Strain measurements at the Vyhne tidal station

L. Brimich
Geophysical Institute of the Slovak Academy of Sciences[1]

Abstract: In the paper the results of the strain measurements at the Vyhne tidal station are presented. From the tidal deformations observed at the Vyhne tidal station the resonance effect of the liquid core of the Earth was confirmed. The thermoelastic deformations due to the annual changes of the air temperature with the observed slow deformations were compared. It was shown that the periodic part of the slow deformations observed at the Vyhne tidal station with the period about 1 year are caused by thermoelastic deformations due to the annual variations of the air temperature.

Key words: Earth's tides, tidal deformations, extensometric measurements, Earth core resonance

1. Introduction

Investigation of the Earth crust deformation is of great interest for studying recent global and geodynamical processes. For this reason a quartz tube extensometer was installed at the tidal station of the Geophysical Institute of the Slovak Academy of Sciences in Vyhne. High precision 10^{-9} – 10^{-11} extensometric (strain) data have been used to study the tidal deformations of the Earth's crust and the slow deformations connected with the local tectonic conditions.

2. Description of the station

The Vyhne tidal station is located in Central Slovakia in the cadastre of the village of Vyhne (about 10 km from Banská Štiavnica) in the St. Anthony of Padua gallery. Its geographic coordinates are:

[1] Dúbravská cesta 9, 845 28 Bratislava, Slovak Republic; e-mail: geofbrim@savba.sk

$\varphi = 48^\circ\, 29'\, 52''$ (latitude),
$\lambda = 18^\circ\, 49'\, 48''$ (longitude),
$h = 420\,\mathrm{m}$ (height).

The neighbourhood of the St. Anthony of Padua gallery is built of Palaeozoic, Mesozoic, Paleogene and Neogene rocks. At the time of the Younger Palaezoic pronounced faults were generated here striking NW-SE. These faults were generated several times in the subsequent periods. In the Mesozoic the fundamental fault zone was generated striking NE-SW. Sketch map of the tidal station in Vyhne is in the Fig. 1 (*Brimich, 1988)*.

The fault line, regenerated tectonically several times, later separated the rising "Hodrušsko-Vyhniansky ostrov" from the neighbouring SE depression. Diorite-gabrodiorite intrusions and an older body of the Vyhne gran-

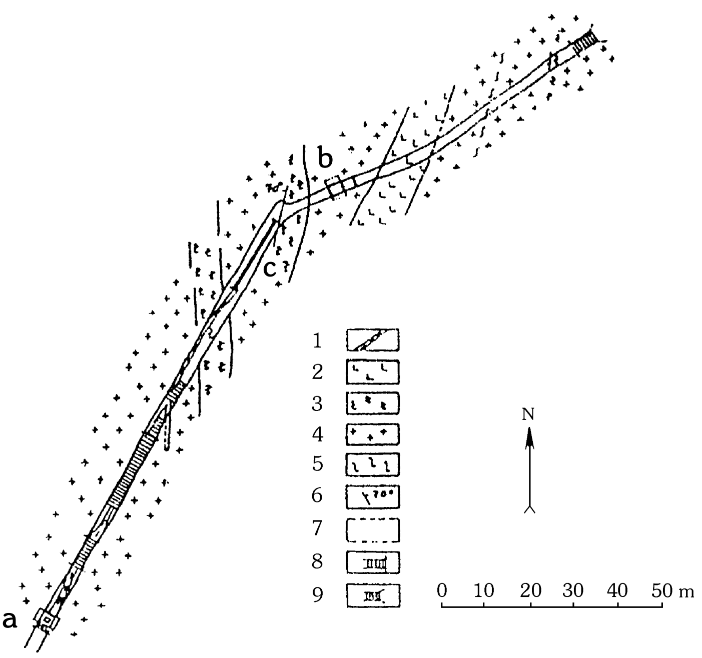

Fig. 1. Tidal station in the Vyhne. Geologic and tectonic situation in the Anton Paduánsky gallery: 1-granite zone with mylonites and quartz; 2-Dacite (Neogene); 3-ancient zone in granite; 4-granite (Vyhne ancient granite - Palaeozoic); 5-fault; 6-fault with its slope; 7-location of instruments (a-tiltmeter chamber, b-recording instrument chamber, c-extensometer); 8-walled-up areas; 9-entrance to gallery.

ite form are a more rigid element in the structure of the complex, whereas the most tectonically mobile zone is situated at its edge. The gallery was mostly driven in the Variscan-age granites, which were disturbed tectonically on two occasions. The first motion caused the mylonitization of the granites, the others were accompanied by young intrusions and mineralizations of the tectonically modified medium (*Dudášová, 1998*).

The relative humidity in the gallery is 80% and air temperature 6.8° C. The seasonal changes of the air temperature between summer and winter at the location, where the extensometer is installed, protected by a polystyrene cover, are ±0.04° C. The relative depth of the extensometer location is 50 m.

The deformations of the Earth's crust at the Vyhne tidal station are recorded by a rod extensometer manufactured in the Institute of Physics of the Earth in Moscow. The overall length of the instrument is 20.5 m, the rod of the extensometer is assembled of three-meter quartz pipes with an external diameter of 40 mm and wall thickness of 3 mm. The separate pipes were joined by epoxy cement. The joints were strengthened by invar sleeves. The rod is supported by suspensions spaced at 2.5 m. The whole instrument is covered by polystyrene. The extensometer is cutting across the line of one of local faults. The azimuth of the quartz rod is $55° \, 27' \, 57''$. The fundamental design of the quartz rod extensometer is on the Fig. 2.

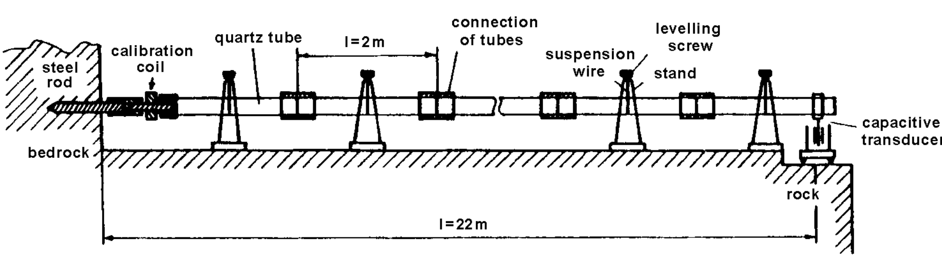

Fig. 2. The fundamental design of the rod extensometer.

Modernization of the Vyhne tidal station was realized in 3 stages. In the first stage in the year 1996 the capacitive transducer constructed at the Geodetic and Geophysical Research Institute of the Hungarian Academy of Sciences in Sopron were installed *(Mentes, 1986, 1995, 1998).* In the second stage in the year 2001 the datalogger CR 10X from Campbell Scientific Ltd.

were installed and in the third stage in the year 2005 online connection of the tidal station in Vyhne with the Geophysical Institute SAS in Bratislava was built.

The results of the analysis of the tidal deformations are presented in the Table 1.

The tidal deformations are characterized by small positive anomalies, that dominate in the regions characterized by high heat flows. The neighbourhood of the tidal station Vyhne belongs to such a region.

3. Liquid core resonance effect

The information about the interaction between the core and the mantle of the Earth which is presented in the liquid core resonance effects in the Earth's tides is contained (*Brimich and Latynina, 1989*). In the past the tidal deformations were not used for the study of this resonance effects because they were influenced by inhomogeneities of the Earth's crust. In specific combination of the Love's number h and Shida's number l of the waves O_1 and K_1 the influences of these inhomogeneities are not contained and it is possible to use it for the study of the resonance effects. This combination is the ratio

$$R = h(O_1)/h(K_1)$$

where: $h(O_1) = h_1 - 2l_1(1 + \cos^2 a)$,
$h(K_1) = h_2 - 2l_2(1 + \cos^2 a)$,

Here h_1, h_2 are Love's numbers of the waves O_1 and K_1, l_1, l_2 are Shida's numbers of the waves O_1 and K_1, and a is azimuth of the extensometer.

The theoretical values $h$and l for the waves K_1 and O_1 for the Molodensky's and Wahr's models are known. The observed wave O_1 and K_1 can be written as follows:

$$A(O_1)\cos\varphi_1) = E[h_1 - 2l_1(1 + \cos^2 a)]\sin 2\Theta,$$
$$A(K_1)\cos\varphi_2) = 1.407E[h_2 - 2l_2(1 + \cos^2 a)]\sin 2\Theta,$$

Table 1. Results of the analysis of the tidal deformations

```
 Program ANALYZE, version 3.40 970921               File:   40010931
####################################################################
# STATION 0931 VYHNE                HORIZONTAL STRAIN              #
#  48 29 52 N   18 49 48 E     H  420 M   P   50M    D 1000KM      #
# GEOPHYSICAL INSTITUTE, SLOVAK ACADEMY OF SCIENCES                #
# QUARTZ TUBE STRAINMETER                                          #
# DIGITAL RECORDING, 10min. SAMPLE RATE                            #
# CALIBRATION                                                      #
# INSTALLATION     L.LATYNINA,L.BRIMICH                            #
# MAINTENANCE      L.BRIMICH, M.BEDNARIK                           #
#                                                                  #
#                                                                  #
####################################################################
 Latitude: 48.4980 deg, longitude: 18.8300 deg, azimuth: 55.4630 deg.
 20010704...20020211     4 blocks. Recorded days in total:    208.583
 Tamura (1987)          TGP, threshold: 0.100E-06      1200 waves.
 UNITY window used for least squares adjustment.
 Sampling interval:     3600. s
 Numerical filter is  PERTZEV59   with   51 coefficients.

 Average noise level at frequency bands in  nstr
 0.1 cpd***********      1.0 cpd   0.267236       2.0 cpd   0.120835
 3.0 cpd   0.081236      4.0 cpd   0.051240    white noise   0.062193

 adjusted tidal parameters :
```

from [cpd]	to [cpd]	wave	theor. ampl. [nstr]	ampl.fac.	stdv.	ph. lead [deg]	stdv. [deg]
0.501370	0.911390	Q1	1.2103	0.81977	0.17164	-0.4706	11.9913
0.911391	0.947991	O1	6.3214	0.72480	0.03349	6.0069	2.6502
0.947992	0.981854	NO1	0.4971	0.94383	0.36863	-54.9950	22.4148
0.981855	1.023622	PSK1	8.8903	0.67705	0.02146	2.6519	1.8175
1.023623	1.054746	J1	0.4971	0.53181	0.41063	-45.9986	44.2528
1.054747	1.470243	OO1	0.2720	0.70524	0.72867	-37.8120	59.2130
1.470244	1.914128	N2	1.1970	0.78806	0.07611	-0.3733	5.5353
1.914129	1.950419	M2	6.2521	0.72790	0.01543	-2.3902	1.2136
1.950420	1.984282	L2	0.1767	1.27312	0.54056	37.0493	24.3213
1.984283	2.451943	S2K2	2.9088	0.66253	0.03330	9.2044	2.8821
2.451944	3.381378	M3	0.0302	2.74192	2.02747	52.7990	42.3850

```
 Standard deviation:                     2.433   nstr
 Degree of freedom:                       4784
 Maximum residual:                      17.141   nstr
 Maximum correlation:                    0.083 X-wave-O1  with Y-wave-Q1
 Condition number of normal equ.         1.290

 Routine GEOEXT. Execution time=    11.800 sec
```

where: $A(O_1), A(K_1)$ – observed amplitudes of the wave O_1 and K_1,
φ_1, φ_2 – observed phases of the wave O_1 and K_1 ,
Θ – colatitude,
$E = 15.863.10^{-9}$ is the combination of the astronomic elements of the wave O_1.

Using the observed amplitudes and phases we can determine the values $h(O_1)$, $h(K_1)$ and the ratio R. In Table 2 the values of the ratio R are given for the Molodensky's model of the Earth, Wahr's model of the Earth, and the observed value at the Vyhne tidal station.

Table 2. The liquid core resonance effect

Model	Wave	Azimuth	*h*	*l*	$\eta(O_1)$	$\eta(K_1)$	$R=\eta(O_1)\eta(K_1)$
Molodenskyi	O_1		0.614	0.0809			
	K_1		0.535	0.0837			
		0°			0.2904	0.2002	1.45
		90°			0.4522	0.3676	1.23
		55.5°			0.4003	0.3139	**1.275**
Wahr	O_1		0.603	0.0841			
	K_1		0.520	0.0867			
		0°			0.2666	0.1728	1.54
		90°			0.4348	0.3764	1.25
		55.5°			0.3820	0.2919	**1.31**
Observed values	O_1						
	K_1						
		55.5°			0.3422	0.2844	**1.20**

4. Thermo-elastic deformations due to the annual temperature variations

We consider the surface region of the rock massif surrounding the gallery with the extensometer as a homogeneous elastic halfspace $z \geq 0$ with a horizontal surface and Lame's elastic constants λ and μ, temperature conductivity κ, and the coefficient of the linear expansion α (Fig. 3). It is also assumed that the temperature on surface $z = 0$ varies harmonically with the time t, ω being the circular frequency and A the amplitude of the temperature wave at the surface, i.e. $T|_{z=0} = A \cos \omega t$.

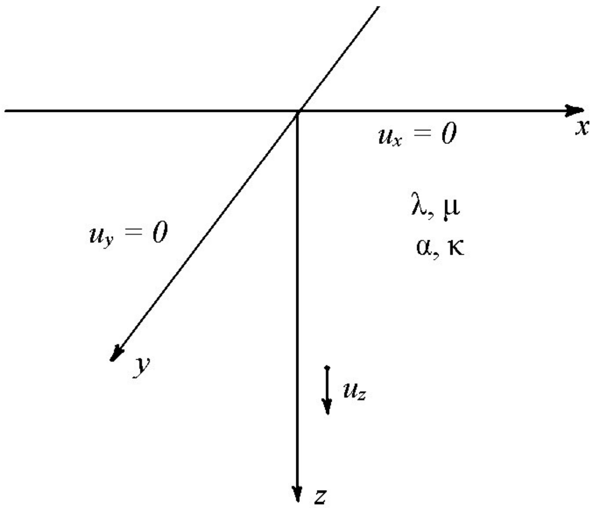

Fig. 3. Model of the horizontal elastic half space.

From the solution of the heat conduction equation we know that the temperature variation at the depth z is given by:

$$T(z,t) = A \exp(-z/\delta) \cos(\omega t - z/\delta),$$

where $\delta = (2\kappa/\omega)^{1/2}$.

The field of elastic deformations due to these temperature variations can be obtained by solving the thermo-elastic equation:

$$(\lambda + \mu) \,\mathrm{grad}\,\mathrm{div}\, \boldsymbol{u} + \mu \,\Delta \boldsymbol{u} - \gamma \,\mathrm{grad}\, \boldsymbol{T} = 0,$$

where $\gamma = (3\lambda + 2\mu)\,\alpha$.

If the temperature $T(z,t)$ is considered to be a complex function

$$T(z,t) = A \exp(-kz) \exp(i\omega t),$$

where $k = (1+i)/\delta = (1+i)(\omega/2\kappa)^{1/2}$, we can express the thermo-elastic equation as:

$$\mathrm{d}^2 u_z/\mathrm{d}z^2 = -\gamma \, k \, A \exp(-kz) \exp(i\omega t)/(\lambda + 2\mu).$$

The resultant solution of this equation will read:

$u_z = -\gamma A \exp(i\omega t)[k^{-1} \exp(-kz) + z]/(\lambda + 2\mu).$

Using the displacement expressed in complex form we are able to estimate the thermo-elastic deformation within the halfspace being considered. The linear deformation in the z direction will read:

$$\varepsilon_{zz} = \mathrm{Re}\{\partial u_z/\partial z\} = -\gamma A\{\cos \omega t - \exp(-z/\delta) \cos(\omega t - z/\delta)\}/(\lambda + 2\mu).$$

With the aid of the solution of the quasi-stationary thermo-elastic equation (the resultant solution for u_z), we can also compute the thermo-elastic deformations inside a slope inclined at angle φ (Fig. 4).

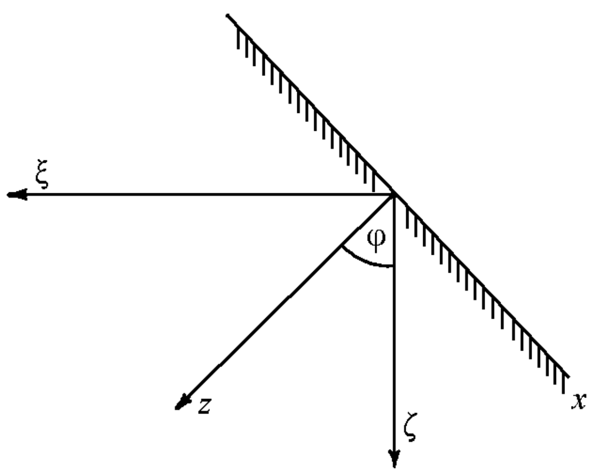

Fig. 4. Model of the inclined elastic half space.

The coordinate axis x and y are assumed to lie in the slope's plane and the axis z is perpendicular to it. To determine the thermo-elastic deformations in a horizontal gallery, we shall introduce a new coordinate system (ζ, η, ξ) whose ξ axis coincides with the gallery axis and ζ-axis makes the angle φ with the z-axis and the η-axis is identical with the y-axis. The displacements along axes ζ and ξ will then be:

$$u_\xi = u_z \sin \varphi, \qquad u_\zeta = u_z \cos \varphi,$$

where $z = \xi \sin\varphi + \zeta \cos\varphi$. Then we shall obtain for the displacement along the ξ axis:

$$u_\xi = -\gamma A \sin\varphi\{k^{-1} - \exp\left[-k(\zeta\cos\varphi + \xi\sin\varphi)\right] + \\ + (\zeta\cos\varphi + \xi\sin\varphi)\}\exp(i\omega t)/(\lambda + 2\mu).$$

The deformation $\varepsilon_{\xi\xi}$ then becomes:

$$\varepsilon_{\xi\xi} = Re\{\partial u_\xi/\partial\xi\} = \gamma A \sin^2\varphi\{\exp\left[-(\zeta\cos\varphi + \xi\sin\varphi)\delta\right] \times \\ \times \cos[\omega t - (\zeta\cos\varphi + \xi\sin\varphi)\delta] - \cos\omega t\}/(\lambda + 2\mu).$$

The numerical calculation were made using the values for granite:

$\lambda = 2.24 \times 10^{10}$ Pa,
$\kappa = 1.1 \times 10^{-6}$ m^2 s^{-1},
$\mu = 2.9 \times 10^{10}$ Pa,
$A = 5^\circ$ K,
$\rho = 2650$ kg m^{-3},
$\alpha = 10^{-6}$ K^{-1}.

Using these values the penetration depth for the annual temperature variation we get as $\delta = 3.32$ m. The amplitude of the annual temperature variation was taken to be only 5° K because the gallery in Vyhne is located beneath an afforested slope and also the soil cover above the rock massif is 30–50 cm thick, which also attenuates the temperature variation at the surface of the rock massif. The angle of the slope above the gallery varies between 16° and 21°.

In Fig. 5 the time variations of the computed elastic deformations for the slope inclined at the angles 10°, 15°, 20° and the slow deformation observed at the Vyhne tidal station are presented. The dependence of amplitudes of the calculated thermo-elastic deformations on the distance from the free boundary for slopes inclined at angles 10°, 15°, 20° are given.

5. Conclusions

The tidal deformations are characterized by small positive anomalies. The positive anomalies dominate in the regions, which are characterized by

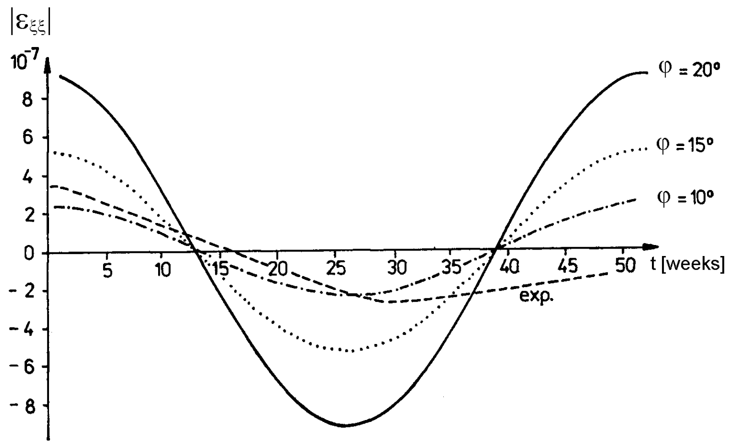

Fig. 5. The time variation of the computed elastic deformations for the slope inclined at angle φ =10° (dot-dashed curve), φ = 15° (dotted curve), φ =20° (solid curve) and the slow deformations observed at the Vyhne station (dashed curve).

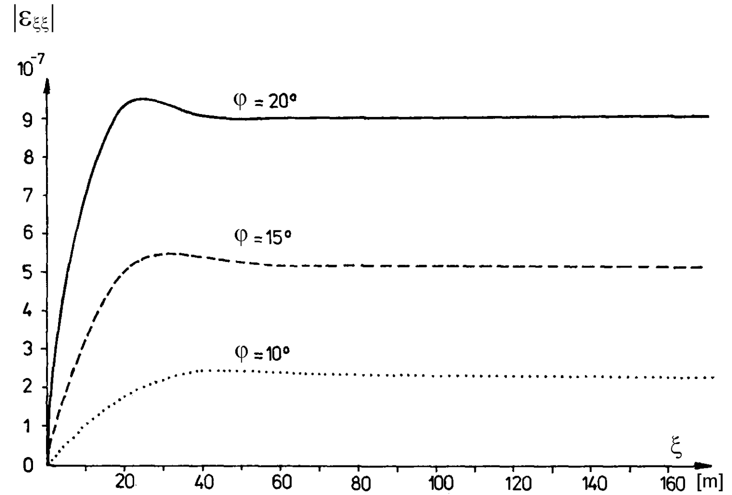

Fig. 6. The dependance of the amplitudes $|\varepsilon_{\xi\xi}|$ of the calculated thermo-elastic deformations on the distance from the free boundary for the slopes inclined at angles φ =10° (dotted curve), φ =15° (dashed curve), φ =20° (solid curve).

high heat flows. The neighbourhood of the tidal station Vyhne belongs to such regions. From the tidal characteristics of the waves K_1 and O_1 the liquid core resonance effect was confirmed.

The periodic part of the slow deformations observed at the Vyhne tidal station with the period of about 1 year are caused by thermoelastic deformations due to the annual variations of the air temperature.

Acknowledgments. The author is grateful to the Science and Technology Assistance Agency under the contract No. APVT-51-002804. This work was also supported by the Slovak Grant Agency VEGA (grants No. 1/3066/06 and 2/6019/06) for the partial support of this work.

References

Brimich L., 1988: Extensometric measurements at the Vyhne tidal station. Contr. Geophys. Inst. Slov. Acad. Sci., **18,** 58–62.

Brimich L., Latynina L. A., 1989: The results of the extensometric observations in Vyhne. In: Proceedings of 6th International Symposium "Geodesy and Physics of the Earth", Potsdam, ZIPE, 239–254.

Dudášová V., 1998: Description of the renovated extensometer at the Vyhne tidal station. Contr. Geophys. Inst. Slov. Acad. Sci., **18**, 197–203.

Mentes G., 1986: An intelligent data acquisition system for recording the tidal signal. Acta Geophys. Mont. Hung., **21**, 21–29.

Mentes G., 1995: High precision calibration of quartz tube extensometers. In: Proceedings of the 12th Internationl Symposium on Earth tides, Science Press, 209–214.

Mentes G., 1998: Calibration of tidal instruments. In: Proceedings of the 13th Internationl Symposium on Earth tides, Observatoire Royal de Belgique, Brussels, 43–50.