Global atmospheric effects on the gravity field quantities

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Abstract: We compile the global maps of atmospheric effects on the gravity field quantities using the spherical harmonic representation of the gravitational field. A simple atmospheric density distribution is assumed within a lower atmosphere ($<$ 6 km). Disregarding temporal and lateral atmospheric density variations, the radial atmospheric density model is defined as a function of the nominal atmospheric density at the sea level and the height. For elevations above 6 km, the atmospheric density distribution from the United States Standard Atmosphere 1976 is adopted. The 5×5 arc-min global elevation data from the ETOPO5 are used to generate the global elevation model coefficients. These coefficients (which represent the geometry of the lower bound of atmospheric masses) are utilized to compute the atmospheric effects with a spectral resolution complete to degree and order 180. The atmospheric effects on gravity disturbances, gravity anomalies and geoid undulations are evaluated globally on a 1×1 arc-deg grid.

Key words: atmosphere, effect, gravity field

1. Introduction

Ecker and Mittermayer (1969) developed a method for computing the atmospheric gravity correction later adopted in the definition of the Geodetic Reference System 1980 (GRS 80). This method, known as the IAG (International Association of Geodesy) approach, is described in *Moritz (1980, 1992)*. According to this method, the normal gravity is generated by the geocentric reference ellipsoid GRS 80 including the earth's atmosphere. The atmospheric gravity correction is then defined as the gravitational attraction of atmospheric masses distributed at elevations above the computation point as if they were uniformly distributed inside the geocentric reference ellipsoid. At the calculation point, atmospheric correction is then subtracted from the normal gravity. This approach allows for a simple evaluation of the atmospheric gravity correction as a function of the height of the computation point. The obtained atmospheric gravity correction varies from 0.869 mGal at the sea level to 0.000 mGal at the elevation of 46 km. *Anderson et al. (1975)* and *Anderson (1976)* were the first to compute globally the atmospheric corrections on gravity and geoid considering the actual topography as the lower atmospheric bound. They modeled the radially distributed atmospheric density by the piecewise linear function with the upper limit at a height of 40 km and assessed that the first- and second-order atmospheric gravity correction is about 0.87 mGal and 0.1 mGal, respectively. They were obviously the first to arrive to conclusion that the attraction of the model atmosphere above the oceans has opposite sign to that above the continents. *Wenzel (1985)* defined a simple formula for computing the atmospheric gravity correction as a function of the height of the computation point, but without considering the Earth topography (see also *Hinze et al., 2005* and *Li et al., 2006*). However, in complex terrain, errors caused by disregarding the actual topography can be even larger than the magnitude of the atmospheric effect on gravity itself. Moreover, the secondary indirect atmospheric effect on the gravity anomaly is completely disregarded in the IAG approach (cf. *Moritz, 1980*). These deficiencies as well as the bias due to the implementation of this approach to the truncated Stokes integral were already emphasized in *Sjöberg (1993).* In the alternative method formulated in *Sjöberg (1998)* and *Sjöberg and Nahavandchi (1999)* the spectral representation of Newton's integral was used to define the atmospheric effects on gravity and geoid taking into account the actual topography. This concept was further investigated in *Sjöberg (1999, 2001)* and *Sjöberg and Nahavandchi (2000). Ramillien (2002)*, using a different approach, arrived at similar results as *Anderson et al. (1975)*, including the signs of the calculated effects. *Nahavandchi (2004)* used a novel approach to compute the direct atmospheric effect for Iran. *Tenzer et al. (2006)* used the spatial representation of Newton's integral for computing the atmospheric effects over the Canadian Rocky Mountains. In the above mentioned studies, the atmospheric effect on gravity is non-negative when evaluated at the earth's surface. *Mikuška et al. (2008)* arrived independently at the conclusion that the atmospheric effect on gravity becomes negative when evaluated offshore and confirmed the findings of *Anderson et al. (1975)*, *Anderson (1976)* and *Ramillien (2002)*. This is confirmed again also by results of this study using quite a different way of calculation. *Sjöberg (2006)* derived expressions for the atmospheric potential and attraction considering the ellipsoidal layering of the earth's atmosphere. *Novák and Grafarend (2006)* proposed a method for computing the gravitational effect of atmospheric masses on spaceborne data based on spherical harmonic approach with a numerical study in North America. *Eshagh and Sjöberg (2009)* computed the atmospheric effect on satellite gravity gradiometry data over Fennoscandia. In this study, we compute globally the atmospheric effects on the gravity field quantities based on the spectral representation of Newton's integral. The expressions for the atmosphere-generated gravitational potential and attraction are derived in the form of spherical height functions which represent the lower bound of the earth's atmosphere (section 2). A radially distributed atmospheric density model is adopted. The coefficients of the Global Elevation Model (GEM) are used to compute globally the atmospheric effects with a spectral resolution complete to degree and order 180. The results are shown in section 3, and the conclusions are given in section 4.

2. Long-wavelength gravitational field generated by the atmosphere

To model the long-wavelength gravitational field components generated by the atmospheric masses, we consider the spherical approximation of the geoid surface and adopt a radially distributed atmospheric density model. After applying the analytical upward continuation, the atmosphere-generated gravitational potential V^a reads

$$V^a(r,\Omega) = V^a(R,\Omega) + \sum_{k=1}^{\infty} \frac{(r-R)^k}{k!} \left. \frac{\partial^k V^a(r,\Omega)}{\partial r^k} \right|_{r=R} . \tag{1}$$

Similarly, the atmosphere-generated gravitational attraction g^a is given by

$$g^a(r,\Omega) = g^a(R,\Omega) + \sum_{k=1}^{\infty} \frac{(r-R)^k}{k!} \left. \frac{\partial^{k+1} V^a(r,\Omega)}{\partial r^{k+1}} \right|_{r=R} . \tag{2}$$

The potential $V^a(R,\Omega)$ and the attraction $g^a(R,\Omega)$ in Eqs. (1) and (2) are evaluated on the geoid surface which is approximated by the mean earth's radius R. The 3-D position is defined in a frame of the geocentric spherical coordinates r, ϕ, λ; r is the geocentric radius, ϕ and λ are the spherical latitude and longitude, $\Omega = (\phi, \lambda)$.

Based on the results of *Ecker and Mittermayer (1969)* which were derived from the US standard atmospheric density model presented in 1961 (USSA61, *Reference Atmosphere Committee, 1961*), *Sjöberg (1998)* proposed the radially distributed atmospheric density model $\rho^a(r)$ in the following form

$$\rho^a(r) = \rho_0^a \left(\frac{R}{r}\right)^{\mu} , \tag{3}$$

where ρ_0^a is the atmospheric density at the sea level, and where the positive integer constant $\mu > 2$ describes the radial atmospheric density distribution model as a function of the height above the sea level. *Novák (2000)* used a second-order polynomial function to approximate the density distribution within a lower atmosphere ($<$ 10 km). This approximation fits the USSA76 (United States Standard Atmosphere 1976) model with the accuracy of about 10^{-3} up to 10 km above the sea level. For elevations above 10 km, he adopted the atmospheric density distribution from the USSA76 model. The second-order polynomial approximation of the atmospheric density up to 10 km described in *Novák (2000)* was later adopted by *Eshagh and Sjöberg (2009)*. For modeling the atmospheric density above 10 km, they used the mathematical model from Eq. (3) with the parameters ρ_0^a and μ modified for the nominal height of 10 km. All these three atmospheric density models assume a spherical stratification. *Sjöberg (2006)* acquired that the errors of about 2 cm in geoid determination can be expected when disregarding the earth's flattening. Therefore, he assumed the ellipsoidally symmetric atmospheric density model, and also introduced physically a more realistic approximation of the atmospheric density taking into account atmospheric density variations due to latitudinal temperature variations.

Adopting the atmospheric density model from Eq. (3), the atmosphere-generated gravitational potential can be expressed by Newton's volume integral

$$V^a(r,\Omega) = G\rho_0^a \iint\limits_{\Omega'\in\Omega_0} \int_{R+H_{\max}}^{r_{\lim}} \left(\frac{R'}{r}\right)^{\mu} \ell^{-1}\left(r,\psi,r'\right) r'^2 \,\mathrm{d}r'\mathrm{d}\Omega' +$$
$$+ G\rho_0^a \iint\limits_{\Omega'\in\Omega_0} \int_{R+H(\Omega')}^{R+H_{\max}} \left(\frac{R}{r'}\right)^{\mu} \ell^{-1}(r,\psi,r') r'^2 \mathrm{d}r'\mathrm{d}\Omega', \tag{4}$$

where G is Newton's gravitational constant, ℓ is the Euclidean spatial distance of two points (r,Ω) and (r',Ω'), $d\Omega' = \cos\phi'\, d\phi'\, d\lambda'$ is the infinitesimal surface element on the unit sphere, and Ω_0 denotes the full solid angle. The volumetric domain of the earth's atmosphere in Eq. (4) is subdivided into the atmospheric spherical shell and the atmospheric spherical roughness term. The atmospheric spherical shell is bounded by the upper limit of topography $H_{\max}$ and the upper limit of atmosphere $(r_{\lim} - R)$ above which the gravitational contribution of atmospheric masses becomes negligible; approximately 50 km. It is known that more than 99.9% of all atmospheric masses are located within 50 km above the sea level (cf. *Ecker and Mittermayer, 1969*). The atmospheric spherical roughness term is enclosed between the earth's surface and the upper limit of topography. The upper limit of topography is defined as the maximum height above the sea level. For $r < R + H_{\max}$, Eq. (4) becomes

$$V^a(r,\Omega) = 4\,\pi\, G\, \rho_0^a\; R^{\mu} \frac{r_{\lim}^{2-\mu} - (R+H_{\max})^{2-\mu}}{2-\mu} +$$
$$+ G\,\rho_0^a \iint\limits_{\Omega'\in\Omega_0} \int_{R+H(\Omega')}^{R+H_{\max}} \left(\frac{R}{r'}\right)^{\mu} \ell^{-1}(r,\psi,r') r'^2 \,\mathrm{d}r'\mathrm{d}\Omega', \tag{5}$$

where the first constituent on the right-hand side of Eq. (5) represents the constant gravitational potential of the atmospheric spherical shell in the interior domain $r < R + H_{\max}$ (cf. *Tenzer, 2005*).

The first and higher radial derivatives of the gravitational potential of the atmospheric spherical shell (of radially distributed density) in the interior domain $r < R + H_{\max}$ equal zero (cf. *MacMillan, 1930*). The radial derivatives of the gravitational potential $\left\{\partial^k\, V^a/\partial\, r^k\; k = 1, 2, ...\right\}$ in Eqs. (1) and

(2) are then defined as

$$\left.\frac{\partial^k V^a(r,\Omega)}{\partial r^k}\right|_{r=R} = G\rho_0^a \iint_{\Omega'\in\Omega_0} \int_{R+H(\Omega')}^{R+H_{\max}} \left(\frac{R}{r'}\right)^{\mu} \left.\frac{\partial^k \ell^{-1}(r,\psi,r')}{\partial r^k}\right|_{r=R} \times \\ \times r'^2 \, \mathrm{d}r' \mathrm{d}\Omega'. \tag{6}$$

The radial derivatives of ℓ^{-1} for $r < r'$ are found to be

$$\frac{\partial^k \ell^{-1}(r,\psi,r')}{r^k} = \frac{k!}{r'^{k+1}} \sum_{n=k}^{\infty} \left(\frac{r}{r'}\right)^{n-k} \binom{n}{k} P_n(\cos\psi), \tag{7}$$

where P_n is the Legendre polynomial of degree n for the argument of cosine of the spherical distance ψ. The substitution from Eq. (7) to Eq. (6) yields

$$\left.\frac{\partial^k V^a(r,\Omega)}{\partial r^k}\right|_{r=R} = G\rho_0^a R^{n-k+\mu} \sum_{n=k}^{\infty} \frac{n!}{(n-k)!} \iint_{\Omega'\in\Omega_0} P_n(\cos\psi) \times \\ \times \int_{R+H(\Omega')}^{R+H_{\max}} \frac{1}{r'^{n+\mu-1}} \mathrm{d}r' \mathrm{d}\Omega'. \tag{8}$$

Since the expansion of Newton's integral kernel into a series of the Legendre polynomials converges uniformly and absolutely, the interchange of summation and integration in Eq. (8) is permissible (cf. *Moritz, 1980*). The radial integral term on the right-hand side of Eq. (8) is found to be

$$\int_{R+H(\Omega')}^{R+H_{\max}} \frac{1}{r'^{n+\mu-1}} \mathrm{d}r' = \frac{(R+H_{\max})^{2-n-\mu} - [R+H(\Omega')]^{2-n-\mu}}{2-n-\mu}. \tag{9}$$

Inserting from Eq. (9) to Eq. (8), we arrive at

$$\left.\frac{\partial^k V^a(r,\Omega)}{\partial r^k}\right|_{r=R} = \frac{G\rho_0^a}{R^{k-2}} \sum_{n=k}^{\infty} \left(\frac{R}{R+H_{\max}}\right)^{n+\mu-2} \frac{n!}{(n-k)!} \quad \frac{1}{2-n-\mu} \times \\ \times \iint_{\Omega'\in\Omega_0} P_n(\cos\psi)\, \mathrm{d}\Omega' - \frac{G\rho_0^a}{R^{k-2}} \sum_{n=k}^{\infty} \left[\frac{R}{R+H(\Omega')}\right]^{n+\mu-2} \times \\ \times \frac{n!}{(n-k)!} \quad \frac{1}{2-n-\mu} \iint_{\Omega'\in\Omega_0} P_n(\cos\psi)\, \mathrm{d}\Omega'. \tag{10}$$

The application of the binomial theorem to $[1 + H(\Omega')/R]^{2-n-\mu}$ results in

$$\left[1+\frac{H\left(\Omega'\right)}{R}\right]^{2-n-\mu} \cong \sum_{i=0}^{\infty} \binom{2-n-\mu}{i} \left(\frac{H\left(\Omega'\right)}{R}\right)^{i}. \tag{11}$$

Disregarding terms higher than the second degree in Eq. (11), we get

$$\left[1+\frac{H\left(\Omega'\right)}{R}\right]^{2-n-\mu} \approx 1+\frac{H\left(\Omega'\right)}{R}\left(2-n-\mu\right)+\frac{1}{2}\left[\frac{H\left(\Omega'\right)}{R}\right]^{2} \times$$
$$\times\left(2-n-\mu\right)\left(1-n-\mu\right). \tag{12}$$

Substituting from Eq. (12) to the second constituent on the right-hand side of Eq. (10), we arrive at

$$\left.\frac{\partial^{k} V^{a}\left(r,\Omega\right)}{\partial r^{k}}\right|_{r=R} = \frac{G\,\rho_0^a}{R^{k-2}} \sum_{n=k}^{\infty} \frac{n!}{(n-k)\,!}\frac{1}{2-n-\mu}\left[\left(\frac{R}{R+H_{\max}}\right)^{n+\mu-2}-1\right] \times$$
$$\times \iint\limits_{\Omega'\in\Omega_0} P_n(\cos\psi)\,\mathrm{d}\Omega' - \frac{G\,\rho_0^a}{R^{k-1}} \sum_{n=k}^{\infty} \frac{n!}{(n-k)\,!} \times$$
$$\times \iint\limits_{\Omega'\in\Omega_0} H\left(\Omega'\right) P_n\left(\cos\psi\right)\,\mathrm{d}\Omega' - \frac{G\,\rho_0^a}{2R^{k}} \sum_{n=k}^{\infty} \frac{n!}{(n-k)\,!} \times$$
$$\times\left(1-n-\mu\right)\iint\limits_{\Omega'\in\Omega_0} H^2\left(\Omega'\right) P_n\left(\cos\psi\right)\,\mathrm{d}\Omega'. \tag{13}$$

Since $\iint\limits_{\Omega'\in\Omega_0} P_n\left(\cos\psi\right)\,d\Omega' = 0$ for $n>0$, Eq. (13) becomes

$$\left.\frac{\partial^{k} V^{a}\left(r,\Omega\right)}{\partial r^{k}}\right|_{r=R} = -\frac{G\,\rho_0^a}{R^{k-1}} \sum_{n=k}^{\infty} \frac{n!}{(n-k)\,!} \iint\limits_{\Omega'\in\Omega_0} H\left(\Omega'\right) P_n\left(\cos\psi\right)\,\mathrm{d}\Omega' -$$
$$-\frac{G\,\rho_0^a}{2R^{k}} \sum_{n=k}^{\infty} \frac{n!}{(n-k)\,!}\left(1-n-\mu\right) \times$$
$$\times \iint\limits_{\Omega'\in\Omega_0} H^2\left(\Omega'\right) P_n\left(\cos\psi\right)\,\mathrm{d}\Omega'. \tag{14}$$

From Eq. (14), the generic expression for radial derivatives of the atmosphere-generated gravitational potential in terms of the surface spherical height functions $H_n(\Omega)$ and the surface spherical squared height functions ${H_n}^2(\Omega)$ is found to be

$$\left.\frac{\partial^k V^a(r,\Omega)}{\partial r^k}\right|_{r=R} = -4\pi \frac{G\rho_0^a}{R^{k-1}} \sum_{n=k}^{\infty} \frac{1}{2n+1} \frac{n!}{(n-k)!} \sum_{m=-n}^{n} H_{n,m} Y_{n,m}(\Omega) - \\ - 2\pi \frac{G\rho_0^a}{R^k} \sum_{n=k}^{\infty} \frac{1}{2n+1} \frac{n!}{(n-k)!} (1-n-\mu) \times \\ \times \sum_{m=-n}^{n} H_{n,m}^2 Y_{n,m}(\Omega) = -2\pi \frac{G\rho_0^a}{R^k} \sum_{n=k}^{\infty} \frac{1}{2n+1} \frac{n!}{(n-k)!} \times \\ \times \sum_{m=-n}^{n} \left[2R H_{n,m} + (1-n-\mu) H_{n,m}^2\right] Y_{n,m}(\Omega). \tag{15}$$

The surface spherical height functions $H_n(\Omega)$ and the surface spherical squared height functions $H_n^2(\Omega)$ in Eq. (15) are defined as follows (e.g., *Novák, 2000*)

$$H_n(\Omega) = \frac{2n+1}{4\pi} \iint_{\Omega' \in \Omega_0} H(\Omega')\, P_n(\cos\psi)\, \mathrm{d}\Omega' = \sum_{m=-n}^{n} H_{n,m} Y_{n,m}(\Omega), \tag{16}$$

$$H_n^2(\Omega) = \frac{2n+1}{4\pi} \iint_{\Omega' \in \Omega_0} H(\Omega')^2\, P_n(\cos\psi)\, \mathrm{d}\Omega' = \sum_{m=-n}^{n} H_{n,m}^2 Y_{n,m}(\Omega). \tag{17}$$

The surface spherical harmonic functions $Y_n(\Omega)$ are given by (e.g., *Heiskanen and Moritz, 1967*)

$$Y_n(\Omega) = \frac{2n+1}{4\pi} \iint_{\Omega' \in \Omega_0} P_n(\cos\psi)\, \mathrm{d}\Omega' = \sum_{m=-n}^{n} Y_{n,m}(\Omega). \tag{18}$$

The substitution from Eq. (15) to Eqs. (1) and (2) yields the expressions for the atmosphere-generated gravitational potential and attraction. For $r < R + H_{\max}$, the atmosphere-generated gravitational potential is introduced in the following form

$$V^a(r,\Omega) = V^a(R,\Omega) - 2\pi G \rho_0^a \frac{H(\Omega)}{R} \sum_{n=1}^{N} \frac{n}{2n+1} \times$$

$$\times \sum_{m=-n}^{n} \left[2R\,H_{n,m} + (1-n-\mu)\,H^2_{n,m} \right] Y_{n,m}(\Omega) - \pi\,G\,\rho_0^a \frac{H^2(\Omega)}{R^2} \times$$
$$\times \sum_{n=2}^{N} \frac{n(n-1)}{2n+1} \sum_{m=-n}^{n} \left[2R\,H_{n,m} + (1-n-\mu)\,H^2_{n,m} \right] Y_{n,m}(\Omega), \quad (19)$$

where $H_{n,m}$ and $H^2_{n,m}$ are the GEM coefficients complete to degree and order N. The gravitational potential generated by the atmospheric spherical shell enclosed between the upper limits of topography and atmosphere (first constituent on the right-hand side of Eq. 5) is included in the expression for the potential $V^a(R,\Omega)$ which is evaluated on the geoid; i.e.,

$$V^a(R,\Omega) = 4\pi\,G\,\rho_0^a\;R^\mu\,\frac{r_{\rm lim}^{2-\mu} - (R+H_{\rm max})^{2-\mu}}{2-\mu} +$$
$$+ G\,\rho_0^a \iint_{\Omega'\in\Omega_0} \int_{R+H(\Omega')}^{R+H_{\rm max}} \left(\frac{R}{r'}\right)^{\mu} \ell^{-1}(R,\psi,r') r'^2\;{\rm d}r'{\rm d}\Omega'. \quad (20)$$

The reciprocal spatial distance in Eq. (20) reads (e.g., *Pick et al., 1973*; Eq. D-14, 4)

$$\ell^{-1}(R,\psi,r') = \frac{1}{r'} \sum_{n=0}^{\infty} \left(\frac{R}{r'}\right)^{n} P_n(\cos\psi) \qquad (r' > R)\,. \quad (21)$$

Substituting from Eq. (21) to Eq. (20), and subsequently integrating with respect to r', we arrive at

$$V^a(R,\Omega) = 4\pi\,G\,\rho_0^a\;R^\mu\,\frac{r_{\rm lim}^{2-\mu} - (R+H_{\rm max})^{2-\mu}}{2-\mu} +$$
$$+ 4\pi\,G\,\rho_0^a\,\frac{R^2}{2-\mu} \left[\left(1+\frac{H_{\rm max}}{R}\right)^{2-\mu} - 1 \right] - 2\pi\,G\,\rho_0^a \sum_{n=0}^{N} \frac{1}{2n+1} \times$$
$$\times \sum_{m=-n}^{n} \left[2R\,H_{n,m} + (1-n-\mu)\,H^2_{n,m} \right] Y_{n,m}(\Omega). \quad (22)$$

The first constituent on the right-hand side of Eq. (22) represents the gravitational potential of the atmospheric spherical shell. The second constituent is a function of the upper bound of the atmospheric spherical roughness term. The last constituent defines the lower atmospheric bound in terms of the surface spherical height functions and the surface spherical squared height functions. The atmospheric density distribution function in Eq. (22) can be specified individually for the atmospheric spherical shell and for the atmospheric spherical roughness term. The gravitational potential of the atmospheric spherical shell can then be computed either using the expression in Eq. (22) or adopting the available atmospheric density model such as the USSA76.

For $r < R + H_{\max}$, the atmosphere-generated gravitational attraction is given by

$$\begin{aligned} g^a(r,\Omega) &= g^a(R,\Omega) - 2\pi\, G\, \rho_0^a \frac{H(\Omega)}{R^2} \sum_{n=2}^{N} \frac{n(n-1)}{2n+1} \sum_{m=-n}^{n} \times \\ &\times \left[2R\, H_{n,m} + (1-n-\mu)\, H_{n,m}^2 \right] Y_{n,m}(\Omega) - \pi\, G\, \rho_0^a \frac{H^2(\Omega)}{R^3} \times \\ &\times \sum_{n=3}^{N} \frac{n(n-1)(n-2)}{2n+1} \sum_{m=-n}^{n} \left[2R\, H_{n,m} + (1-n-\mu)\, H_{n,m}^2 \right] \times \\ &\times Y_{n,m}(\Omega), \end{aligned} \tag{23}$$

where the attraction $g^a(R,\Omega)$ which is evaluated on the geoid reads

$$\begin{aligned} g^a(R,\Omega) &= -2\pi \frac{G\, \rho_0^a}{R} \sum_{n=1}^{N} \frac{n}{2n+1} \sum_{m=-n}^{n} \times \\ &\times \left[2R\, H_{n,m} + (1-n-\mu)\, H_{n,m}^2 \right] Y_{n,m}(\Omega). \end{aligned} \tag{24}$$

The expressions for the atmosphere-generated gravitational attraction in Eqs. (23) and (24) comprise only terms for the lower atmospheric bound. Equations (19) and (22) through (24) are used in the next section for the global modeling of atmospheric corrections.

3. Global atmospheric effects

The 5×5 arc-min elevation data from the ETOPO5 (provided by the NOAA's National Geophysical Data Centre) are used to generate the GEM coefficients. These coefficients are utilized to compute the atmospheric effects with a spectral resolution complete to degree and order 180. The

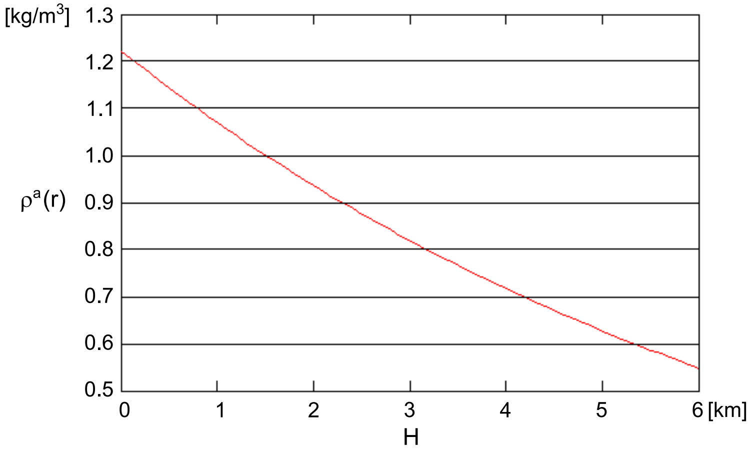

Fig. 1. The atmospheric density model $\rho^a(r)$ up to 6 km above the sea level computed using Eq. (3) for the parameters: $\rho^{\mathrm{a}}_{\mathrm{o}} = 1.2227$ kg/m^3, $\mu = 850$ and R = 6378137 m.

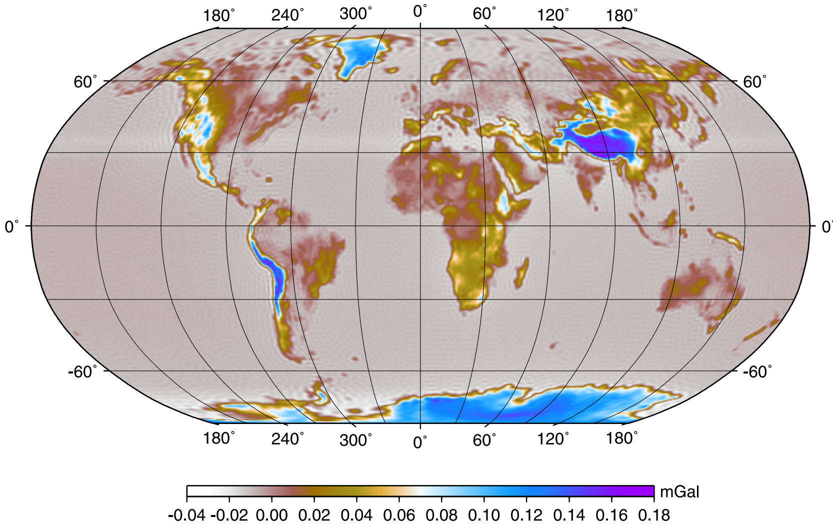

Fig. 2. The global atmospheric effect on gravity disturbances evaluated on a 1×1 arc-deg grid at the earth's surface.

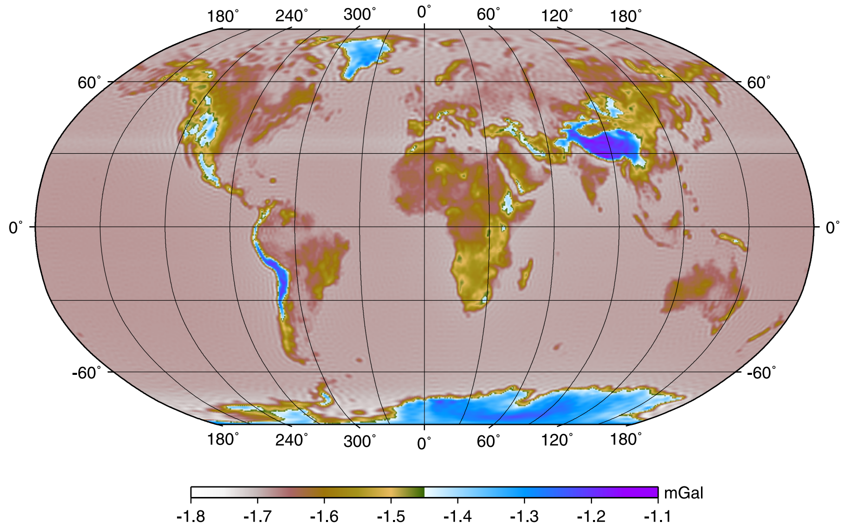

Fig. 3. The global complete atmospheric correction on gravity anomalies evaluated on a 1×1 arc-deg grid at the earth's surface.

atmospheric effects on gravity disturbances, gravity anomalies and geoid undulations are evaluated globally on a 1×1 arc-deg geographical grid. The maximum mean heights of the global elevation model with 180-degree spectral resolution reach 6 km. The atmospheric density distribution up to the upper limit of topography (6 km above the sea level) is defined according to Eq. (3) for the chosen parameters $\rho_0^a = 1.2227$ kg/m^3, $\mu = 850$ and $R = 6378137$ m (cf. *Sjöberg, 1998*). The density within the lower atmosphere (< 6 km) is shown in Fig. 1. For the elevations above 6 km, the atmospheric density distribution from the USSA76 model is adopted.

The atmospheric effect on gravity disturbances evaluated at the earth's surface using Eq. (23) is shown in Fig. 2. It globally varies from –0.03 to 0.18 mGal with the mean of 0.01 mGal, and the standard deviation is 0.04 mGal. The complete atmospheric effect on gravity anomalies evaluated at the earth's surface is shown in Fig. 3. It is everywhere negative and globally varies from –1.76 to –1.13 mGal with the mean of –1.63 mGal, and the standard deviation is 0.11 mGal. The complete atmospheric effect on gravity anomalies comprises not only the direct atmospheric effect on gravity but also the secondary indirect atmospheric effect (cf. e.g., *Tenzer* *et al., 2006*). The secondary indirect atmospheric effect is defined as the product of the atmosphere-generated gravitational potential $V^a(r, \Omega)$ and the term $2r^{-1}$. The atmosphere-generated gravitational potential $V^a(r, \Omega)$ is evaluated at the earth's surface using Eq. (19). The secondary indirect atmospheric effect is shown in Fig. 4. It varies from 1.30 to 1.72 mGal with the mean of 1.65 mGal, and the standard deviation is 0.07 mGal.

4. Summary and conclusions

We have computed globally the atmospheric effects on the gravity field quantities using the expressions for spectral analysis of gravitational field. Disregarding temporal and lateral atmospheric density variations, the density distribution model within the lower atmosphere up to 6 km above the sea level is defined as a function of the nominal atmospheric density at the sea level and the height. For elevations above 6 km, we adopted the USSA76 atmospheric density model. From the error analysis in *Tenzer et al. (2006)*, it follows that the accuracy of used atmospheric density distribution model determines the resulting accuracy of computed atmospheric effects on gravity and potential, while the errors due to inaccuracies of the digital elevation models are considerably smaller. Relatively large errors are thus expected in forward modeling the global atmospheric effects when using a simple radially distributed atmospheric density model especially due to neglecting the latitudinal density variations within the lower atmosphere. The main global atmospheric density variations are attributed to the average annual temperature latitudinal variations between equatorial and polar regions of about 300 to 260 K (cf. *Wallace and Hobbs, 1977*; see also *Sjöberg, 2006*). These latitudinal temperature variations represent some 15% higher atmospheric densities at the polar areas and correspond to errors of computing the atmospheric potential and attraction up to 3 m^2/s^2 and 0.03 mGal, respectively. The seasonal atmospheric density variations of about 10% are not considered in the total error budget.

The results in section 3 revealed that the atmospheric effects are strongly correlated with the geometry of the lower atmospheric bound. There is a stronger correlation of the atmospheric-attraction related correction term (direct atmospheric effect) compared to the atmospheric-potential related

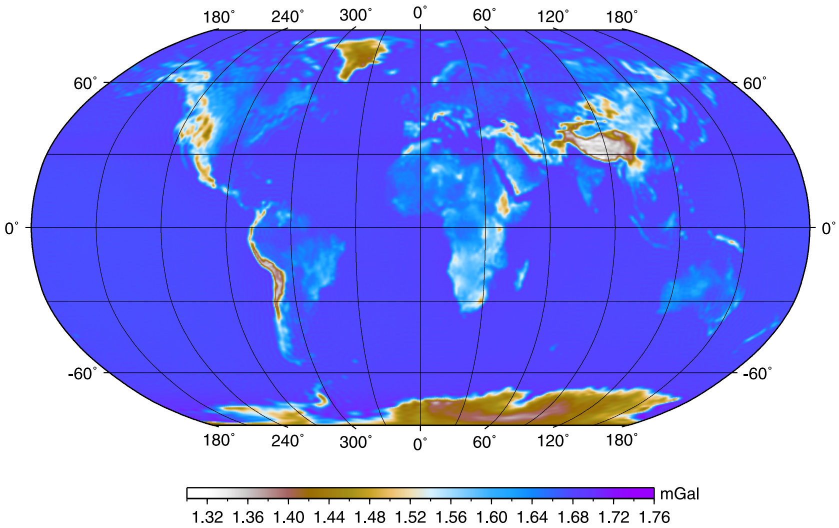

Fig. 4. The global secondary indirect atmospheric effect evaluated on a 1×1 arc-deg grid at the earth's surface.

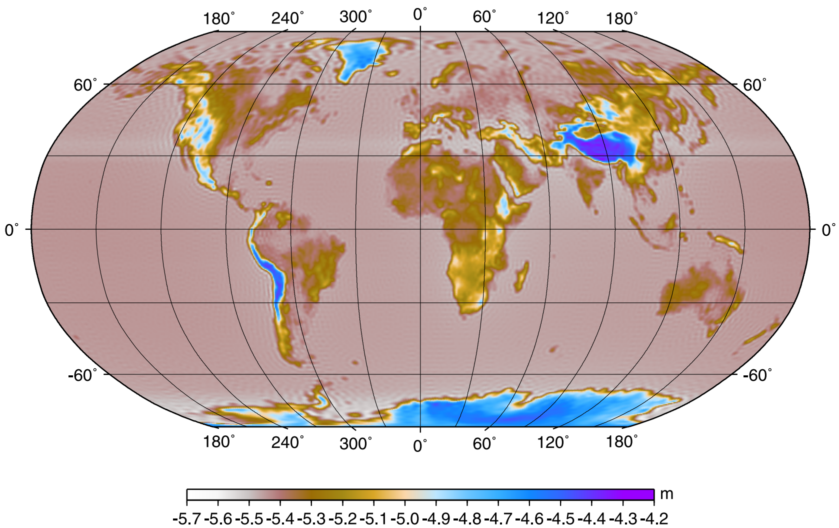

Fig. 5. The global atmospheric effect on geoid undulations evaluated on a 1×1 arc-deg grid.

correction terms (primary and secondary indirect atmospheric effects). The maximum absolute values of all atmospheric effects are located mainly over the mountainous regions. As also can be seen from Figs. 2 and 3, the complete atmospheric effect on gravity anomalies is at least one-order of magnitude larger than the atmospheric effect on gravity disturbances, due to the fact that the complete effect on gravity anomalies comprises not only the direct atmospheric effect on gravity but also the secondary indirect atmospheric effect.

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