Contributions to Geophysics and Geodesy Vol. 37/1, 2007

The boundary integral numerical modelling of the D.C. geoelectric field in a two-layered earth with a 3-D block inhomogeneity bounded by sloped faces

M. Hvoždara
Geophysical Institute of the Slovak Academy of Sciences[1]

Abstract: The paper presents an algorithm and numerical results for the boundary integral (B.I.E.) method of the forward D.C. geoelectric problem for a two-layered earth which contains a 3-D block inhomogeneity in the superficial layer. In comparison with previous numerical calculation the is algorithm is presented for the calculation of the boundary integrals for the cases with sloped planar boundary faces of the prism. Although the numerical calculations are more complicated in comparison with faces orthogonal to some of the x, y, z axis, this generalization to the sloped faces enables the treatment of the anomalous fields for the bodies of more general shapes than rectangular prisms. The graphs with numerical results present isoline maps of the potential, electric field components as well as the dipole profiling apparent resistivities when the source is the pair of D.C. electrodes at the surface of the earth.

Key word: geoelectric potential field theory, boundary integral methods, double-layer potential calculation, Schlumberger apparent resistivity for laterally inhomogeneous media

1. Introduction

The method of B.I.E. developed in the last 25 years has been shown as a very effective for solving geoelectric potential fields in the layered medium containing a 3-D or 2-D perturbing body; see e.g. *Lee (1975); Okabe (1981); Hvoždara (1982, 1983); Eloranta (1986); Furness (1992).* In our earlier papers *(Hvoždara, 1983, 1984, 1990)* we have paid our attention to the cases of a uniform exciting electric field, which approximates a telluric field for long periods. These general boundary integral formulae can be easily

[1] Address: Dúbravská cesta 9, 845 28 Bratislava, Slovak Republic; geofhvoz@savba.sk

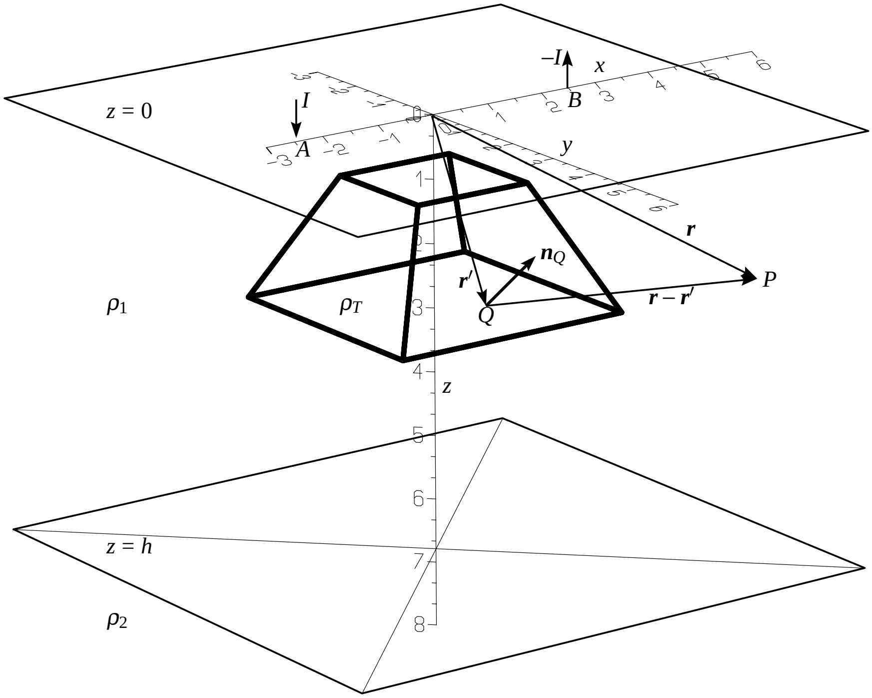


Fig. 1. Model of a 3-D prism with sloped faces buried in the first layer of the two-layer earth.

adopted for the cases of non-uniform exciting electric field, which is e.g. due to the point source electrode on the surface of the earth, or by the pair of such electrodes. In our paper *Hvoždara (1995)* we presented detailed extension of the B.I.E. method to the more complicated cases: the 3-D body embedded in the superficial layer of 2-layered earth, including its possible contact with the lower or/and upper boundary of the layer, while the source electrode can be situated even on the surface of the outcropping body. The present study is directed towards a generalization of our numerical studies to the cases of 3-D block bodies bounded by sloped faces, in contrast to the cases when the faces are orthogonal to some of the co-ordinate axis x, y, z.

2. Boundary integral expressions for potentials and electric field

Theoretical formulae for our B.I.E. analysis are identical as those in *Hvoždara (1995)*, but for better clarity of explanation we repeat them also now. We consider the two-layered earth represented by the superficial layer $z \in [0, h]$ of resistivity ρ_1 and substratum $z > h$ of resistivity ρ_2. In the layer we shall consider a 3-D disturbing body Ω_T of resistivity ρ_T, bounded by the surface S with a piecewise continuous outer normal $\boldsymbol{n}$ (see Fig. 1). In the absence of disturbing body the D.C. current source excites potentials $V_1(P)$ in the layer (medium "1") or $V_2(P)$ in the substratum (medium "2"), but due to the presence of the perturbing body Ω_T these potentials change and result in the total potentials $U_1(P)$ and $U_2(P)$, respectively. The total potential inside Ω_T is denoted by $U_T(P)$. According to the previous theory presented in (*Hvoždara, 1995*), we can write expressions for total potentials $U_1(P), U_2(P), U_T(P)$ in the form of sum of unperturbed potentials $V_1(P), V_2(P)$ and generalized double layer potentials (given by the boundary integrals), namely:

$$U_1(P) = V_1(P) + \frac{1}{4\pi} \int\limits_S f(Q) \frac{\partial}{\partial n_Q} G_1(P,Q) \,\mathrm{d}\, S_Q \,, \quad P \in \Omega_1 \,, \quad P \notin \Omega_T \,, \tag{1}$$

$$U_2(P) = V_2(P) + \frac{1}{4\pi} \int\limits_S f(Q) \frac{\partial}{\partial n_Q} G_2(P,Q) \,\mathrm{d}\, S_Q \,, \quad P \in \Omega_2 \,, \tag{2}$$

$$U_T(P) = \frac{\rho_T}{\rho_1} \left[V_1(P) - v_0 + \frac{1}{4\pi} \int\limits_S f(Q) \frac{\partial}{\partial n_Q} G_1(P,Q) \,\mathrm{d}\, S_Q \right] + v_0 \,, \ P \in \Omega_T \,. \tag{3}$$

Here $G_1(P,Q), G_2(P,Q)$ are Green's functions for the two-layered earth. They correspond to the potential of the point source electrode, situated at the point $Q \equiv (x', y', z') \in S$, calculated for the point $P \equiv (x, y, z)$, but instead of source factor $I\rho_1/(4\pi)$ we must put dimensionless factor=1. The function $G_1(P,Q)$ obeys the Poisson equation in the layer Ω_1:

$$\nabla^2 G_1(P,Q) = -4\pi\delta(P,Q) \,, \quad P \in \Omega_1 \,, \quad Q \in S \,, \tag{4}$$

while $G_2(P,Q)$ is a harmonic function in the whole region $\Omega_2 (z \geq h)$, i.e.:

$$\nabla^2 G_2(P,Q) = 0. \tag{5}$$

On the surfaces $z = 0$ and $z = h$ boundary conditions must be satisfied:

$$[\partial G_1(P,Q)/\partial z]_{z=0} = 0\,, \tag{6}$$

$$[G_1(P,Q)]_{z=h} = [G_2(P,Q)]_{z=h}\ , \tag{7}$$

$$\rho_1^{-1}\,[\partial G_1(P,Q)/\partial z]_{z=h} = \rho_2^{-1}\,[\partial G_2(P,Q)/\partial z]_{z=h}\,. \tag{8}$$

Both G_1 and G_2 must have zero limit for $\overline{PQ} \to +\infty$. Using known treatment given in our previous papers, with some modification (since in *Hvoždara, 1982* the body was considered in the substratum), we can obtain the following expressions for $G_1(P,Q), G_2(P,Q)$:

$$\begin{aligned} G_1(P,Q) = R^{-1} + R_+^{-1} + \sum_{m=1}^{\infty} k_{12}^m \Big\{ &\left[r^2 + (2mh + z + z')^2\right]^{-1/2} + \\ &+ \left[r^2 + (2mh + z - z')^2\right]^{-1/2} + \left[r^2 + (2mh - z + z')^2\right]^{-1/2} + \\ &+ \left[r^2 + (2mh - z - z')^2\right]^{-1/2} \Big\}\ , \end{aligned} \tag{9}$$

$$\begin{aligned} G_2(P,Q) = (1 + k_{12}) \Big\{ R^{-1} + R_+^{-1} + \sum_{m=1}^{\infty} k_{12}^m \Big[&\left[r^2 + (2mh + z + z')^2\right]^{-1/2} + \\ &+ \left[r^2 + (2mh + z - z')^2\right]^{-1/2} \Big] \Big\}\ , \end{aligned} \tag{10}$$

where $k_{12} = (1 - \rho_1/\rho_2)/(1 + \rho_1/\rho_2)$ and $r^2 = (x - x')^2 + (y - y')^2$, $R = \left[r^2 + (z - z')^2\right]^{1/2}$, $R_+ = \left[r^2 + (z + z')^2\right]^{1/2}$. Both these functions occur in integrals of formulae (1) through (3) in the form of their derivatives with respect to the outer normal $\boldsymbol{n}_Q \equiv (n'_x, n'_y, n'_z)$ at the point $Q \in S$, which means:

$$\begin{aligned} \frac{\partial G_k(P,Q)}{\partial n_Q} &\equiv \boldsymbol{n}_Q \cdot \mathrm{grad}_Q\, G_k(P,Q) = \\ &= \left(n'_x \frac{\partial}{\partial x'} + n'_y \frac{\partial}{\partial y'} + n'_z \frac{\partial}{\partial z'} \right) G_k(x,y,z;x',y',z')\,, k = 1,2. \end{aligned} \tag{11}$$

In formulae (1)–(3) these normal derivatives are integrated being multiplied by the function $f(Q)$ which represents the density of the double layer distributed over the surface S. This double layer density has to be determined by solving the boundary integral equation which holds true for points $P \in S$:

$$f(P) = 2\beta\,[V_1(P) - v_0] + \frac{\beta}{2\pi} \int\!\!\!\!\!\!\diagdown_{S} f(Q) \frac{\partial}{\partial n_Q} G_1(P,Q)\,\mathrm{d}S_Q\,, \quad P \in S\,, \tag{12}$$

where $\beta = (1 - \rho_1/\rho_T)/(1 + \rho_1/\rho_T)$ and

$$v_0 = \frac{1}{|S|} \int_S V_1(P)\,\mathrm{d}S_Q \tag{13}$$

is the mean value of the exciting potential on the surface S. The B.I.E. (12) is the Fredholm integral equation of the second kind with a weakly singular kernel $K(P,Q) = \partial G_1(P,Q)/\partial n_Q$. Its singularity is due to the term $R^{-1} = \left[(x-x')^2 + (y-y')^2 + (z-z')^2\right]^{-1/2}$ in $G_1(P,Q)$. This term becomes singular when $P \to Q$. Fortunately, the surface integral in (12) must be performed in a sense of the principal value (which is denoted by the back slash) and means that a small surface element ΔS_p around the singular point P is excluded from integration. The result reads:

$$\begin{aligned} \int\!\!\!\!\!\!\diagdown_{S} f(Q) \frac{\partial}{\partial n_Q} G_1(P,Q)\,\mathrm{d}S_Q = & \int_{S-\Delta S_p} f(Q) \frac{\partial}{\partial n_Q} (R^{-1})\,\mathrm{d}S_Q + \\ & + \int_S f(Q) \frac{\partial}{\partial n_Q} H_1(P,Q)\,\mathrm{d}\,S_Q\,, \end{aligned} \tag{14}$$

where $H_1(P,Q) = G_1(P,Q) - R^{-1}$ is the non-singular part of the Green's function. The primary potentials $V_1(P)$ and $V_2(P)$ for a single point electrode supplied with the current I and situated on the surface $z = 0$ can be expressed by $G_1(P,Q_A)$ or $G_2(P,Q_A)$ as follows:

$$V_1(P) = \frac{I\rho_1}{4\pi} G_1(P,Q_A)\,, \quad P \in \Omega_1\,, \tag{15}$$

$$V_2(P) = \frac{I\rho_1}{4\pi} G_2(P,Q_A)\,, \quad P \in \Omega_2\,, \tag{16}$$

where $Q_A \equiv (x_A, y_A, 0)$ is the point where the electrode is buried. The solution of B.I.E. (12) can be performed analytically only for some simple cases, e.g. spherical body embedded in the unbounded conducting space *(Hvoždara, 1994)*. In this paper we have proved the coincidence of the B.I.E. solution with the solution by means of spherical harmonic functions. The numerical solution is possible by means of collocation method briefly described in *Hvoždara (1983)*. Let us note that according to *Hvoždara (1982)* the double-layer density $f(P)$ is in linear relation to the values of the potential $U_T(P)$ on the surface S:

$$f(P) = (1 - \rho_1/\rho_T)[U_T(P) - v_0] \,, \quad P \in S \,. \tag{17}$$

Having solved the B.I.E. (12) we can calculate the potential on the surface of the earth according to the formula (1). Then the electric field is

$$\boldsymbol{E}_1(P) = -\mathrm{grad}\, U_1(P) \,, \tag{18}$$

its components on the surface being:

$$(E_{1x})_{z=0} = -\frac{\partial V_1}{\partial x} - \frac{1}{4\pi} \int\limits_S f(Q) \frac{\partial}{\partial x} \left[\frac{\partial G_1(P,Q)}{\partial n_Q} \right]_{z=0} \mathrm{d}S_Q \,, \tag{19}$$

$$(E_{1y})_{z=0} = -\frac{\partial V_1}{\partial y} - \frac{1}{4\pi} \int\limits_S f(Q) \frac{\partial}{\partial y} \left[\frac{\partial G_1(P,Q)}{\partial n_Q} \right]_{z=0} \mathrm{d}S_Q \,, \tag{20}$$

while $(E_{1z})_{z=0} = 0$ satisfying the well-known boundary condition. Let us stress that the above formulae are valid when the body Ω_T does not touch the bottom boundary $z = h$ or the surface $z = 0$. Such contact cases must be considered separately as it was shown in *Hvoždara (1995)*.

3. Calculation of the solid angle of view for the triangle subarea with general orientation of its normal

In the numerical calculations of B.I.E. the calculation of integrals with the kernel of type of the double-layer potential: $\boldsymbol{n}_Q \cdot (\boldsymbol{r} - \boldsymbol{r}') \cdot |\boldsymbol{r} - \boldsymbol{r}'|^{-3}$ over a small subsurface ΔF_j plays fundamental role, which is the part of surface S of the perturbing body Ω_T. In the paper *Ivan (1994)* we can find explanation for the reliable calculation of such integrals for the triangle planar subarea ΔF_j:

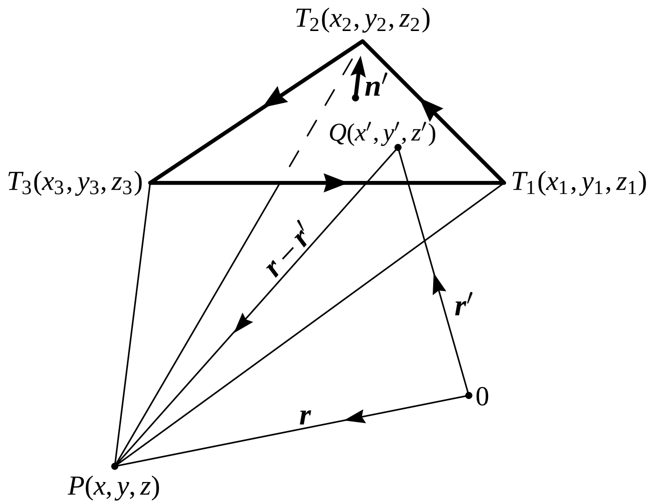


Fig. 2. Sketch of the geometrical parameters for the calculation of the solid angle of view from the point $P(\boldsymbol{r})$ onto a triangle T_1, T_2, T_3 with outer normal $\boldsymbol{n}' \equiv \boldsymbol{n}_Q$.

$$\Delta A_j = \int\limits_{\Delta F_j} \frac{\boldsymbol{n}_Q \cdot (\boldsymbol{r}-\boldsymbol{r}')}{|\boldsymbol{r}-\boldsymbol{r}'|^3}\,\mathrm{d}\,S_Q = -\Delta\Phi_j \tag{21}$$

where $\Delta\Phi_j$ is the solid angle of view from the point $P(\boldsymbol{r})$ onto planar triangle subarea ΔF_j with outer normal $\boldsymbol{n}' \equiv (n'_x, n'_y, n'_z) \equiv \boldsymbol{n}_Q$. The formula given by *Ivan (1994)* is:

$$\begin{aligned}\Delta A_j &= 2A_{123} = \\ &= 2\sum_{1,2,3} \operatorname{arctg} \frac{2w_{12}d_{12}}{(R_1+R_2+d_{12})\,|R_1+R_2-d_{12}|+2q(R_1+R_2)}.\end{aligned} \tag{22}$$

Geometrical parameters for the formulae (21) (22) are depicted in Fig. 2. The summation in (22) must be performed for three vertices of the T_1, T_2, T_3 in the counterclockwise sense. The components of the unit other normal $\boldsymbol{n}_Q$ are denoted in the formula (22) as A, B, C

$$\boldsymbol{n}_Q \equiv (A, B, C), \qquad \sqrt{A^2+B^2+C^2} = 1, \tag{23}$$

because the components of $\boldsymbol{n}_Q$ are directional cosines of the unit vector. The triangle ΔF_j is situated in the plane $t(x, y, z)$ with analytical equation:

$$Ax + By + Cz + D = 0, \tag{24}$$

while D we can calculate by using co-ordinates of some vertice, e.g. $T_1 \equiv (x_1, y_1, z_1)$:

$$D = -(Ax_1 + By_1 + Cz_1). \tag{25}$$

Next we calculate the distance of the point $P(x, y, z)$ from the plane $t(x, y, z)$ of triangle:

$$q = |Ax + By + Cz + D|. \tag{26}$$

This distance will be non-zero if the point does not lie in the plane $t(x, y, z)$ and in this case it will hold for the scalar product: $\boldsymbol{n}_Q \cdot (\boldsymbol{r} - \boldsymbol{r}') \neq 0$. If the point P is situated in the plane of the triangle there is $\boldsymbol{n}_Q \cdot (\boldsymbol{r} - \boldsymbol{r}') = 0$ and the solid angle $\Delta\Phi_j$ will be zero. In the Ivan's formula (22) there must be used also the following quantities for the neighbouring points T_1, T_2 and $P(x, y, z)$:

$$|\boldsymbol{T}_1\boldsymbol{T}_2| = d_{12} = \left[(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2\right]^{1/2},$$

$$R_1 = \left[(x_1 - x)^2 + (y_1 - y)^2 + (z_1 - z)^2\right]^{1/2},$$

$$R_2 = \left[(x_2 - x)^2 + (y_2 - y)^2 + (z_2 - z)^2\right]^{1/2}.$$

Then we use the unit vector $\boldsymbol{t}_{12}$ in the direction $\boldsymbol{T}_1\boldsymbol{T}_2$ and vector $\boldsymbol{PT}_1$ with components:

$$\boldsymbol{t}_{12} \equiv (x_2 - x_1, y_2 - y_1, z_2 - z_1)/d_{12}, \quad \boldsymbol{PT}_1 \equiv (x_1 - x, y_1 - y, z_1 - z).$$

In the next steps we obtain:

$$PD_1 = \boldsymbol{PT}_1 \cdot \boldsymbol{t}_{12}, \quad d_2 = PD_1 + d_{12}, \quad \boldsymbol{e}_{12} = \boldsymbol{t}_{12} \times \boldsymbol{n}_\Delta, \quad w_{12} = \boldsymbol{PT}_1 \cdot \boldsymbol{e}_{12}.$$

This procedure is repeated in the cycle for vertices T_2 and T_3, so we obtain necessary values of the formula (22). It must be stressed that this algorithm, when applied to the whole closed boundary S (with piecewise continuous normal $\boldsymbol{n}_Q$), must give with high precision, better than 10^{-3}, the well known fundamental values of the Gauss integral:

$$\int_S \frac{\partial}{\partial n_Q} \frac{1}{|\boldsymbol{r}-\boldsymbol{r}'|} \,\mathrm{d}\, s' = \int_S \frac{\boldsymbol{n}' \cdot (\boldsymbol{r}-\boldsymbol{r}')}{|\boldsymbol{r}-\boldsymbol{r}'|^3} \,\mathrm{d}\, s' = \begin{cases} 0, & P(\boldsymbol{r}) \in \mathrm{Ext}(S) \\ -2\pi, & P(\boldsymbol{r}) \in S \\ -4\pi, & P(\boldsymbol{r}) \in \mathrm{Int}(S) \end{cases} \tag{27}$$

For the calculation of integral (21) by means of (22) we successfully realized our original subroutine SLAGIV3 and tested it for the 6-faces prisms, like that shown in Fig. 1. We have found that the subdivision of the sloped planar faces of the prism into a set of triangle subareas is rather akward and leads to large number of subareas. So we decided to improve the Ivan's algorithm into quadrilateral subareas ΔS_j, with four vertices T_1, T_2, T_3, T_4, while normal $\boldsymbol{n}_Q$ is constant for the whole face of the prism. In this manner we decrease the number of subareas into one half in comparison with triangle case ΔF_j. The algorithm of the subdivision for each of 6 faces into quadrilateral subareas is much simpler and faster. The subroutine SLAGIV4 when applied to the sum of $\Delta\Phi_j$ due to all subareas ΔS_j gives values of the Gauss integral (30) i.e. $(-2\pi, -4\pi, 0)$ with the accuracy of at least 4 decimal digits. This subroutine was used for numerical calculations of the forward geoelectrical problem for the anomalous field due to a prism with 6 sloped faces. Let us note that the demands on the computing time and memory were greater than for the similar problem with rectangular faces, because of the more complicated algorithm of calculations of the solid angle $\Delta\Phi_j$.

4. Numerical calculations and discussion

The numerical calculations were performed in a similar way as in *Hvoždara (1983, 1995)* regarding that the Green's function $G_1(P,Q)$ is now given by the infinite series (9). Nevertheless, the principal terms are again R^{-1}, R_+^{-1} and $R_h^{-1} = [r^2 + (2h - z - z')^2]^{-1/2}$. The special cases when the perturbing body Ω_T touches the bottom and/or upper plane of the layer must be treated similarly as in *Hvoždara (1995)*. The B.I.E. (12) can be solved by the collocation method. It means that the surface S of the perturbing body is discretized into M subareas ΔS_j whose centres are denoted as P_m or Q_j. It is also assumed that each subarea is small enough to put $f(Q) = f(Q_j) =$ const. on it. So we introduce the constant approximation of an unknown function $f(Q)$ on ΔS_j. Putting the number M sufficiently large, we can express the B.I.E. (34) in its discretized form:

$$f(P_m) = 2\gamma[V_1(P_m) - v_0] + \sum_{j=1}^{M} f(Q_j)W(P_m, Q_j)\,, \tag{28}$$

where $\gamma = \beta$ if the body does not touch at the point P_m the planar boundary of the surrounding layer and attains slightly changed values as given in *Hvoždara (1995).* The weighting coefficients $W(P_m, Q_j)$ are given by the formula:

$$W(P_m, Q_j) = \frac{\gamma}{2\pi} \int\!\!\!\!\!\!\!\diagup_{\Delta S_j} \frac{\partial}{\partial n_Q} G_1(P, Q)\,\mathrm{d}\,S_Q\,. \tag{29}$$

The integration in the principal value sense was explained in the previous section. It ensures that $W(P_m, Q_j)$ cannot be infinite even if $P_m \equiv Q_m$.

In fact, the formula (39) is the system of M linear equations for the unknown values $f(Q_j)$. This system can be expressed as follows:

$$\sum_{j=1}^{M} [\delta_{mj} - W(P_m, Q_j)]\, f(Q_j) = 2\gamma\,[V_1(P_m) - v_0]\;, \qquad m = 1, 2, ..., M\;, \tag{30}$$

where δ_{mj} is the Kronecker symbol. This system of equations can be solved using known methods of linear algebra. Once the system (41) is solved, we can calculate the potential and the intensity of the electric field and the other geoelectric characteristics, e.g. apparent resistivity.

We checked out this algorithm for a 3-D body with planar upper and bottom faces and four sloped faces which close the upper and bottom rectangle into 3-D block. The upper face in the form of rectangle is at the depth z_1, the bottom rectangle is at the depth $z_2 = h$, so the prism is in contact with bottom substratum. The central depth plane of the block is $h_T = (z_1 + z_2)/2$ and we must keep conditions: $z_1 > 0$, $z_2 \le h$. This block is situated in the first layer (of resistivity ρ_1), its thickness being h. The resistivity of the block we put $\rho_T = \rho_2$, while the resistivity of the layer is ρ_1. In the case $\rho_T/\rho_1 >> 1$ the block represents a high resistive dyke of the substratum into the layer, and in the case $\rho_T/\rho_1 << 1$ the block and substratum are of lower resistivity. In our numerical calculations we put

$\rho_1 = 100\,\Omega$m and $\rho_2 = \rho_T = 1000\,\Omega$m or $10\,\Omega$m.

The subdivision of each face was performed by introducing numbers of subdivisions (> 5) for edges of each pair of opposite sides of the trapezoid, which is a general form of some face of the prism. The x, y, z coordinates of vertices for each subarea in the form of quadrangle are stored, since they are used as vertices T_1, T_2, T_3, T_4 for repeatedly called calculations of the solid angle of view by means of subroutine SLAGIV4. The direction cosines of the unit normal $\boldsymbol{n}_Q$ remain constant for each trapezoidal planar face of the prism. Let us note that for solving the system linear equations (33) for each of the central points P_m there must be calculated weighting coefficients $W(P_m, Q_j)$ for all sets of point Q_j, while in Green's function we must treat by using SLAGIV4 at least contributions by terms with R^{-1}, R_+^{-1} and for P_m from the bottom face also from R_h^{-1}. If we choose the subdivision of each trapezoidal face into 64 quadrangle subareas, we obtain $6 \times 64 = 384 = M$ surface elements ΔS_j, which contribute into summation approximation of the boundary integrals.

We assume that the unperturbed potential $V_1(P)$ in the layer “1” is due to the configuration of the $+I$ source electrode at the point $(x_A, 0, z_A)$ and $-I$ electrode at the point $(x_B, 0, z_B)$, $x_B > x_A$, being accepted. In order to avoid singularities of the surface primary potential we considered “slightly buried” source electrodes putting $z_A = z_B = z_1/10$. Hence, $V_1(P)$ is expressed by the formula:

$$
\begin{aligned}
V_1(P) = {} & \frac{I\rho_1}{4\pi}\left[G_1(P, Q_A) - G_1(P, Q_B)\right] = \\
& = \frac{I\rho_1}{2\pi}\left\{\frac{1}{R_{A0}} - \frac{1}{R_{B0}} + \sum_{m=1}^{\infty} k_{12}^m \left[\left[r_A^2 + (2mh + z_A)^2\right]^{-1/2} + \right.\right. \\
& + \left[r_A^2 + (2mh - z_A)^2\right]^{-1/2} - \left[r_B^2 + (2mh + z_B)^2\right]^{-1/2} - \\
& \left.\left. - \left[r_B^2 + (2mh - z_B)^2\right]^{-1/2}\right]\right\}, \qquad (31)
\end{aligned}
$$

where $r_A^2 = (x - x_A)^2 + y^2$, $r_B^2 = (x - x_B)^2 + y^2$ are squares of horizontal distances from the $+I$, $-I$ electrode, respectively. This unperturbed potential is symmetric with respect to y-coordinate,but the resulting potential must not be of the same property of symmetry if the prism is not symmetric with respect to the y-coordinate. In this manner we obtain a greater system of linear equations than in *Hvoždara (1995)*.

Introducing this potential $V_1(P_m)$ calculated for points $P_m \in S$ we can solve the system of Eq. (30) and then calculate the potential $U_1(P)$ on the surface of the earth by means of the formula (1) in discretized approximation of the boundary integral, and also the intensity of the electric field:

$$\begin{aligned}\boldsymbol{E}_1(P) &= -\mathrm{grad}\, U_1(P) = \\ &= -\mathrm{grad} V_1(P) - \frac{1}{4\pi} \int\limits_S f(Q) \mathrm{grad}_P \left[\boldsymbol{n}_Q \cdot \mathrm{grad}_Q G_1(P,Q) \right] \mathrm{d}S_Q \,. \end{aligned} \tag{32}$$

From the practical point of view the most interesting is the electric field on the straight line connecting the electrodes, which in view of symmetry has the non-zero E_x component only:

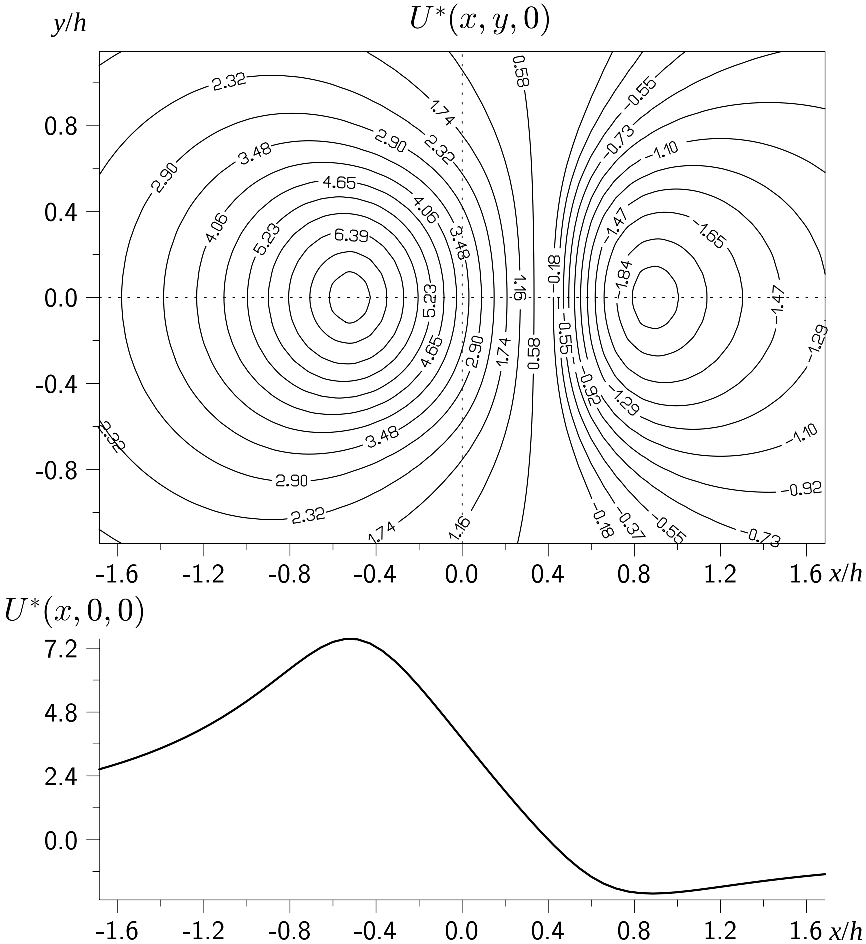
$$\begin{aligned} E_{1x}\big|_{\substack{z=0\\y=0}} &= \frac{I\rho_1}{4\pi} \left\{ \left[-\frac{\partial G_1(P,Q_A)}{\partial x} \right]_{\substack{z=0\\y=0}} + \left[\frac{\partial G_1(P,Q_B)}{\partial x} \right]_{\substack{z=0\\y=0}} \right\} - \\ &- \frac{1}{4\pi} \int\limits_S f(Q) \frac{\partial}{\partial x} \left[\boldsymbol{n}_Q \cdot \mathrm{grad}_Q G_1(P,Q) \right]_{\substack{z=0\\y=0}} \mathrm{d}S_Q \,. \end{aligned} \tag{33}$$

The absolute value of this electric field intensity can be used for calculating the practically needed characteristics – the apparent resistivity ρ_a (normalized to the ρ_1 value):

$$\left(\frac{\rho_a}{\rho_1} \right)_x = \frac{2\pi}{I\rho_1} \left[|E_{1x}| \right]_{\substack{z=0\\y=0}} \left\{ \left| \frac{x - x_A}{R_A^3} - \frac{x - x_B}{R_B^3} \right| \right\}^{-1} , \tag{34}$$

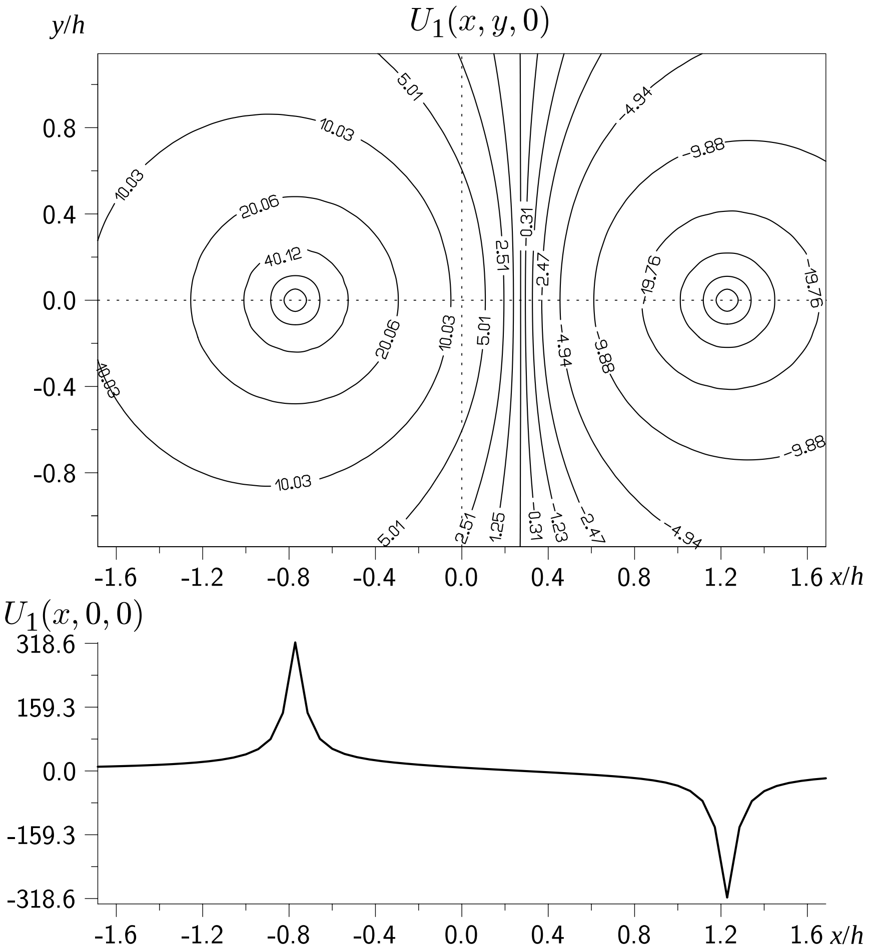
where x is the point where the intensity E_{1x} is calculated and $R_A^2 = r_A^2 + (z_A)^2$, $R_B^2 = r_B^2 + (z_B)^2$. Two points, where the $+I$ and $-I$ electrodes are situated, are excluded from calculations because the intensity is singular there.

As an example of results we present Figs 3a-d and 4a-d, where the source electrodes are situated at points $x_A = -1.6$, $x_B = 2.4$, while h=3 m. In Figs. 3a-d there are plotted results for the high resistive prismatic dyke of the substratum ($\rho_T/\rho_1 = 10$), while in Figs. 4a–d there are results for low resistive prism $\rho_T/\rho_1 = 0.1$. The isoline maps of the anomalous potential U_1^*, total potential U_1, E_x and E_y-components of the electric field at the surface $z = 0$ are presented. The curves at the bottom of the isoline maps in



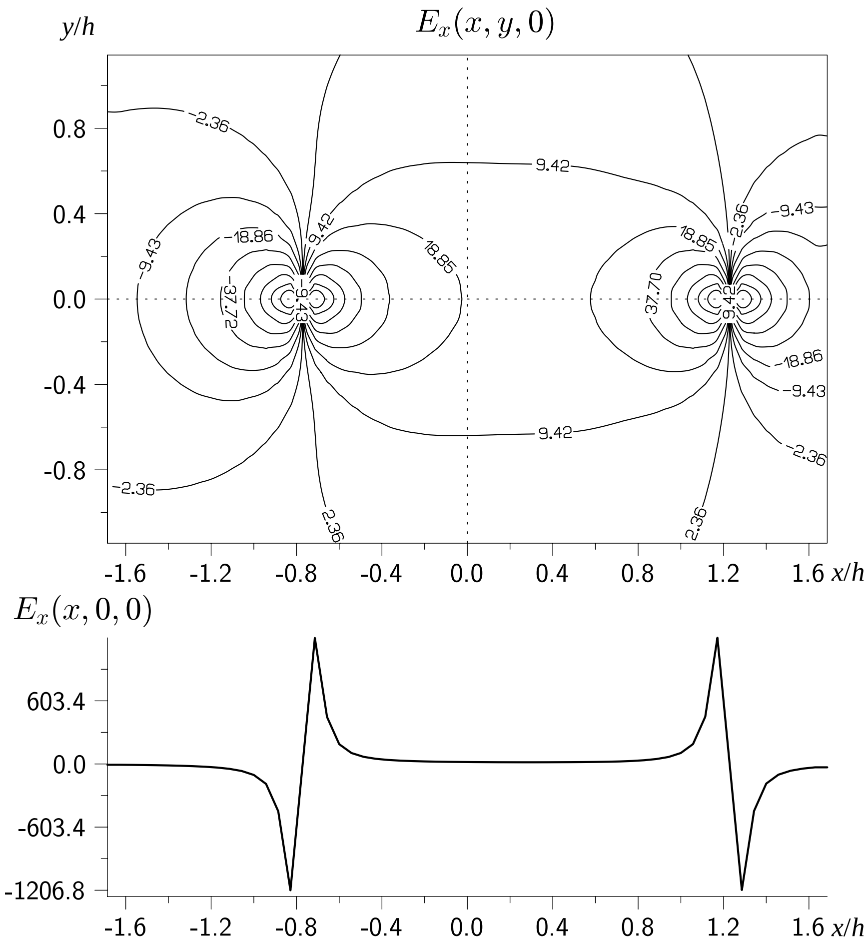
z_1, xl, xr, yl, yr = .5, -1.0, 1.0, -1.0, 1.0, m
z_2, xl, xr, yl, yr = 3.0, -2.0, 2.0, -2.0, 2.0, m
h_T, h = 1.8, 3.0, m
x_A, y_A, z_A = -1.35, .00, .05, m
x_B, y_B, z_B = 2.15, .00, .05, m
ρ_1 = 100., ρ_2 = 1000., ρ_T = 1000. Ω m

Fig. 3a. Isoline map and profile curve for the anomalous potential U^*.



z_1, xl, xr, yl, yr = .5, -1.0, 1.0, -1.0, 1.0, m
z_2, xl, xr, yl, yr = 3.0, -2.0, 2.0, -2.0, 2.0, m
h_T, h = 1.8, 3.0, m
x_A, y_A, z_A = -1.35, .00, .05, m
x_B, y_B, z_B = 2.15, .00, .05, m
ρ_1 = 100., ρ_2 = 1000., ρ_T = 1000. Ω m

Fig. 3b. Isoline map and profile curve for the total potential U_1 on the surface $z = 0$.



z_1, xl, xr, yl, yr = .5, -1.0, 1.0, -1.0, 1.0, m
z_2, xl, xr, yl, yr = 3.0, -2.0, 2.0, -2.0, 2.0, m
h_T, h = 1.8, 3.0, m
x_A, y_A, z_A = -1.35, .00, .05, m
x_B, y_B, z_B = 2.15, .00, .05, m
ρ_1 = 100., ρ_2 = 1000., ρ_T = 1000. Ω m

Fig. 3c. Isoline map and profile curve for the electric component E_x on the surface $z = 0$.

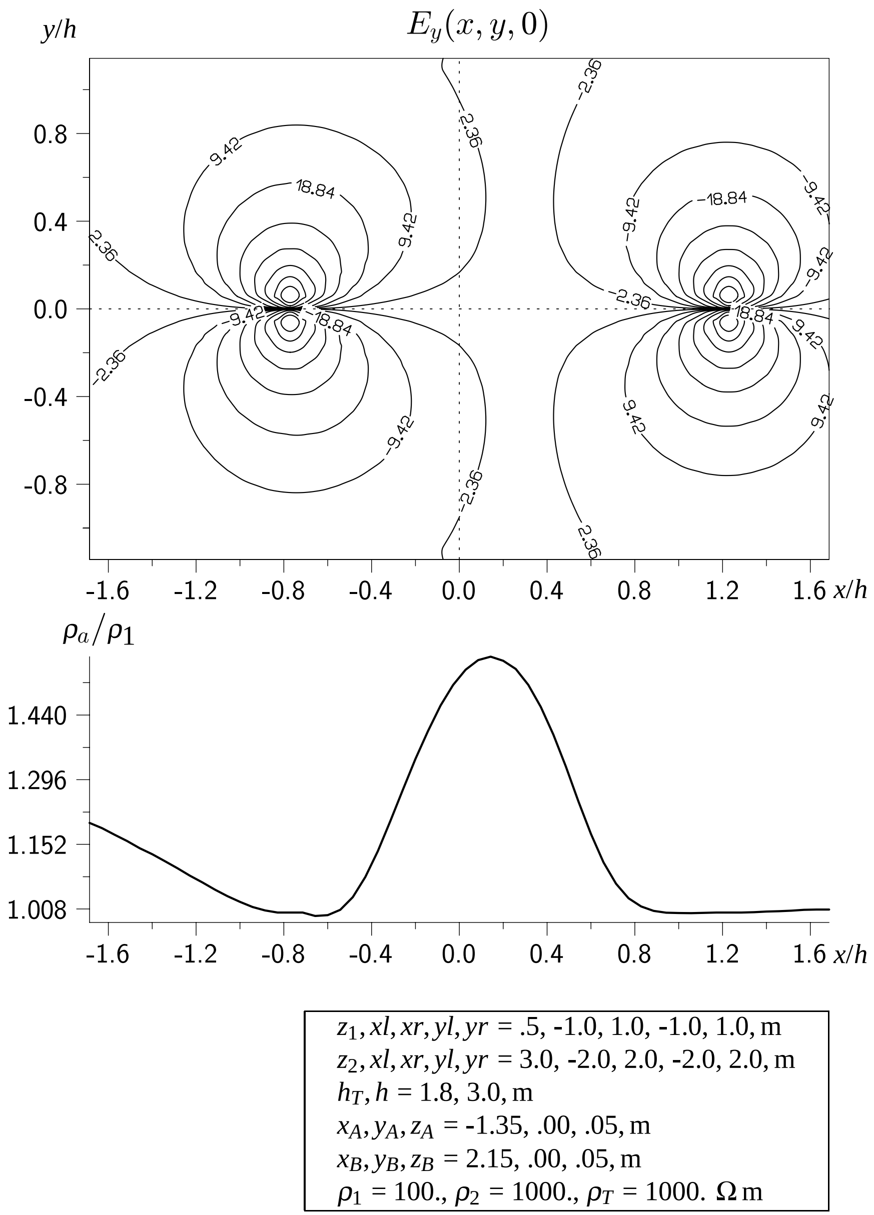
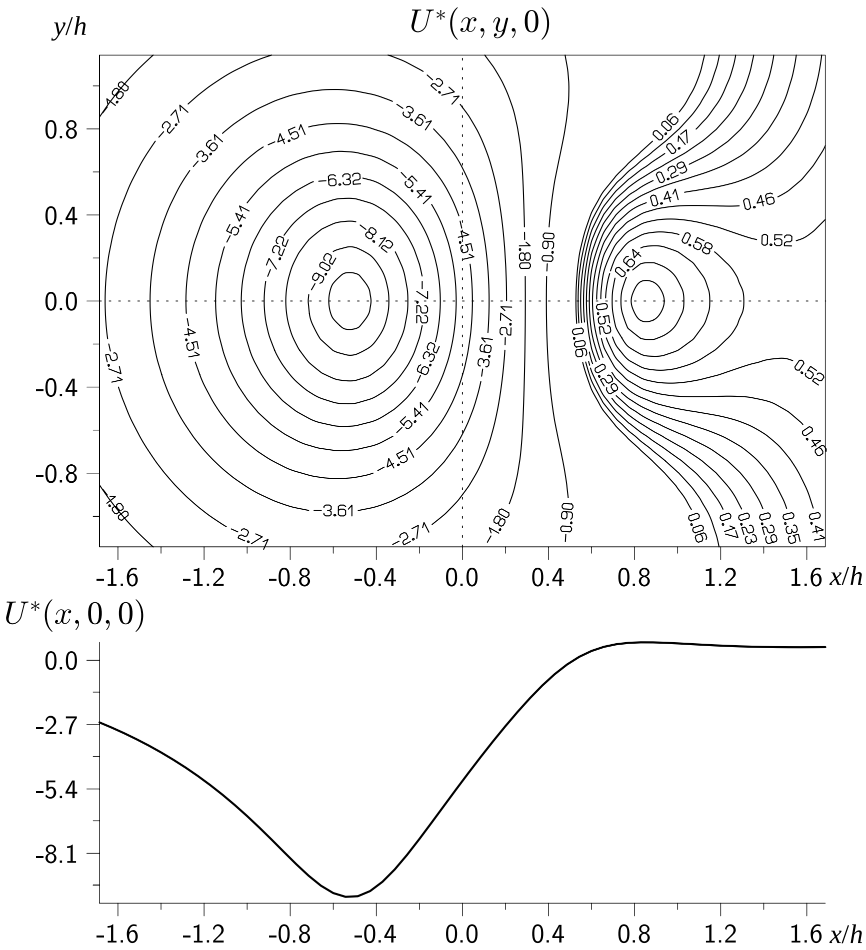
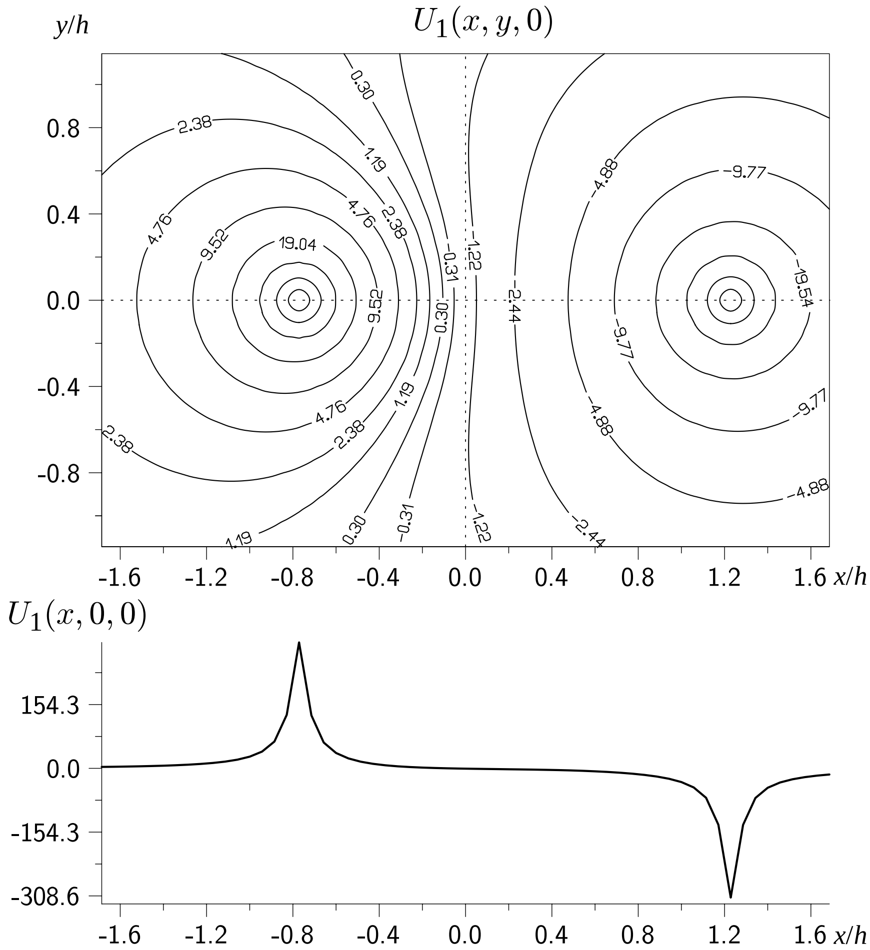


Fig. 3d. Isoline map for the electric component E_y on the surface $z = 0$, the profile curve shows apparent resistivity.



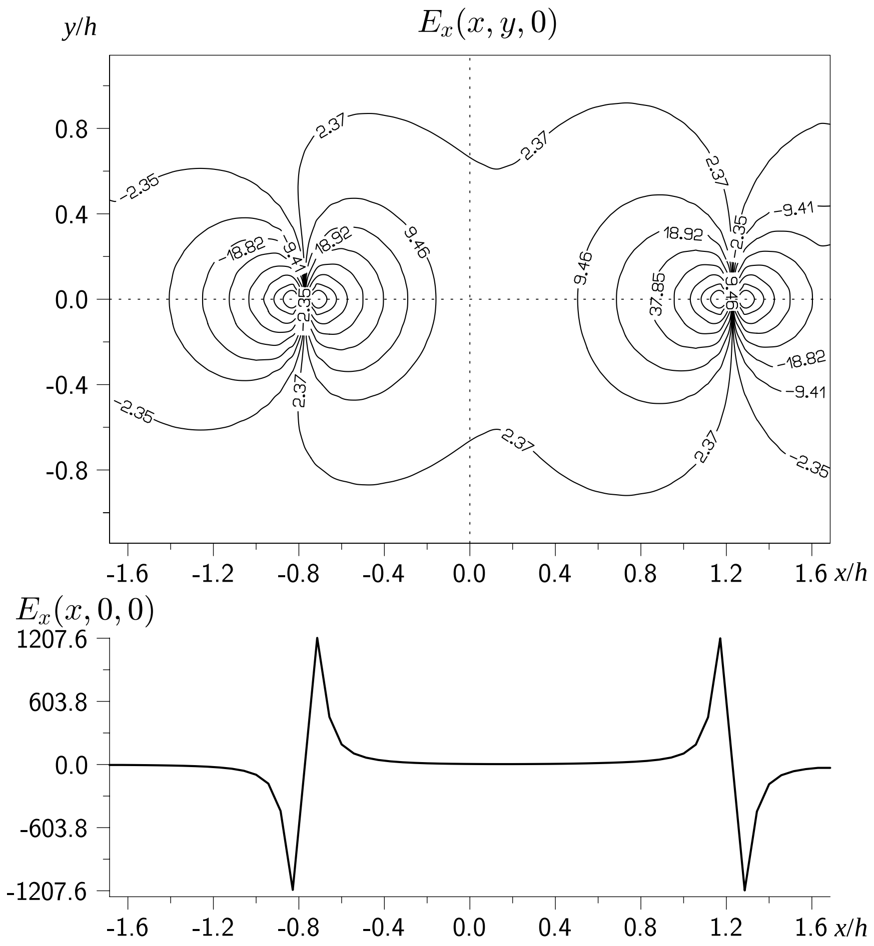
z_1, xl, xr, yl, yr = .5, -1.0, 1.0, -1.0, 1.0, m
z_2, xl, xr, yl, yr = 3.0, -2.0, 2.0, -2.0, 2.0, m
h_T, h = 1.8, 3.0, m
x_A, y_A, z_A = -1.35, .00, .05, m
x_B, y_B, z_B = 2.15, .00, .05, m
ρ_1 = 100., ρ_2 = 10., ρ_T = 10. Ω m

Fig. 4a. Isoline map and profile curve for the anomalous potential U^*.



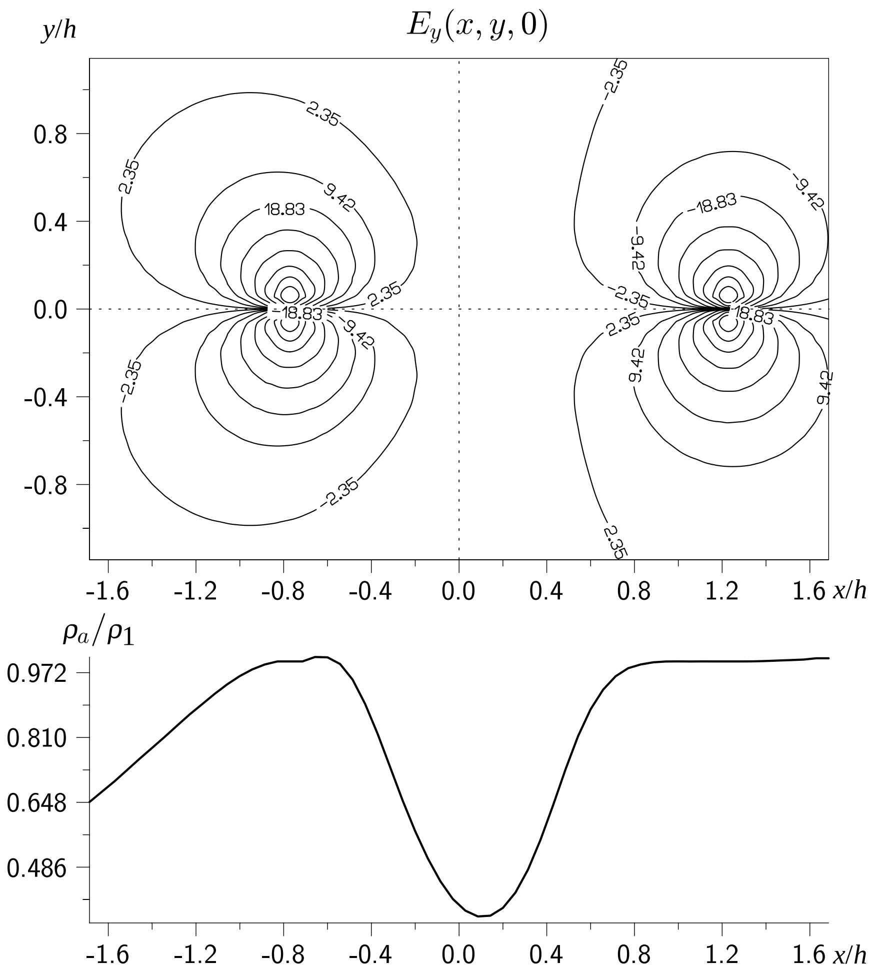
z_1, xl, xr, yl, yr = .5, -1.0, 1.0, -1.0, 1.0, m
z_2, xl, xr, yl, yr = 3.0, -2.0, 2.0, -2.0, 2.0, m
h_T, h = 1.8, 3.0, m
x_A, y_A, z_A = -1.35, .00, .05, m
x_B, y_B, z_B = 2.15, .00, .05, m
ρ_1 = 100., ρ_2 = 10., ρ_T = 10. Ω m

Fig. 4b. Isoline map and profile curve for the total potential U_1 on the surface $z = 0$.



z_1, xl, xr, yl, yr = .5, -1.0, 1.0, -1.0, 1.0, m
z_2, xl, xr, yl, yr = 3.0, -2.0, 2.0, -2.0, 2.0, m
h_T, h = 1.8, 3.0, m
x_A, y_A, z_A = -1.35, .00, .05, m
x_B, y_B, z_B = 2.15, .00, .05, m
ρ_1 = 100., ρ_2 = 10., ρ_T = 10. Ω m

Fig. 4c. Isoline map and profile curve for the electric component E_x on the surface $z = 0$.



z_1, xl, xr, yl, yr = .5, -1.0, 1.0, -1.0, 1.0, m
z_2, xl, xr, yl, yr = 3.0, -2.0, 2.0, -2.0, 2.0, m
h_T, h = 1.8, 3.0, m
x_A, y_A, z_A = -1.35, .00, .05, m
x_B, y_B, z_B = 2.15, .00, .05, m
ρ_1 = 100., ρ_2 = 10., ρ_T = 10. Ω m

Fig. 4d. Isoline map for the electric component E_y on the surface $z = 0$, the profile curve shows apparent resistivity.

Figs. 3a–c or Figs. 4a–c depict the courses of isoline quantities along the axis x (connecting source electrodes). The profile curves in Fig. 3d and Fig. 4d show values of apparent resistivity ρ_a/ρ_1 for the dipole profiling along the x axis calculated by means of formula (34). The parameters of the model prism are given in box tables, namely: z_1, xl, xr, yl, yr are coordinates (left, right) of the upper rectangle surface of the prism at the depth z_1; similar values z_2, xl, xr, yl, yr concern the bottom rectangle of the prism at the depth $z_2 = h$. The $+I$ electrode is situated at the point (x_A, y_A, z_A) and the $-I$ electrode in the point (x_B, y_B, z_B). Resistivities ρ_1, ρ_2, ρ_T are given in the box. Since we suppose $z_2 = h$ and $\rho_T = \rho_2$ the prism represents an elevation (outcrop) of the substratum into the first layer. In this curve ρ_a/ρ_1 in Fig. 3d we can see that the presence of the high resistive prism is clearly pronounced by increased values of ρ_a/ρ_1 till 30% in comparison to "near electrode" values (= 1). The resistivity profile in Fig. 4d shows clear decrease of the apparent resistivity due to the low resistive prism. Also the anomalous potential U_1^* in Fig. 4a values are negative when comparing with Fig. 3a.

In conclusion, it can be stated that our algorithm and computer program work reliably for both high- or low-resistive buried prismatic 3-D dykes with sloped faces, which enables us to calculate more general families of geoelectrical potential problems in non-uniform media.

Acknowledgments. The author is grateful to the Slovak Grant Agency VEGA for partial support of the work by means of project No. 2/7008/27.

References

Eloranta E. H., 1986: The solution of a stationary electric current problem with the aid of Fredholm's integral equation of the second kind formulated for the potential. Geophys. Prosp., **34**, 856–871.

Furness P., 1992: Modelling of spontaneous mineralization potentials with a new integral equation. Journ. Appl. Geoph., **29**, 143–155.

Hvoždara M., 1982: Potential field of a stationary electric current in a stratified medium with a three-dimensional perturbing body. Studia geophys. et geod., **26**, 160–172.

Hvoždara M., 1983: Electric and magnetic field of a stacionary current in a stratified medium with a three-dimensional inhomogeneity. Studia geophys. et geod., **27**, 59–84.

Hvoždara M., 1984: Solutions of the stationary approximations for MT field in the layered Earth with 3-D or 2-D inhomogeneities. J. Geophys., **55**, 214–227.

Hvoždara M., 1990: Stationary approximation of the magnetotelluric field in a halfspace with vertical contact and three-dimensional disturbing body. Studia geophys. et geod., **34**, 10–31.

Hvoždara M., 1994: Analytical solution of the boundary integral equation for potential of a point source electrode near by a spherical body. Contr. Geophys. Inst. SAS, **24**, 24–38.

Hvoždara M., 1995: The boundary integral calculation of the D.C. geoelectric field due to a point current source on the surface of 2-layered earth with a 3-D perturbing body buried or outcropping. Contr. Geophys. Inst. SAS, **25**, 7–25.

Ivan M., 1994: Upward continuation of potential fields from a polyhedral surface. Geophys. Prosp., **42**, 391–404.

Lee T., 1975: An integral equation and its solution for some two- and three-dimensional problems in resistivity and induced polarization. Geophys. J. Roy. Astr. Soc., **42**, 81–95.

Okabe M., 1981: Boundary element method for the arbitrary inhomogeneities problem in electrical prospecting. Geophys. Prosp., **29**, 39–59.