# Truncation Filtering Methodology: Input gravity data and pattern matching

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Abstract: The compilation of proper input gravity data for the Truncation Filtering Methodology (TFM) from observed gravity is discussed. The aim of the TFM interpretation is to determine the anomalous density distribution, or at least some of its characteristics, below the earth's surface in a studied region. It implies that the input data must be equal to the gravity effect (attraction) of all such anomalous masses of interest. Furthermore, the TFM requires that the input gravity data be given on a level reference surface, the position of which is further constrained by the requirement to stay outside all the anomalous masses, hence above all the terrain, in order to avoid downward continuation through anomalous masses. Such a requirement is imposed by the fact, that the TFM is a pattern recognition technique and the knowledge of patterns comes from synthetic modeling on a level surface without topography. Consequently the requirements imply that the input data needed are the gravity disturbances, corrected for the effects of topography and bathymetry, harmonically upward continued to a level surface tightly enveloping the topo-surface in the area of interest. Numerical procedures and several approximations in compiling such data are discussed.

**Key words:** TFM, gravity disturbance, gravity interpretation, gravimetric inversion, pattern recognition

## 1. Introduction

The Truncation Filtering Methodology (TFM) was introduced in 1995 *(Vajda, 1995; Vajda and Vaníček, 1999; 2002)* as a tool for interpreting

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gravity data. The methodology uses specially designed filters, by which gravity data are transformed into other quantities. The truncation filter has one free parameter (the truncation parameter), therefore the transformation results in a sequence of profiles or surfaces *(Vajda and Vaníček, 1997; 2002)* of the "post-filtered quantity", when this parameter is varied monotonically within a selected interval of values. When the sequence of such profiles or surfaces is animated, dynamic patterns are observed. These patterns are signatures of the anomalous masses generating the gravity data. The TFM may be classified as a data enhancement and pattern recognition technique. The patterns often have an onset (a value of the truncation parameter at which they appear). From the onset of a pattern the depth of some characteristic feature of the anomalous mass density distribution may be determined. To be able to interpret such dynamic patterns, the relationship between the observed pattern and the source (anomalous masses) must be known apriori. This may be established by modeling (synthetic simulations) and case studies. Work is in progress to establish such a know–how or databank of patterns. So far this relationship is known only for point sources *(Vajda and Vaníček, 1997; 2002).* Point sources produce dimple patterns. The relationship between the depth of a point source and the onset of the dimple pattern was established not only by computer simulations, but also by analytical derivations *(Vajda and Vaníček, 1999; 2002).*

Originally, at the very beginning of developing the TFM, when the truncation filter in use was the Truncated Stokes Transform *(Vajda and Vaníček, 1997; 1999)*, the input data were the gravity anomalies. However, such approximations were adopted in computer simulations and analytical derivations, that the gravity anomalies were actually approximated by gravity disturbances without explicitly declaring it. This was caused by taking the geoidal undulation (the separation between reference ellipsoid and geoid) to be zero in the first approximation. Later, the use of the Truncated Stokes Transform was extended to using a general truncation filter (the Stokes function as the kernel of the integral was replaced by an arbitrary function of the angular distance between the computation point and the integration point), and the gravity anomaly was replaced by the gravity disturbance *(Vajda and Vaníček, 2002).*

In computer simulations the synthetic gravity data are computed either on a plane bounding the halfspace containing anomalous masses (planar approximation) or on a reference sphere bounding the anomalous masses (spherical approximation). Topography of the earth's surface is not considered. Synthetic TFM sequences are computed from synthetic gravity data and the knowledge of "synthetic" TFM patterns is built based on these computer simulations. However, we want to interpret gravity data pertinent to the real world by means of TFM pattern recognition. In real world the observed gravity data are: (1) typically given on the topo-surface, as opposed to the reference plane or sphere, and (2) impacted by the effect of topographic masses, and eventually of the bathymetric density contrast. In this paper we shall examine how to handle these two issues in order to be able to interpret observed gravity data by means of the TFM, or in other words, in order to be able to match 'observed TFM patterns' with 'synthetic TFM patterns'.

## 2. Background

Points and surfaces in our study are positioned using geographical coordinates (geocentric geodetic [Gauss-ellipsoidal] coordinates), vertical position being given by ellipsoidal (geodetic) height $h$ and horizontal position $\Omega \equiv (\phi, \lambda)$ by geodetic latitude and longitude, respectively. The International Reference Ellipsoid, such as GRS'80, is chosen both as the *datum* for the geodetic coordinates and as the *normal ellipsoid*, i.e., the reference body for normal potential, *normal gravity*, and a *model normal density distribution*. Gravity data are assumed already properly corrected for the gravitational effects of the atmosphere, tides, and other known temporal effects. The discussed topographic and bathymetric corrections to a gravity disturbance are given by Newton-type volume integrals for attraction. Newton-type volume integrals herein are written in spherical approximation (e.g., *Moritz, 1980,* p. 349; *Vajda et al., 2004; Novák and Grafarend, 2005*) expressed in geodetic (not spherical) coordinates, taking the mean earth radius $R = 6\ 371$ km. The $J$ kernel of the volume integrals for the vertical component of attraction is defined here as the negative vertical derivative of the reciprocal Euclidean distance $L$ between computation $(h, \Omega)$ and integration $(h', \Omega')$ points (e.g., *Vajda et al., 2006*)

$$L(h, \Omega, h', \Omega') = \sqrt{(R+h)^2 + (R+h')^2 - 2(R+h)(R+h')\cos\psi}\,, \qquad (1)$$

Table 1. Surfaces of particular relevance in our study

| **surface** | **definition** | **domain** |
|---|---|---|
| reference ellipsoid (RE) | $h(\Omega) = 0$ | $\Omega_0$ |
| topo-surface onshore (relief) | $h(\Omega) = h_T(\Omega)$ | $\Omega_L$ |
| topo-surface offshore (geoid) | $h(\Omega) = N(\Omega)$ | $\Omega_S$ |
| sea bottom | $h(\Omega) = h_B(\Omega)$ | $\Omega_S$ |

$$\cos\psi = \sin\phi\ \sin\phi' + \cos\phi\cos\phi'\cos\left(\lambda - \lambda'\right) , \tag{2}$$

$$J(h,\Omega,h',\Omega') = \left[(R+h) - (R+h')\cos\psi\right] L^{-3}(h,\Omega,h',\Omega') . \tag{3}$$

The topographic and bathymetric corrections to the scalar gravity disturbance are then the negative vertical component of the gravitational attraction of the topography and of the bathymetric contrast given by the volume integral of the Newtonian type. The convolution integrals of the truncation filters are evaluated, either in geographical coordinates or in local polar coordinates, on the reference ellipsoid in spherical approximation. The surface increment at the unit sphere in geographical coordinates reads $d\Omega' = \cos\phi'\ d\phi'\ d\lambda'$ and in local polar coordinates $d\Omega' = (1/R)\sin(s/R)\,ds\ d\alpha$, where $s$ is the spherical distance from the local pole and $\alpha$ is azimuth. The surface increment on the RE is $d\sigma' = R^2 d\Omega'$, while at a sphere of radius $r = R + h'$ it is $d\sigma' = (R+h')^2\, d\Omega'$. The volume increment is $d\vartheta' = (R+h')^2\, d\Omega'\ dh'$.

The extension towards evaluating the Newton volume integrals in ellipsoidal geometry would be carried out by replacing the $J$ kernel and the volume increment $d\vartheta'$ (of the volume integrals) by their respective (and more cumbersome) expressions in Gauss- or Jacobi- ellipsoidal coordinates *(Vajda et al., 2004; Novák and Grafarend, 2005)*. For brevity, we will write the $J$ kernel and the volume increment of the Newton volume integrals herein without their position arguments. The following surfaces, listed in Tab. 1, are particularly relevant in our study. The onshore and offshore regions of the globe ($\Omega_0$) are denoted as $\Omega_L$ and $\Omega_S$, respectively, $\Omega_0 = \Omega_L \cup \Omega_S$.

Both the relief and sea bottom in the topo-correction and the bathymetric correction are reckoned from the RE, i.e., positioned (referred to) in geodetic heights (not heights above sea level). When not directly available, these are obtained by adding geoidal (or quasigeoidal) heights to the orthometric (or normal) heights, e.g., from a local or regional geoid model in the vicinity of the station and from a global geoid solution such as EGM96 *(Lemoine, 1998)* in the remainder to $\Omega_0$. We shall use the term 'reference ellipsoid' (RE) for both the body and its surface, trusting that the meaning will be clear from the context.

We presuppose that a model normal density distribution $\rho_N(h,\Omega)$ inside the RE, meeting the constraint of generating normal gravitational potential in the exterior of the RE, generating normal gravity (both inside and outside the RE), which in addition is geophysically meaningful, can be found to at least a satisfactory approximation *(Tscherning and Sünkel, 1981; Moritz, 1968; 1973; 1990)* in the form of an ellipsoidally stratified normal density distribution with a PREM-like "radial" behavior (PREM being an acronym of the Preliminary Reference Earth Model), consisting of the top layer of an average crustal density $\rho_0$ at least 11 km thick.

## 3. Gravity data interpretation or inversion

The objective of inverting or interpreting gravity data is to determine the *anomalous density* distribution $\delta\rho$ ("anomalous masses") below the topographic surface onshore or below the sea bottom offshore. To do that a physical link between the observables (gravity) and the unknowns (anomalous density) must be established. That can be achieved by the decomposition of the earth gravitational potential (e.g., *Vajda et al., 2006; 2008*) resulting in

$$\delta g^{BT}(h,\Omega) = \delta A^{BT}(h,\Omega)\,. \tag{4}$$

Above, $\delta g^{BT}$, called the 'BT gravity disturbance', is a gravity disturbance corrected for the (global) effects of topography and bathymetry, both effects being referred to the reference ellipsoid, cf. (*Vajda et al., 2008*, Sects. 5 and 6)

$$\delta g^{BT}(h,\Omega) = g(h,\Omega) - \gamma(h,\Omega) - A^{BT}(h,\Omega)\,, \tag{5}$$

where $g$ is actual gravity observed at the observation point, $\gamma$ is normal gravity at the same point, and $(-A^{BT})$ is the "*topographic and bathymetric correction*" based on the RE

$$A^{BT}(h,\Omega) = G\rho_0 \int\limits_{0}^{h_T(\Omega')} \iint\limits_{\Omega_L} J\, d\vartheta' + G\rho_W \int\limits_{0}^{N(\Omega')} \iint\limits_{\Omega_S} J\, d\vartheta' + G\delta\rho_0 \int\limits_{h_B(\Omega')}^{0} \iint\limits_{\Omega_S} J\, d\vartheta', \quad (6)$$

$\rho_0$ being average crustal density, $\rho_W$ density of sea water, and $\delta\rho_0 = \rho_W - \rho_0$. In Eq. (6) the first volume integral is the topographic effect of the so-called "solid topography" onshore, the second is the topographic effect of the so-called "liquid topography" offshore, and the third is the effect of the bathymetric density contrast offshore. All three effects are relative to the RE.

On the right hand side of Eq. (4) we have the gravitational attraction of the unknown and sought (global) mass anomalies below the topographic surface onshore and below the sea bottom offshore

$$\delta A^{BT}(h,\Omega) = G \int\limits_{-R}^{h_T(\Omega')} \iint\limits_{\Omega_L} \delta\rho(h',\Omega') J\, d\vartheta' + G \int\limits_{-R}^{h_B(\Omega')} \iint\limits_{\Omega_S} \delta\rho(h',\Omega') J\, d\vartheta'. \quad (7)$$

When the gravimetric inverse problem, formulated in terms of gravity disturbances by means of Eq. (4), is solved by direct inversion, the volume integral for the attraction of the unknown anomalous density distribution is discretised to turn the problem into a system of linear equations. The problem is non-unique and ill-posed. When the gravimetric inverse problem is solved by the forward modeling techniques, then the left-hand-side of Eq. (4) represents observed (observed and compiled) gravity data, while its right-hand-side represents synthetic (modeled) gravity data, computed from a selected initial anomalous density by evaluating the volume integrals of Eq. (7) (direct problem solution), which is iteratively fine-tuned so that the synthetic gravity matches the observed gravity. The problem is non-unique and ill-posed, but in forward modeling additional geophysical or geological constraints may be more readily adopted.

Matching synthetic to observed gravity data is the core procedure in forward modeling techniques for solving the gravimetric inverse problem. The synthetic gravity data are matched with the observed ones and the model is iteratively tuned to minimize the misfit. Matching the observed with synthetic gravity data takes place at stations (observation points). The location of the stations is arbitrary, cf. Eq. (4). Therefore, the matching can take place on the relief, at the sea surface, on the sea bottom, or at the points of the trajectory of an airplane, when dealing with airborne gravity data, etc. Upward or downward continuation of gravity data is not necessary. The continuation of the gravity data to a level (reference) surface is necessary only if the gravity data are further processed by means of integral transformations defined at a level surface, or in pattern recognition techniques, where the knowledge of a pattern is respective to a level surface, just as in our case of the TFM.

## 4. TFM pattern recognition – synthetic TFM patterns

The knowledge of the TFM patterns can be acquired through modeling – synthetic studies (computer simulations). A simplified model of a given geological setting is constructed in terms of the synthetic (model) anomalous density distribution $\delta\rho$. This model assumes no topography, all anomalous masses dwell within a sphere (spherical approximation) or a halfspace (planar approximation). This model anomalous density generates the synthetic gravity data, i.e., attraction $\delta A$, cf. Eq. (7), dropping the superscript "BT", as there is no relief or sea bottom present in the modeling. These synthetic gravity data are computed on a reference sphere (spherical approximation), or a level plane (planar approximation), $h = 0$. These synthetic gravity data are in the next step truncation filtered, i.e., they enter as input gravity data the integral transforms (convolution surface integrals) of the truncation filters to produce the respective synthetic truncation sequences *(Vajda and Vaníček, 2002)*. The truncation filter

$$Z^{s_0}_{syn}(\Omega) = \iint_{cap(s_0)} \delta A(h' = 0, \Omega') w\left[s(\Omega, \Omega')\right] d\sigma' =$$

$$= R \int_0^{s_0} \int_0^{2\pi} \delta A(h' = 0, s, \alpha) w(s) \sin\left(\frac{s}{R}\right) d\alpha \, ds \tag{8}$$

is a surface integral convolving the gravity data with an isotropic kernel (acting as a weight function). The kind of the kernel ($w$) specifies the kind of the truncation filter (ibid). This kernel is a preselected function of the spherical distance $s$ (on the reference sphere) between the computation point $\Omega$ and the integration point $\Omega'$. The gravity data are convolved on a spherical cap of radius $s_0$, called the truncation parameter, which is a free parameter of the transform. For a sequence of values of the truncation parameter a sequence of surfaces of the post-filter gravity quantity ($Z_{syn}$) is computed and animated (the so-called "truncation sequence" or "TFM sequence"). Subscript "syn" stands here for "synthetic". In Eq. (8) the transform is written first in geographic coordinates, where the surface increment is $d\sigma' = R^2 d\Omega' \; = R^2 \cos\phi' \; d\phi' \; d\lambda'$, and next in the local polar coordinates of the computation point, where $s$ is spherical distance and $\alpha$ is azimuth, and where the surface increment is $d\sigma' \; = R \sin(s/R)\, ds \;\; d\alpha$. The spherical distance reads

$$s\left(\Omega, \Omega'\right) = R \; \arccos \; \left[\sin\phi \sin\phi' + \cos\phi \cos\phi' \cos\left(\lambda - \lambda'\right)\right]. \qquad (9)$$

Since the kernel of the truncation filter is isotropic, in numerical evaluation of a truncation filter there is no need to compute the azimuths of the running (integration) points. The numerical integration can be more conveniently performed in geographic coordinates.

Dynamic patterns in the synthetic truncation sequences $Z_{syn}$ are studied. These patterns are qualitatively, and – if possible – quantitatively related to the model (synthetic) anomalous density distribution (of the studied geological model). This simulation procedure is repeated for many more-or-less simplistic anomalous density distributions (models) to build a databank of known TFM patterns. Different kinds of truncation filters (various kernels) may produce different patterns for the same model. The development of the databank of dynamic TFM patterns and the investigation on which filters are sensitive to which features of realistic geological situations are subject to our research in progress, which is a task for a long run.

## 5. TFM pattern recognition – observed TFM patterns

Suppose we already have knowledge of dynamic TFM patterns that we gained by studying synthetic truncation sequences (of post-filtered gravity quantities $Z_{syn}$) generated by model anomalous density distributions (synthetic geological models), as described in Sec. 4. Let us now move to the realm of observed gravity data. Recalling Eq. (4) we realize that the match to the synthetic gravity data are the "observed" BT gravity disturbances, compiled from observed gravity by means of Eq. (5). Since our objective now is to "match" the *synthetic TFM patterns* (dynamic patterns in synthetic truncation sequences) with *observed TFM patterns* (dynamic patterns in truncation sequences computed by truncation filtering the observed gravity data, i.e., the BT gravity disturbances), in order to apply the pattern recognition techniques to solving the inverse problem, we have to compute the "observed truncation sequences". However, at this moment a little complication arises.

The knowledge of the dynamic TFM patterns was gained by synthetic modeling the gravity data on a reference sphere, not at the topo-surface. On the other hand, our observed gravity data are most commonly given at stations on the earth's surface. However, integral transforms defining truncation filters are surface convolution integrals – they require the input gravity data be given on a reference sphere (in spherical approximation), or on a level plane (in planar approximation). Consequently, the BT gravity disturbances must be continued to a reference sphere (or a level plane). Harmonic downward continuation to the sphere $h = 0$ is not possible, because the BT gravity disturbances are not harmonic below the topo-surface *(Vajda et al., 2006; 2008).* They must be harmonically upward continued to a reference sphere $h = h_R$, where the reference geodetic height $h_R$ is such, that the reference sphere resides above the topo-surface everywhere in the region of our gravity data interpretation. We want to have it just above the highest point in our region, but not higher, due to the attenuation of the useful signal with height in the upward continuation.

The harmonic upward continuation can be performed for instance by using the Poisson integral (e.g., *Hofmann-Wellenhof and Moritz, 2006*, p. 247, Eq. [6–44]). The function that is harmonic and that is continued is $(R+h)$ $\delta g^{BT}(h,\Omega)$. There is a further complication due to the fact that we want to continue from an irregular topo-surface to a reference sphere, not vice versa. Another option to carry out the continuation is to use an *equivalent sources method* (e.g. *Xia et al., 1993; Ivan, 1994; Meurers and Pail, 1998*), where the equivalent sources may be either polyhedra or point masses, or those in general represented by Spherical Radial Basis Functions (*Klees et al., 2007* and references quoted in Introduction; *Tenzer et al., 2008*). The detailed treatment of the harmonic upward continuation of the BT gravity disturbances is considered outside the scope of this paper. We have initiated a numerical study to compare the performance of various methods for the continuation that shall be presented in a separate work.

The BT gravity disturbances upward continued to the reference sphere $h = h_R$,

$$\delta g^{BT}\left(h = h_R, \Omega\right) = \delta g^{BT}[h = h_T(\Omega), \Omega] + D\delta g^{BT}[h_R, h_T(\Omega), \Omega], \tag{10}$$

where the $D\delta g^{BT}$ term is the *harmonic upward continuation correction* to the BT gravity disturbance, are exactly what we need as the input gravity data for the truncation filters to compute the "observed" truncation sequences

$$Z^{s_0}_{obs}\left(\Omega\right) = \iint\limits_{cap(s_0)} \delta g^{BT}(h' = h_R, \Omega') w\left[s\left(\Omega, \Omega'\right)\right] d\sigma' = \\ = R \int\limits_0^{s_0} \int\limits_0^{2\pi} \delta g^{BT}(h' = h_R, s, \alpha) w(s) \, \sin\left(\frac{s}{R}\right) d\alpha \, ds \,. \tag{11}$$

Here the subscript "obs" stands for "observed".

Now we come to the rationale of the TFM pattern recognition technique. When a dynamic TFM pattern is observed in a truncation sequence ($Z_{obs}$) computed from observed BT gravity disturbances, and this pattern is recognized as known (from synthetic simulations), then this pattern relates to the sought unknown real anomalous masses in the same way, as the synthetic (known) pattern relates to the known model (synthetic) anomalous masses. Hence, the model anomalous masses (respective to the TFM pattern) become one possible solution to the unknown sought real anomalous masses. The fact that it is only one possible solution is implied by the non-uniqueness of the gravimetric inverse problem. If the dynamic TFM pattern has an onset that can be quantitatively related to the depth of a characteristic feature (element) of the model geology, in synthetic studies yielding a depth $d*$, then the depth of that element in the real geology, yielded by the observed TFM pattern, will obviously become $d * +h_R$ reckoned from the reference surface $h = h_R$ , or $d*$ reckoned from the RE.

# 6. Additional approximations – regional and local applications

## 6.1. Truncating the volume integral for the attraction of anomalous masses

Let us first discuss the truncation of the Newton-type volume integral over anomalous density for a general case, when it is evaluated on the topo-surface. The case, when it is evaluated on the reference sphere, is then easily derived by putting $h_T(\Omega') = 0$. In regional and local studies the synthetic gravity data, i.e., the attraction of the anomalous masses given by Eq. (7), which is also used in our simulations for building the databank of synthetic TFM patterns, may be evaluated as the contribution of the anomalous masses from a "near domain", due to (a) the integral being evaluated over anomalous density, and (b) the fast attenuation of the $J$ kernel with spherical distance (e.g., Fig. A1 in *Vajda et al., 2007*). The "near domain" is horizontally defined as a "near zone", i.e., a spherical cap of the radius equal to a preselected maximum spherical distance $s_M$. Vertically the "near domain" is bound from above by the topo-surface $h = h_T(\Omega')$, and from below by some preselected maximum depth of interest $h = -d_M$. The "near domain" is respective to the computation point, i.e., it moves with the computation point. Written in local polar coordinates of the computation point, the volume integral reads

$$\delta A(h,\Omega) \approx G \int\limits_{-d_M}^{h_T(s,\alpha)} \int\limits_{0}^{s_M} \int\limits_{0}^{2\pi} \delta\rho(h',s,\alpha)\, J(h,h',s,\alpha) \;\times$$

$$\times\; \frac{(R+h')^2}{R} \sin\left(\frac{s}{R}\right)\; d\alpha\; ds\; dh'. \qquad (12)$$

Let us repeat that the "near domain" (respective to and horizontally moving with each computation point) is a 3D domain defined as $h' \in \langle -d_M; h_T(\Omega')\rangle$, $s \in \langle 0; s_M\rangle$, and $\alpha \in \langle 0;\, 2\pi)$, where $d_M$ and $s_M$ are respectively the maximum depth (below the reference ellipsoid) and the maximum spherical distance to which the contribution of anomalous masses is considered. In the case of gravimetric forward modeling techniques, that match synthetic with observed data on the topo-surface, the upper boundary of the "near domain" is the topo-surface. In our case of synthetic TFM modeling, considering no topography in the simulations, $h_T(s, \alpha) = 0$. It is more convenient to numerically evaluate the above volume integral in geographic coordinates, in order to avoid transforming the horizontal coordinates of running integration points into local polar coordinates, doing that for each computation point. When computing numerically the volume integral in geographic coordinates, all it takes is testing each integration grid cell whether it lies within the "near domain". Recall that the "near domain" moves (horizontally) with the computation point. Therefore, if we want to compute synthetic gravity on a computation grid $\phi \in \langle \phi_{\min}; \phi_{\max} \rangle$, $\lambda \in \langle \lambda_{\min}; \lambda_{\max} \rangle$, we must have the anomalous density distribution given on a 3D integration grid which horizontally envelopes the computation grid in a spherical distance everywhere by $s_M$. To fulfill these requirements we must be cautious of the convergence of the meridians on the globe.

A different approach is often taken in the practice. A "local domain", in terms of "integration grid", is selected as $h' \in \langle -d_M; h_T(\Omega') \rangle$, $\phi' \in \langle \phi'_{\min}; \phi'_{\max} \rangle$, $\lambda' \in \langle \lambda'_{\min}; \lambda'_{\max} \rangle$ on which anomalous density is given. The attraction of anomalous density outside of the "local domain" is disregarded. Then the volume integral is computed in geographic coordinates in such a fashion, that each grid cell of the "local domain" contributes into the volume integral for each computation point of the computation grid. The "local domain" is fixed. The result of the "local domain" approach differs from the result of the "near domain" approach in terms of edge effects.

In local studies, in the case of small enough $s_M$, the volume integral of Eq. (12) may be evaluated even in planar approximation (cf. Sec. 6.4). For our synthetic TFM modeling we are currently using the Mod3D modeling software *(Cerovský et al., 2004)*, which adopts planar approximation and "local domain" approach.

### 6.2. Truncating the volume integrals of the "topographic and bathymetric correction"

Rigorously speaking the *topographic and bathymetric correction* to gravity disturbance, Eq. (6), is to be evaluated over the entire globe. In regional and local studies the integration may be truncated to a spherical cap (the so-called near-zone), i.e., to some maximum spherical distance $s_M$ from the computation point (such as the Hayford-Bowie limit of 167 km, or a different one), if the truncation error (the contribution to the volume integrals from the so-called far-zone – the remainder to the full sphere) can be neglected as trend of no interest. In addition, the near-zone may be split into sub-zones with different grid steps – the finer the closer we are to the computation point, due to the shape of the $J$ kernel (which tapers off sharply with distance from the computation point). Also in the inner-most zone the volume integral may be expressed in planar approximation (cf. Sec. 6.4). A lot has been published about numerical aspects of evaluating the volume integrals of topographic corrections in geophysical and geodetic literature (e.g., *LaFehr, 1991; Talwani, 1998; Novák et al., 2001; Grand et al., 2004; Hinze et al., 2005; Janák et al., 2006; Mikuška et al., 2006*; and references quoted therein). We do not wish to repeat those concepts here. Although most of the published work regards topographic corrections referred to the geoid, the same numerical procedures apply also to our volume integrals in Eq. (6).

### 6.3. Approximating BT gravity disturbance by Bouguer anomaly

In the cases, when in local or regional applications the *geophysical indirect effect* (*Chapman and Bodine, 1979; Vogel, 1982; Jung and Rabinowitz, 1988; Meurers, 1992; Talwani, 1998; Li and Götze, 2001; Hackney and Featherstone, 2003; Hinze et al., 2005; Vajda et al., 2006*) can be neglected as trend of no interest, then the *BT gravity disturbance* may be approximated by the *Bouguer gravity anomaly* (cf. *Vajda et al., 2006*, Sec. 9).

### 6.4. Planar approximation

In planar approximation the computation or integration points given by the triplet of geodetic coordinates $(h, \phi, \lambda)$ become referred to in local Cartesian coordinates $(z, y, x)$ where the x-y plane is tangential to the reference sphere at the origin of the local Cartesian coordinate system $(\phi_0, \lambda_0)$, while the z-axis points upwards. If the y-axis is chosen as directed towards the north (y coordinates being the "northing") and the x-axis as directed towards the east (x coordinates being the "easting"), the coordinate system becomes right-handed. The local Cartesian coordinates (of both computation and integration points) are obtained by the following mapping of the geodetic coordinates

$$(x, y, z)^T = (R\cos(\phi_0)(\lambda - \lambda_0),\ \ R(\phi - \phi_0),\ \ h)^T\ . \tag{13}$$

In planar approximation the spherical distance becomes planar distance

$$s\left(x, y, x', y'\right) = \sqrt{(x - x')^2 + (y - y')^2}\,. \tag{14}$$

The Euclidean (spatial) distance becomes

$$L\left(x, y, z, x', y', z'\right) = \sqrt{s^2 + (z - z')^2}\,. \tag{15}$$

The negative vertical derivative of the reciprocal Euclidean distance, the $J$ kernel, becomes

$$J\left(x, y, z, x', y', z'\right) = \left(z - z'\right) L^{-3}\left(x, y, z, x', y', z'\right)\ . \tag{16}$$

The surface increment becomes

$$d\sigma' \ = R^2 \cos\phi'\ d\phi'\ d\lambda' = R\ \sin\left(s/R\right) d\alpha\ ds \approx s\ \ d\alpha\ ds = dx'dy'\,, \tag{17}$$

and the volume increment becomes

$$d\vartheta' = \frac{(R + h')^2}{R} \sin\left(\frac{s}{R}\right) d\alpha\ ds\ dh' \approx s\ d\alpha\ ds\ dz' = dx'dy'dz'. \tag{18}$$

The truncation filter in planar approximation becomes

$$Z_{syn}^{s_0}\left(x, y\right) = \iint\limits_{cap(s_0)} \ \delta A(z' = 0, x', y') w\left[s\left(x, y, x', y'\right)\right] dx'dy' \ = $$

$$= \int\limits_0^{s_0}\int\limits_0^{2\pi} \delta A(z' = 0, s, \alpha) w(s)\ s\ d\alpha\ ds\,. \tag{19}$$

For observed gravity, in the above surface integrals, the $\delta A(z' = 0, x', y')$ is replaced by $\delta g^{BT}(z' = h_R, x', y')$ and subscript "syn" is replaced by "obs". Also the harmonic upward continuation of the BT gravity disturbances can be performed in planar approximation (e.g., *Hofmann-Wellenhof and Moritz, 2006*, p. 248, Eq. [6–53]). In planar approximation the gravity disturbance itself, $\delta g^{BT}$, is a harmonic function.

## 7. Conclusion

The aim of the TFM interpretation is to determine the mass anomalies (anomalous density distribution) below the relief onshore and below the sea bottom offshore, or at least some of their characteristics, including depth estimates. Therefore the input data must be constructed such, that they are equal to the attraction of the anomalous density distribution of interest. It is the BT gravity disturbances that are equal to such an attraction, as proved by *Vajda et al. (2006; 2008)*. As the truncation filters are surface convolution integrals, the TFM requires that the input gravity data be given on a "level" surface (reference sphere in spherical approximation or level plane in planar approximation). The position of the level surface is further constrained by the requirement to stay above all the anomalous masses, hence above the topographic surface in the region of interest. All these requirements lead to a necessity to compile the BT gravity disturbances and to continue them harmonically to a level surface tightly enclosing (from above) the terrain in the region of interest. The upward continuation also assures the feasibility of matching observed TFM patterns with synthetic TFM patterns that represent the know-how of the TFM pattern recognition technique.

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