By means of these three integrating technologies the *quasigeoidal deviation parameters* $\{\eta, \xi, \zeta\}$ at every BGN-points can be determined. Thus the lack of detailed and accurate quasigeoid can be overcome. In the frame of a *consistent system* the comparison of results from these three technologies with the data from the existing national networks can be realized without transition to the separately derived independent RS^S.

For determination of the direction of the verticals in BGN-stations a geometrical IP can be applied (*Grafarend and Awange, 2000*) or an optimal calculation of $\{\eta, \xi\}$ in terms of the deflections along arbitrary azimuths (*Soler et al., 1989*). The two approaches use a radial configuration of baselines at the central BGN-stations without performing astronomic observations. A high-precision astrogeodetic observation system (*Hirt and Buerki, 2002*) for real-time measurement of $\{\eta, \xi\}$, using modern CCD-technology for imaging stars and a GPS-receiver, is the most perspective integrating technology in the realization of BGN. Naturally, it is also appropriate here to perform one IC_p between these components of the deflection of the vertical and their astronomic equivalent in the astronomic BGN-points. At the same time, it can be added the value ζ determined by integrating technology GPS/levelling or by the proposed integrating model in Section 6. In this way for every BGN-point we have two sets of *deviation parameters* - astrogeodetic and "technological", and they can be compared to third set from gravimetric equivalencies. As a consequence the *transforming surface model* for *deviation parameters* can be derived basically from IC_p between these three sets.

The newly determined GPS-coordinates in ITRS in combination with *parameters* $\{\eta, \xi, \zeta\}$ are the premise for smaller distortions of BGN and more regular distribution of the random errors. In this way the BGN-points will play a role of "collocation" stations for low-order networks. The BGN-stations as a carrier of *basic quasigeoid* are free from systematic errors of its gravimetric determination. In these stations the close connection have to be realized between the triangulation, levelling, astronomic and gravimetric existing and newly measured information. The time/space-variable connections should be established between old and *integrating* datums and data.

6. Adjustment of 1D heterogeneous height data for the basic geodetic network

The vertical integrating modelling has to be applied to all possible heterogeneous height data in derivation of RVS (*Kotsakis and Sideris, 1999*). Here this problem will be solved in the absence of reliable quasigeoidal information. We shall propose an integrating model for adjustment of levelling, absolute mareograph and GPS-heights with the attendant assessment of ZP and determination of a discrete *basic quasigeoid* in BGS-stations. The results will be achieved without determination of one correcting proofed 2D-background, provoked from datums and systematic deformations. The aim is to determine one homogeneous and optimal *height system* from heterogeneous height data. This *system* is materialized by one compound 1D network. This is not a *hybrid network* consisting of the elements of one RS only. The uniqueness of the proposed integrating adjusted model is the optimal unification of the geometrical GPS-heights and physical quantities -

leveled, related to MSL, SST or to $\mathrm{RS_f VD}$. The suggested integrating model will be "anchored" to the geometrical GPS-heights by reason of their high absolute precision.

6.1 Heterogeneous height data

Let's have at our disposal two types of height information in the BGN-stations - GPS-ellipsoidal heights and levelling/mareograph/geopotential values. These heights differ in 1) their own physical essence, 2) the definition and realization of respective $\mathrm{RS_f VD}$, 3) the observation methods and their precision, 4) the datums to which they refer, 5) two types of $\mathrm{RS^S}$ to which they refer.

Two height types are affected by random noise, distortions (as a result of inconsistency of the datums), systematic deformations and influence of geodynamic effects. The normal heights are affected with the same type of mistakes. The approximations in physical height calculation and estimation of instrumental and other errors of mareograph registrations load additionally the normal heights.

The network of duplicated GPS-stations with levelling benchmarks we denote as GLN. When the tide gauge stations are added to GLN, the network is denoted as GLTGN. These networks must be related to the unified *integrating geodetic datum*.

For the respective data in force are

$$h_{\mathrm{i}}^{o} = h_{\mathrm{i}} + ds_{\mathrm{i}}^{\mathrm{h}} + \upsilon_{\mathrm{i}}^{\mathrm{h}}, \tag{3}$$

$$H_{\mathrm{i}}^{o} = H_{\mathrm{i}} + ds_{\mathrm{i}}^{\mathrm{H}} + \upsilon_{\mathrm{i}}^{\mathrm{H}}, \quad \text{or} \quad W_{\mathrm{i}}^{o} = W_{\mathrm{i}} + ds_{\mathrm{i}}^{\mathrm{W}} + \upsilon_{\mathrm{i}}^{\mathrm{W}}, \quad i = 1, 2, \ldots n, \tag{4}$$

$$_{\mathrm{SST}}H_{\mathrm{k}}^{o} =_{\mathrm{SST}} H_{\mathrm{k}} + ds_{\mathrm{k}}^{\mathrm{SST}} + \upsilon_{\mathrm{k}}^{\mathrm{SST}}, \quad k = 1, 2, 3, 4, \quad \text{(accepted for simplicity)}, \tag{5}$$

where h_{i}^{o}, H_{i}^{o}, W_{i}^{o} and $_{\mathrm{SST}}H_{\mathrm{k}}^{o}$ are the observed values of the ellipsoidal height h, normal height H, gravity potential W and normal height $_{\mathrm{SST}}H$ of SST; h_{i}, H_{i}, W_{i} and $_{\mathrm{SST}}H_{\mathrm{k}}$ are the corresponding true values with respect to *integrating geodetic datum*; $ds_{\mathrm{i}}^{\mathrm{h}}$, $ds_{\mathrm{i}}^{\mathrm{H}}$, $ds_{\mathrm{i}}^{\mathrm{W}}$ and $ds_{\mathrm{k}}^{\mathrm{SST}}$ are the quantities, which correspond to all necessary reductions, which must be applied to original heights and geopotential data to eliminate the datum inconsistencies and systematical errors; $\upsilon_{\rm i}^{\rm h}$, $\upsilon_{\rm i}^{\rm H}$, $\upsilon_{\rm i}^{\rm W}$ and $\upsilon_{\rm k}^{\rm SST}$ are the corresponding values describing the random zero-mean errors, for which the second-order stochastic model is in force

$$E\{\boldsymbol{\upsilon}_{\rm h}\boldsymbol{\upsilon}_{\rm h}^{\rm T}\} = \mathbf{C}_{\rm hh}; \quad E\{\boldsymbol{\upsilon}_{\rm H}\boldsymbol{\upsilon}_{\rm H}^{\rm T}\} = \mathbf{C}_{\rm HH}; \quad E\{\boldsymbol{\upsilon}_{\rm W}\boldsymbol{\upsilon}_{\rm W}^{\rm T}\} = \mathbf{C}_{\rm WW};$$
$$E\{\boldsymbol{\upsilon}_{\rm SST}\boldsymbol{\upsilon}_{\rm SST}^{\rm T}\} = \mathbf{C}_{\rm SSTSST}. \tag{6}$$

The integration of altimetry in the VD-problem depends on the orbit error model. Upon the implementation of this integration in the subsequent stage, it is imperative to use the fixed number of mareographs, which stabilize the inner geometry of the altimeter solution. In order to realize this IP between the altimetric, mareograph and marine GPS-measurements on buoys, four tide gauge stations are necessary at least (*Groten and Mûller, 1990*).

As a result of the adjustment (using products of *International GPS Service for geodynamics*) of GPS-measurements of BGN, the covariance matrix $\mathbf{C}_{\rm hh}$ is known. As a result of the preliminary adjustment of levelling layer of BGN, the covariance matrix $\mathbf{C}_{\rm HH}$, respectively $\mathbf{C}_{\rm WW}$ is known. The determined heights ${}_{\rm SST}H_{\rm k}$ in four tide gauge stations with the corresponding covariance matrix $\mathbf{C}_{\rm SSTSST}$ will be included in the forming of VD.

In the proposed integrating model we put the requirement for determination of VD by fixing it with respect to the HD, which is the conception of derivation of *integrating geodetic datum*. It's desirable that the new *datum* will be free from distortions in the two height data sets (4) and (5) as a result of overconstrained adjustment in the initial benchmarks. The constraints from this nature must be embedded in the proposed model only in the mareograph stations forming the VD.

6.2. Adjustment of 1D heterogeneous height data with estimation of the basic quasigeoid

Here one fitting of levelling network to the GPS-stations of GLN will be realized in order to eliminate the long-wavelet levelling errors. Fortunately this fitting will effect local deformations of levelling network if the combined adjustment model is applied only in its capacity of the fitted procedure. The suggested integrating model combines the high relative precision of levelling with high absolute precision of ellipsoidal heights.

The following model for an estimation of the quasigeoid is proposed

$$W^o_{\rm ij} - v^{\rm W}_{\rm ij} + \gamma^{\rm m}_{\rm j}(h_{\rm j} - \zeta_{\rm j}) - \gamma^{\rm m}_{\rm i}(h_{\rm i} - \zeta_{\rm i}) = 0, \quad i \neq j, \quad i, j = 1, 2, \ldots, n, \qquad (7)$$

where $W_{\rm ij} = W_{\rm j} - W_{\rm i}$ are the differences of the gravity potential in two GLN-points, presented with their observed values $W^o_{\rm ij}$ and with the residuals $v^{\rm W}_{\rm ij}$, $\gamma^{\rm m}_{\rm i}$ – the mean integral value of the normal gravity at point i, $\zeta_{\rm i}$ – height anomaly which will be deduced as a result of this adjustment, referring to the non-equipotential $\mathrm{RS_f}VD^{\rm GLN}$.

The quantity $v^{\rm W}_{\rm ij}$ is influenced mainly by random errors in the optical levelling and gravimetry, realized over the loops of the GLN, so that the observations $W^o_{\rm ij}$ can easily be weighted.

In the model (7) we put the values $h_{\rm i}$ from the three-dimensional adjustment model of GPS-measurements. One possibility is that $h_{\rm i}$ will be treated as elements of a constant vector $\mathbf{h}$ which has a zero variance, so that $\mathbf{h}$ does not have change in the adjustment. On the opposite of this we treat $\mathbf{h}$ as a "observations" with a corresponding cofactor matrix $\mathbf{Q}_{\rm hh}$ and a residual vector $\boldsymbol{v}_{\rm h}$. In this way the usual role of the *constraints* is replaced by their rendering in *conditions*. Thus at the beginning all variables, included into the mathematical formulation, are considered as "observations".

We write (7) into matrix form

$$\mathbf{W} + \boldsymbol{v}_{\rm W} + \mathbf{B}(\mathbf{h} - \boldsymbol{\zeta}) = \mathbf{0}, \qquad (8)$$

where $\boldsymbol{\zeta}^{\rm T} = [\zeta_1, \zeta_2, \ldots, \zeta_{\rm n}]$ is the column vector of the "corrections"; $\mathbf{W}$ – the observed vector of order $n^2 \times 1$ with a-priori cofactor matrix $\mathbf{Q}_{\rm WW}$; $\mathbf{B}$ – the known matrix of order $n^2 \times n$, containing the values $\gamma^{\rm m}_{\rm i}$.

The most important here is the fact that $\boldsymbol{\zeta}$ takes the role of "residuals" (*Mikhail, 1976*). In spite of the fact that $\boldsymbol{v}_{\rm W}$ is the vector of the residuals, $\boldsymbol{\zeta}$ is the "residual" for the "observations" $\mathbf{h}$, i.e. we have the a-priori "observations" of the parameters. We can take into consideration all of this if at the end of adjustment the value of the estimated parameter $\hat{\mathbf{h}}$ is set to be identical to the "estimated observation" $\mathbf{h}$, i.e.

$$\hat{\mathbf{h}} = \mathbf{h} - \boldsymbol{\zeta} = \mathbf{h} + \boldsymbol{v}_{\rm h} \qquad (9)$$

or

$$\boldsymbol{\upsilon}_{\mathrm{h}} + \boldsymbol{\zeta} = \mathbf{0}. \tag{10}$$

From (8) and (10) we have

$$\acute{\boldsymbol{\upsilon}} + \acute{\mathbf{A}}\boldsymbol{\zeta} = \mathbf{f} \tag{11}$$

with total a-priori cofactor matrix

$$\acute{\mathbf{Q}} = \begin{bmatrix} \mathbf{Q}_{\mathrm{WW}} & \mathbf{0} \\ \mathbf{0} & \mathbf{Q}_{\mathrm{hh}} \end{bmatrix}, \tag{12}$$

where

$$\acute{\boldsymbol{\upsilon}} = \begin{bmatrix} \boldsymbol{\upsilon}_{\mathrm{W}} \\ \boldsymbol{\upsilon}_{\mathrm{h}} \end{bmatrix}, \quad \acute{\mathbf{A}} = \begin{bmatrix} -\mathbf{B} \\ \mathbf{I} \end{bmatrix}, \quad \mathbf{f} = \begin{bmatrix} -\bar{\mathbf{f}} \\ \mathbf{0} \end{bmatrix}, \quad \bar{\mathbf{f}} = \mathbf{W} + \mathbf{B}\mathbf{h}. \tag{13}$$

The least squares solution for equation (11) is

$$\boldsymbol{\zeta} = \tilde{\mathbf{N}}^{-1}\mathbf{B}^{\mathrm{T}}\mathbf{Q}_{\mathrm{WW}}^{-1}\bar{\mathbf{f}}, \tag{14}$$

$$\acute{\mathbf{N}} = \acute{\mathbf{A}}^{\mathrm{T}}\acute{\mathbf{Q}}^{-1}\acute{\mathbf{A}} = (\mathbf{N} + \mathbf{Q}_{\mathrm{hh}}^{-1}), \tag{15}$$

where $\mathbf{N}$ is the normal matrix of (8).

Thus by means of (14) and (15) the height anomalies vector $\boldsymbol{\zeta}$ are evaluated in the GLN-points while the a-priory value $\mathbf{h}$ stay unchanged.

Naturally one possibility to control the calculations of the normal heights with relation to $\mathrm{RS_fVD^{GLN}}$ by means of the adjusted geopotential differences can be achieved with the geometrical *integrating constrain* of kind (1).

6.3. Adjustment of 1D heterogeneous height data with evaluation of the basic quasigeoid and the vertical origin

It is desirable that the deduced estimations for the absolute heights ${}_{\mathrm{SST}}H_{\mathrm{k}}$ will be processed together with the levelling and GPS-information by means of an optimal integrating model taking into consideration the difference between the MSL and SST in all mareograph stations.

The most representative mareograph station - reliable instrument with long series in the stable technogenical and geodynamical situation, can be selected as a reference station. Let it be the mareograph one with number 2. The three geopotential differences calculated in reference to it are

$$R_{2q} =_{SST} H_2\gamma_2^m -_{SST} H_q\gamma_q^m, \quad q = 1, 3, 4. \tag{16}$$

From (4) and (16) we obtain

$$R_{2q} + \upsilon_{2q}^R = R_{2q}^o, \tag{17}$$

where the residual

$$\upsilon_{2q}^R = \gamma_2^m \upsilon_2^{SST} - \gamma_q^m \upsilon_q^{SST} \tag{18}$$

has the same physical meaning as υ_{ij}^W from (4) and

$$R_{2q}^o = \gamma_2^m \, {}_{SST}H_2^o - \gamma_q^m \, {}_{SST}H_2^o. \tag{19}$$

We denote with $\mathbf{R}_o$ the vector composed from (19) and with $\boldsymbol{\upsilon}_R$ the vector composed from the residuals (18). The vector of unknown parameters - the true values of those three geopotential differences related to RS_f VD^{GLTGN}, is denoted with $\mathbf{R}$. Then the observation equations are

$$\boldsymbol{\upsilon}_R + \mathbf{CR} = \mathbf{R}_o, \tag{20}$$

with corresponding cofactor matrix $\mathbf{Q}_{RR}$.

The least squares solution of (20) and (11) with corresponding weight matrix

$$\mathbf{P} = \begin{bmatrix} \mathbf{Q}_{RR}^{-1} & \mathbf{0} \\ \mathbf{0} & \acute{\mathbf{Q}}^{-1} \end{bmatrix} \tag{21}$$

is reduced to the normal equation

$$(\mathbf{C}^T\mathbf{Q}_{RR}^{-1}\mathbf{C} + \acute{\mathbf{A}}^T\acute{\mathbf{Q}}^{-1}\acute{\mathbf{A}}) \begin{bmatrix} \mathbf{R} \\ \varsigma \end{bmatrix} = \mathbf{C}^T\mathbf{Q}_{RR}^{-1}\mathbf{R}_o + \acute{\mathbf{A}}^T\acute{\mathbf{Q}}^{-1}\mathbf{f}. \tag{22}$$

At the end it's desirable to transfer the heights related to non-equipotential RS_fVD^{GLTGN} to the heights related to the C_vERS_f W_0 = const, passing through ZP at the reference mareograph 2, by the derived value of ZP-height above RS_fVD^{GLTGN} by means of special methodic.

The estimated differences determined between the reference and remaining mareograph stations play the role of *weighted conditions* in the adjustment of GLTGN heights, in contrast to the *strict constraints* which can be imposed so that (16) will be equal to zero, i.e. the mareographs will be considered located on one equipotential surface. This contradicts to the recent practice which takes into consideration the deviation of SST from one equipotential surface. That is way, the static integrating model (11), (12), (20) and (21) is proposed for processing of the BGN-height heterogeneous data for determination of the *basic quasiqeoid* and the normal heights with respect to C_vERS_f.

7. Conclusion

The idea for the "common points", underlying in the construction of BGN as carriers of all information, creates premises for applying the corresponding *integrating processes* in the design, optimization and modelling of the network. The realizations of these *processes* provide 1) the optimal measuring schemes, technologies and projects, 2) the consistency and modelling of heterogeneous data, 3) the evaluation of the common integrated reference parameters, *surfaces* and *datums.* If it is derived an integrated local *geodetic system* by the proposed mode, then the prerequisites exist for its integration with the remaining national *geodetic systems* in one regional or continental *system.* The suggested adjustment model for heterogeneous height data can be applied to derivation of one regional *vertical datum* and corresponded to it *basic quasigeoid* when it does not have one a-priori available with the necessary precision information about the quasigeoid. This demonstrates the suitability of the GLTGN as one component of a fundamental framework for a new regional *vertical datum* taking into consideration the errors associated with the height component of the GPS-solution.

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