Modeling of electromagnetic field in the Earth-ionosphere resonator (Transmission Line Method)

P. Kostecký, A. Ondrášková, L. Rosenberg
Department of Astronomy, Physics of the Earth and Meteorology, Faculty of Mathematics, Physics and Informatics, Comenius University[1]

I. Kohút
Geophysical Institute of the Slovak Academy of Sciences[2]

Abstract: The Schumann resonances – electromagnetic eigenmodes of the resonator bounded by the Earth's surface and lower ionosphere, permanently excited by the global lightning activity – are now widely monitored experimentally. To relate the obtained data with ionospheric parameters the computer modeling of such a resonator is necessary. The various approaches to this task are surveyed and some new insights are suggested.

Key words: electromagnetic field, ionosphere, Schumann resonances, Transmission Line Method

1. Introduction

The electromagnetic (E.M.) resonator, formed by the Earth's surface and lower ionospheric layers (separated by practically insulating tropo- and stratospheric air), is permanently excited by global lightning activity. There are about 200 discharges every second over the whole Earth – the majority at the three principal thunderstorm foci, namely central Africa, Indonesian archipelago and the Amazonia. Because the lower part of clouds obviously bears negative charge with respect to ground, a return current must be excited from the ground upwards. In fair weather, a quasistatic current flows with an average current density of the order of 10^{-12} A.m^{-2}.

[1] Mlynská dolina F-1, 842 48 Bratislava, Slovak Republic
e-mail: kostecky@fmph.uniba.sk
[2] Dúbravská cesta 9, 845 28 Bratislava 45, Slovak Republic; e-mail: geofkohi@savba.sk

Global electromagnetic resonances were predicted and theoretically explained by W. O. Schumann (*Schumann, 1952*) who calculated the eigenfrequencies of the Earth-ionosphere cavity and suggested that global lightning activity was the source of these oscillations. First experimental evidence was obtained by Balser and Wagner (*Balser and Wagner, 1960*) in the form of separate peaks in the spectrum.

The eigenfrequencies fall into the ELF band: the fundamental mode at about 7.8 Hz, next – 14.1, 20.3, 26.0, 32.5 Hz and so on. The measuring and data acquisition technique for monitoring Schumann Resonance (SR) components (magnetic and electric) is described in (*Kostecký et al., 2000*). The SR modes are regularly monitored at several observatories worldwide, for example at the Modra observatory, and their main parameters – peak frequency, damping factor and amplitude – have been evaluated.

The SR spectra from the Modra observatory (the Astronomical and Geophysical Observatory of the Comenius University, Faculty of Mathematics, Physics and Informatics, at Modra) can be viewed in real time at: `http://147.175.143.11/`.

2. Geophysical significance of SR

At present, the geophysical significance of SR monitoring has been established by numerous researchers. involving the following items:

- the monitoring of global lightning activity and its intensity,
- the determination of electrical parameters of lower ionosphere,
- the influence of Solar activity, especially solar flares and solar proton events,
- the global variations of temperature and humidity in troposphere.

The principal parameters of SR eigenmodes (the peak frequency f_n, the amplitude A and the quality factor Q) exhibit many variations – quasiperiodical (diurnal, semiannual, annual) and also irregular. Typical daily variation of the first mode peak frequency based on Modra observatory measurements is presented in Fig. 1. Numerous literature – e.g. *Price (1993)*, *Greenberg and Price (2004)* – is devoted to relations between global geophysical quantities (asymmetric width of the cavity, presence of geomagnetic field, anisotropic conductivity of the ionosphere) and SR parameters.

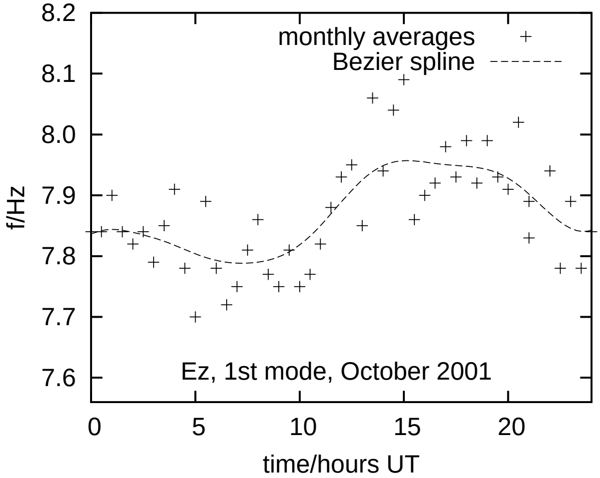

Fig. 1. Diurnal variation of the first SR mode obtained by averaging from October 2001 data.

But, for determination of these relations, it is necessary to compare SR mode parameters (quoted above) to theoretical predictions. Therefore, it is necessary to propose and numerically compute (as precise as possible) models of real Earth-ionosphere resonator.

3. Model of the ideal resonator

In the simplest model, both resonator boundaries (Earth's surface and the lower ionospheric boundary) were considered as perfectly conducting and spherical, the space inside the cavity as perfect insulator. Even at such simplification, the computation of eigenfrequencies is not so simple. Because the resonator is composed of two separate conducting surfaces (there is no lower cut-off frequency), the eigenmodes can be transverse electric or transverse magnetic ones. The detailed analysis (*Bliokh et al., 1977*) predicts the eigenfrequencies of TM (magnetic transverse – $H_r = 0$) modes as roots of transcendental equation:

$$j_n'(kR)\, y_n'[k(R+h)] - j_n'[k(R+h)]\, y_n'(kR) = 0, \qquad (1)$$

where the R is the Earth's radius, h is the height of the lower ionospheric boundary and propagation constant $k = 2\pi/\lambda$ (the frequency is given by usual $f = ck/2\pi$). The spherical Bessel functions j_n and y_n of index n are differentiated with respect to complete argument.

Under the assumption $h \ll R$ it is possible to solve (1) approximately and the theoretical eigenfrequencies for the first five modes can be obtained as 10.54, 18.26, 25.84, 33.34 and 40.82 Hz – well over the observed ones: 7.8, 14.1, 20.3, 26.2 and 32.5 Hz (average values). The principal cause of this discrepancy is the resonator damping, mostly due to the finite ionospheric conductance.

Due to the perfect symmetry of ideal resonator, the eigenmodes - which can be classified by (n, m)- couple of indices $(n = 1, 2, \dots ; m = -n, \dots, n)$ – are completely degenerated with respect to m.

4. The lossy resonator

In the real Earth-ionosphere resonator, the damping is very high compared to technical resonators. The corresponding Q-factor for individual eigenmodes is of the order $5-10$ only. If we calculate the eigenfrequencies (physically fictitious!), their values will be complex with comparable magnitudes of real and imaginary parts.

The simplest way to model such a situation is to develop real field amplitudes into the series of ideal resonator eigenmodes:

$$\vec{E} = \sum_{n=1}^{\infty} \sum_{m=-n}^{n} \alpha_{n,m} . \vec{E}_{n,m}$$
$$\vec{H} = \sum_{n=1}^{\infty} \sum_{m=-n}^{n} \beta_{n,m} . \vec{H}_{n,m} . \tag{2}$$

Using the Leontovich impedance condition at the outer (ionospheric) resonator boundary, a following (doubly infinite) linear system for the "mode mixing coefficients" $\alpha_{n,m}$ and $\beta_{n,m}$ (*Bliokh et al., 1977*) can be written:

$$\omega \alpha_{n,m} - \omega_n \beta_{n,m} = 0,$$
$$\omega_n \alpha_{n,m} - \omega \beta_{n,m} + ic \sum_{p=1}^{\infty} \sum_{q=-p}^{p} \beta_{p,q} L_{n,m,p,q} = 0, \tag{3}$$

(dimension of the system is $n^2 + 2.n$, if truncation at $n = n_{max}$ is used), where ω_n are the frequencies of subsequent eigenmodes of the ideal resonator and the term

$$L_{n,m,p,q} = \iint\limits_{r=(R+h)} \vec{H}_{n,m} \cdot \overline{\overline{\mathcal{Z}}} \cdot \left[\vec{n}_r \times \left[\vec{H}_{n,m} \times \vec{n}_r \right] \right] \mathrm{dS} \tag{4}$$

reflects the mutual coupling between (n, m) and (p, q) ideal resonator eigenmodes. Vector $\vec{n}_r$ is the unit radial vector and integration is performed over the ionospheric boundary in the spherical coordinate system and $\overline{\overline{\mathcal{Z}}}$ is the impedance tensor. The cumulative influence of such terms in (3) is responsible for the energy loss at the lower ionosphere boundary, which is prevailing (the dissipative properties of Earth's surface and tropospheric air are in most cases much less significant).

As described in Eq. (4) the dissipative properties of the lower ionosphere can be simply characterised by the dimensionless surface impedance parameter $\mathcal{Z} = \epsilon^{-1/2}$ (ϵ stands for the relative permittivity at the ionosphere boundary). In general, this quantity must be complex to take into account the conductivity and will be tensorial, if the geomagnetic field is considered and the ionospheric plasma exhibits a gyrotropic properties. This approach is, in principle, an application of the perturbation theory. The doubly infinite linear system (3) must be taken as finite by suitably choosing the upper limit values of (n, m). Because the system of the equations (3) is homogeneous (the excitation of resonator is not considered yet), the eigenvalues of its matrix will give us the new (complex) eigenfrequencies of lossy resonator, taking some preliminary assumptions about quantity $\mathcal{Z}$. If there is some kind of angular symmetry of the ionospheric conductivity, some of the "coupling factors" $L_{n,m,p,q}$ become exactly zero.

In numerous literature, e.g. (*Nickolaenko and Hayakawa, 2002*), there were corroborated various models (angularly non-homogeneous, but partly symmetric), such as ionospheric polar caps model, day-night asymmetry model, or models taking into account the geomagnetic field (symmetric or asymmetric dipole models). As a rule, the real parts of eigenmode frequencies result satisfactory close to measured values, but the damping factors were obviously overestimated. Moreover, when the geomagnetic field is considered, the degeneracy in m is completely vanishing (Zeeman effect!) and the value of fundamental mode frequency splitting is of real order of magnitude. It is worth mentioning that, due to the quadratic terms in matrix elements of the system (3), resulting amplitude-frequency characteristics for all modes are of Lorentzian type. The typical graphs are given in *Kostecký et al., (2005)*. After fitting of FFT spectrum (obtained from raw data) by Lorentz function, the three basic parameters (peak frequency, amplitude and Q-factor) can be determined for each mode. A typical FFT spectrum of raw data is given in Fig. 2.

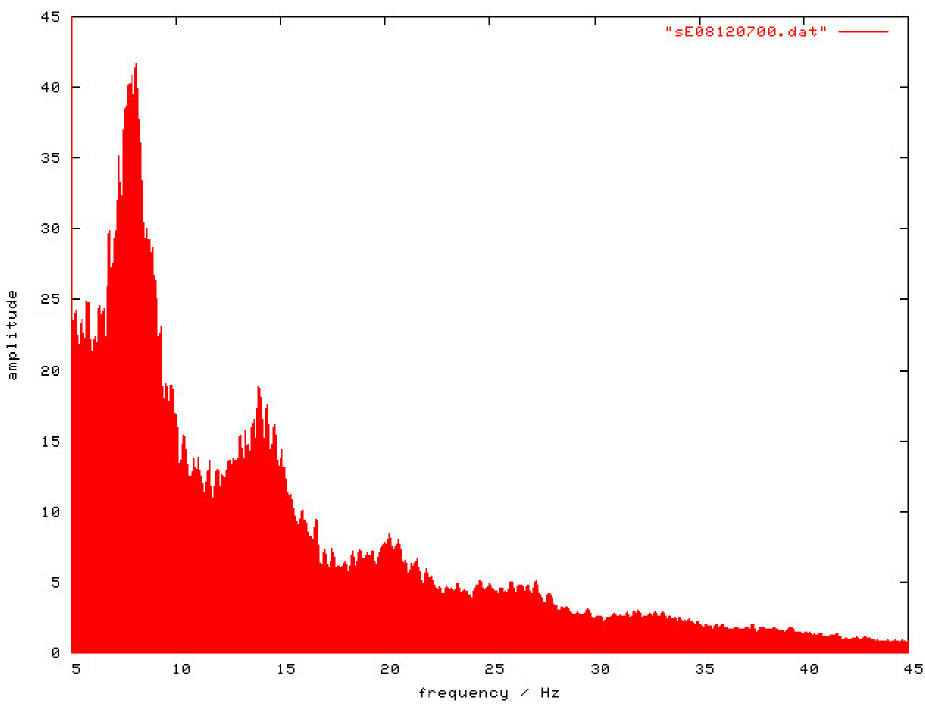

Fig. 2. A typical spectrum of the electric SR component obtained at Modra observatory August 12, 2004, 07:00 UT.

The result of Lorentzian line fitting is depicted in Fig. 3. for the first four subsequent modes calculated from the electric component measurements at Modra observatory. The principal parameters are given, too. Analogical result of fitting of the first four modes from magnetic component measurements can be found in *Price and Melnikov (2004)*.

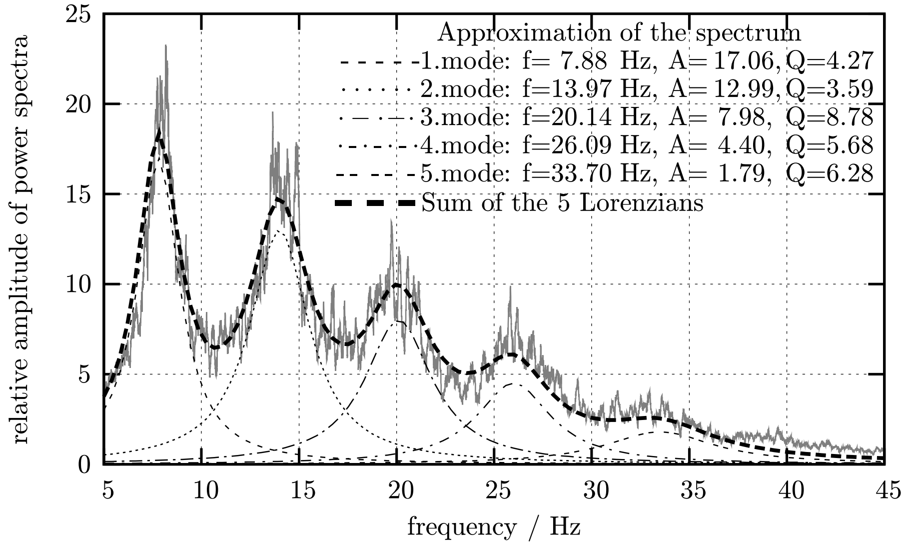

Fig. 3. Fitting of the first five electric SchR eigenmodes by Lorentz line profiles performed on electric component data from Modra observatory.

5. Excitation of the resonator

In this theoretical survey, the excitation conditions have not been considered yet. A single (radial) current $\vec{J}(\vec{r}) = J(r) \cdot \vec{n}_r$ can be presumed as the simplest source of excitation. In this case, the first set of linear equations in (3) will have a non-zero right-hand side, namely:

$$\omega\alpha_{n,m} - \omega_n\beta_{n,m} = 4\pi i \iiint\limits_{R\leq r\leq(R+h)} J_\omega(\vec{r}).\vec{E}^*_{n,m}\mathrm{d}V, \qquad (5)$$

where the current term under the integral represents the component of excitation current, the asterisk denotes the complex conjugate.

As the illustrative example, let us consider the excitation current as a vertical line (of length $l \ll h$) situated at the pole of coordinate system $(\theta = 0)$. In this very simple case, only two field components of each (n-th) mode are non-zero:

– radial electric E_r, with angular dependence as $\mathrm{d}/\mathrm{d}\theta[P_n(\cos\theta)]$;
– tangential magnetic H_φ, proportional directly to $P_n(\cos\theta)$.

The expressions for both field amplitudes contain in the denominator the Lorentzian factor (*Nickolaenko and Hayakawa, 2002*):

$$(\omega_n^2 - \omega^2) + i\omega c \frac{\mathcal{Z}(\omega)}{h} \tag{6}$$

It must be stressed that – in the case of a resonator characterised by a very low quality factor – the physical notion of *"eigenmodes"* is partly obscured. Depending on excitation, the density of field energy as a function of frequency can vary from point to point. It would be more realistic to quote as *"eigenfrequency"* a value of ω for a maximum of total (whole volume) field energy density. Unfortunately, this value is immeasurable.

The spatial dependence of *"local" eigenfrequencies* is discussed, for example, in *Morente et al. (2004).*

6. The more complex models

The approach shown above gives, in some sense, a satisfactory reflection of real Earth-ionosphere resonator properties. But it suffers from general limitations of perturbation theory, namely the implicit assumption of perturbation smallness and the oversimplification of the boundary condition at the ionosphere. Moreover, for even a moderate precision of numerical calculation it is necessary to maintain the order of linear system (2) high, because we express the longitudinal (TM) waves by means of radial ones (spatial harmonics).

The direct numerical solution of Maxwell's equations with appropriate boundary conditions, considering all inhomogeneities in the model would be only physically adequate. Just the boundary conditions complicate the calculations in the finite ionospheric conductance case. The tangential (θ, φ) components of $\vec{E}$ must be zero at the Earth's surface, taken as a perfect conductor. On the contrary, at the lower ionospheric boundary $(r = R + h)$ the tangential components of both $\vec{E}$ and $\vec{H}$ must remain continuous, and by the Sommerfeld radiation condition the field amplitudes must vanish for $r \to \infty$.

The precise formulation of conditions at finite conductive boundary is practically unfeasible and often would be approximated by surface impedance condition, as mentioned above.

The full solution of the problem is principally possible by the FDTD method – for example (*Yang and Pasko, 2005*) and (*Simpson and Taflove, 2002*), or by the FEM (Finite Element Method). In the FEM, the volume of resonator must be discretized into subdomains (*finite elements*) and unknown field amplitudes in each element are expressed by linear combination of prescribed *shape functions* (usually of polynomial type). The coefficients of these linear combinations are the unknowns (*degrees of freedom*) and can be recovered by solving very large linear system of equations for which the system matrix is, as a rule, sparse and with band structure. Naturally, the inter-element continuity (of C^0 or C^1 type) and the conditions at the overall domain boundary are fulfilled.

Respecting the spherical geometry, the natural shape of finite element would be "a spherical brick": $\Delta V = r^2 \sin\theta \Delta r \Delta\theta \Delta\varphi$. The situation complicates at the poles of the sphere – it is possible to use special-type elements there, or simply neglect the very small "polar caps".

In the classical FEM, the shape functions and degrees of freedom are scalar quantities, separately for all components of the vector fields. Therefore, the boundary conditions have very complicated formulation. A remedy was found in the vector finite element formulation (*Kostecký and Kohút, 2002; Harutyunyan et al., 2004*). In this approach, the shape functions and degrees of freedom are vectors, which simplifies the formulation of boundary conditions. Moreover, the vectorial character of E.M. fields is naturally incorporated into the model (the *div* and *rot* conditions).

7. The Transmission Line Method (TLM)

TLM is a numerical method availing of close analogy between the description of E.M. phenomena by field quantities (E, H) and by means of electrical circuit quantities (U, I – voltages and currents). The distributed parameter system (say, E.M. resonator) is substituted by the lumped parameter one (RLC circuit of very complicated topology).

In a simpler variant of TLM model (*Madden and Thompson, 1965*), the spatial domain in question (the spherical shells of Earth's surface and ionosphere, together with resonator inner region) is subdivided into "spherical bricks" or "cells" of thickness Δr and width $\Delta\theta$ and $\Delta\varphi$ in angular coordinates.

The geometric center of each cell is considered to be a node of electric circuit. This circuit (of complicated, essentially non-planar topology) is formed by connecting adjacent nodes by branches. The circuit branches in various directions (parallel lines, meridional and vertical) are composed of inductive, capacitive and real (resistive) impedances, as can be seen in Fig. 4. The values of equivalent circuit elements – the inductances L_k, the capacitances C_k and the conductances G_k are determined by geometrical parameters of the k-th cell (the coordinates of its central point and its dimensions) and by physical properties of the material inside (permittivity ϵ, permeability μ and conductivity σ). In the simple geometry described above, these values are given by:

$$\begin{aligned} L_k &= \mu \frac{r_k^2}{\Delta r} \sin\theta_k \Delta\theta\Delta\varphi, \\ C_k &= \epsilon \frac{r_k^2}{\Delta r} \sin\theta_k \Delta\theta\Delta\varphi, \\ G_k &= \sigma \frac{r_k^2}{\Delta r} \sin\theta_k \Delta\theta\Delta\varphi, \end{aligned} \tag{7}$$

where (r_k, θ_k) stand for the coordinates of the cell center, and $(\Delta r, \Delta\theta, \Delta\varphi)$ for the cell dimensions.

The complex electrical circuit generated by this procedure can be excited by a prescribed voltage waveform between two selected nodes – or by a current impulse forced into one selected branch. The standard procedures and codes for electrical circuit analysis allow to compute the voltage or current waveforms resulting in any part of circuit and corresponding frequency spectra.

In this model, the spatial inhomogeneities can be taken into account naturally, by putting various values of ϵ, μ and (or) σ in different cells of resonator volume. It is worth mentioning that even the gyrotropic character of the ionosphere conductivity can be taken into account, if we align the axis of coordinate system parallel with the magnetic dipole axis and take the different values of inductances into the meridional ($\varphi' =$ const.) and longitudinal ($\theta' =$ const.) branches (the primes denote the new coordinates).

If the division of the resonator volume into cells is sufficiently fine, the computed peak frequencies for subsequent SR eigenmodes (the frequencies of maximum $\vec{E}$ or $\vec{H}$ amplitudes) will be pretty close to real values, as well

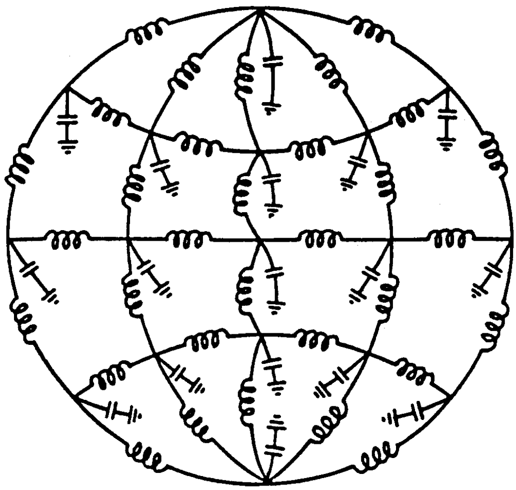

Fig. 4. A part (one spherical shell) of a distributed parameter model of a spherical resonator (*Madden and Thompson, 1965*).

as their variations over the entire globe (these variations can be of the order 0.2 Hz for the fundamental SR mode). This simple variant of the TLM method can be programmed relatively easily and allows using the standard electrical circuit analysis packages. Some – but not insurmountable – complications may arise at the poles of the sphere. In this method, the implicit discretization of field equations is only spatial. The time variable is considered continuous, due to transfer to frequency domain through the Fourier transform. The principal disadvantage of this method is in the artificial (non-physical) frequency dispersion, which is an inevitable consequence of the lumped parameters model in general (the same appears, in more or less degree, in every variant of the FDTD computation).

Another difficulty arises in the necessary transition from node voltages and branch currents – results of model calculation – to intensities of electric and magnetic field at selected points. While restoration of the vertical electric field intensity is relatively simple, it is not so for the horizontal magnetic intensity.

In a more elaborate variant of the TLM method (*Morente et al., 2003*), the implicit discretization of field equations is performed as spatial and temporal, too. The division of the domain under question (resonator volume) will be performed analogically (into "spherical bricks"). Model nodes are selected in the same manner. But, their connections are made not by simple branches containing model impedances (L, C, and G, as was described above). Instead, nodes are connected by sections of the transmission lines.

This allows to make true discretization both in space and time. The series of equidistant (in time) voltage (or current) pulses are injected into the excitation node (nodes). Their time difference is equivalent to the time step (Δt) in the FDTD method.

In order to fulfil the time synchronism between excitation in adjacent nodes, the section of transmission lines connecting nodes must be of equal length (ΔL) and equal characteristic (wave) impedance Z_0. In such configuration alone, it would be impossible to consider different values of material parameters (ϵ, μ and σ) in different subdomains (cells). This can be overcome by connecting new (additional) transmission line sections to each node. Each section (stub) is connected only to a single node. Their second ends are open, short-circuited or (formally) infinitely long. The input impedance of such stubs simulates capacitive, inductive or real (resistive) impedance, maintaining these parameters (C, L or G) at each node in accordance with material parameters of appropriate subdomain (cell).

The detailed description of model construction and computation will be given in the article in preparation.

8. Conclusion

In this article, a survey of modeling techniques appropriate to the Earth-ionosphere resonator problem is given. The FDTD method and the TLM method (in the more elaborate variant) have common roots and can be mutually transformed. The FEM approach seems to be physically more straightforward, at the expense of computing labour. An optimal compromise would be the first variant of the TLM method, employing the lumped elements approximation in the global circuit (if the space discretization will be sufficiently fine).

At the Modra observatory of Comenius University, the monitoring of the SR parameters has been performed for a rather long interval (more than 4 years, for the electric component). This amount of data will be used for the Earth-ionosphere resonator modeling, which is in preparation.

Acknowledgments. This work was supported by the grant 1/2033/05 of the Scientific Grant Agency VEGA.

References

Balser M., Wagner C. A., 1960: Observation of Earth-Ionosphere Cavity Resonances. Nature, 188, 638–641.

Bliokh P. V., Nickolaenko A. P., Filippov J. F., 1977: The Global Electromagnetic Resonances in the Earth-Ionosphere Cavity. Naukova Dumka, Kiev, 198 p. (in Russian).

Greenberg E., Price C., 2004: A global lightning location algorithm based the electromagnetic signature in the Schumann resonance band. J. Geophys Res. –Atmospheres, 109 (D21) D21111.

Harutyunyan D., Botchev M. A., van der Vegt J. J. W., 2004: Finite Element Methods for Maxwell's Equations. MPCM seminar 26th February 2004, University of Twente, The Netherlands, 29 p. also on web: `http://www.home.math.utwente.nl/ harutyunyan/presentation.pdf`

Kostecký P., Kohút I., 2002: Vector finite elements and their use in the field computation. Acta Astron. et Geophys., XXIV, 41–50.

Kostecký P., Ondrášková A., Rosenberg L., Turňa Ľ., 2000: Experimental setup for the monitoring of Schumann resonance electric and magnetic field variations at the Geophysical Observatory at Modra-Piesok. Acta Astron. et Geophys., XXI–XXII, 71–92.

Kostecký P., Ondrášková A., Ševčík S., Rosenberg L., Kohút I., 2005: Observations of the anomalous third Schumann resonance peak during October 21 to 25, 2004 at Modra Observatory and a possible explanation. Contr. Geophys. Geod., **35**, 1, 33–51.

Madden T., Thompson W., 1965: Low-Frequency Electromagnetic Oscillations of the Earth-Ionosphere Cavity, Reviews of Geophysics, **3**, 211–254.

Morente J. A., Portí J. A., Salinas A., Molina-Cuberos G. J., Lichtenegger H., Besser B. P., Schwingenschuh K., 2004: Do Schumann resonance frequencies depend on altitude? J. Geophys. Res., **109**, A5, A05306.

Morente J. A., Molina-Cuberos J., Portí J. A., Besser B. P., Salinas A., Schwingenschuh K., Lichtenegger H., 2003: A numerical simulation of Earth's electromagnetic cavity with the transmission Line Matrix method: Schumann resonances. J. Geophys. Res., **108**, A5, 1195 p.

Nickolaenko A. P., Hayakawa M., 2002: Resonances in the Earth-Ionosphere Cavity. Kluwer, Dordrecht, 378 p.

Price C., 1993: Global surface temperatures and the atmospheric electrical circuit. Geophys. Res. Lett., **20**, 1363–1366.

Price C., Melnikov A., 2004: Diurnal, seasonal and inter-annual variations in the Schumann resonance parameters. JASTP, 66, 1179–1185.

Simpson J. J., Taflove A., 2002: Two-Dimensional FDTD Model of Antipodal ELF Propagation and Schumann Resonance of the Earth. IEEE Antennas and Wireless Propagation Letters, **1**, 53–56.

Schumann W. O., 1952: On the Free Oscillations of a Conducting Sphere which is surrounded by an Air Layer and a Ionosphere Shell. Zs. Naturforschung, 7a, 149–154 (in German).

Yang H., Pasko V. P., 2005: Three-dimensional finite difference time domain modeling of the Earth-ionosphere cavity resonances. Geophys. Res. Lett., **32**, L03114.