Gravimetric estimation of the Eötvös matrix components

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Abstract: The components of the Eötvös matrix are useful for various geodetic applications, such as interpolation of the elements of the deflection of the vertical, determination of gravity anomalies and determination of geoid heights. A torsion balance instrument is customarily used to determine the Eötvös components. In this work, we show that it is possible to estimate the Eötvös components at a point on the Earth's physical surface using gravity measurements at three nearby points, comprising a very small network. In the first part, we present the method in detail, while in the second part we demonstrate a numerical example. We conclude that this method is able to estimate the components of the Eötvös matrix with satisfactory accuracy.

Key words: Eötvös matrix, gravity measurements, simulations

1. Introduction

The Eötvös matrix is the second order derivative of the Earth's gravity potential W at a point S on the Earth's physical surface. This means that the Eötvös matrix consists of the second order partial derivatives of the gravity potential expressed in a local Cartesian system (x, y, z) which is centred at point S (more details will be presented in the next section). The components of the Eötvös matrix – except the component of vertical gradient W_{zz} – can be measured with a torsion balance instrument. The first measurements (*Völgyesi, 2001*) were made by Lorand Eötvös in 1889 in Hungary and the first successful geophysical exploration in 1916.

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The Eötvös matrix components are divided into two groups: a) the curvature group – or the curvature data – which includes the second order partial derivatives W_{xx}, W_{xy} and W_{yy} and b) the horizontal gradient group – or the horizontal gradient data – W_{xz} and W_{yz}. The components of the Eötvös matrix which belong to the first group are necessary for the determination of the mean curvature, the Gauss curvature of the actual equipotential surface at a point and the azimuth of the maximum sectional curvature. The components of the Eötvös matrix which belong to the second group are necessary to describe the deviation between two neighbouring equipotential surfaces, since equipotentials are not parallel.

Conventionally, the unit used for expressing the values of the Eötvös matrix components is the Eötvös unit ($1\,\mathrm{E} = 10^{-9}\ \mathrm{sec}^{-2}$), in honour of Lorand Eötvös. The precision of these values from torsion balance measurements is approximately $\pm 1\,\mathrm{E}$, while the standard deviation varies: for the first group the standard deviation is ± 2 to $\pm 4\,\mathrm{E}$ and for the second group ± 1 to $\pm 2\,\mathrm{E}$.

The determination of the components of the Eötvös matrix is significant to many applications. These applications are related either to one of the previous mentioned groups or they are related to the whole Eötvös matrix. For example, the "Geodetic Singularity Problem" is related to all components of the Eötvös matrix: if the determinant of the Eötvös matrix at a point P is equal to zero, then the Eötvös matrix is rank deficient and this classifies point P as a singular point. This means that it is not possible to replace (pseudo)differentials of unholonomic coordinate systems, which are related to moving local astronomical frames, with differentials of holonomic coordinate systems (*Grafarend, 1971; Livieratos, 1976*).

Applications related to the first group (*Völgyesi, 1993, 1998, 2001 and 2015*) include the determination of the geoid undulation by an alternative solution for the astrogeodetic levelling and the interpolation of the components of the deflection of the vertical. Applications which are related to the second group (*Völgyesi et al., 2005*) include the determination of gravity and gravity anomaly, mainly for geophysical purposes, and the determination of vertical gradients. In addition, the reconstruction of the gravity potential function, namely determination of the function of the gravity potential and its first and second order partial derivatives, including the vertical gradient, is worth mentioning (*Völgyesi, 2015; Toth et al, 2001*). Finally, the appreciation of the Eötvös matrix has recently increased, since a large number of torsion balance measurements are carried out around the world (*Völgyesi, 2015*), in order to detect lateral underground mass inhomogeneities and geological fault structures.

The aim of the present work is to develop a method for the estimation of the Eötvös components using a gravimeter instead of a torsion balance instrument. The components will be estimated at a chosen point S on the Earth's physical surface, using gravity measurements at S and three nearby points, comprising a very small network. The proposed method will be described in detail in the next sections.

2. Methodology

2.1. Estimation of the values of the Eötvös matrix components except W_{xy}

Let S be a point on the Earth's physical surface with known geodetic coordinates, gravity value and geometric height. The Earth's gravity potential is expressed in a local Cartesian system (x, y, z), which is centred at point S (point of interest), the z-axis is perpendicular to the equipotential surface passing through point S pointing upwards, the x-axis is tangent to the equipotential surface passing through point S pointing North and the y-axis is tangent to the aforementioned equipotential surface pointing East.

In addition, let A, B, C be three points in the neighbourhood of point S (within a few metres) with known local Cartesian coordinates and gravity values g_A, g_B, g_C and g_S. Point A is taken on the x-axis (x_A, $y_A = 0$, z_A), point B on the y-axis ($x_B = 0$, y_B, z_B) and point C on the z-axis ($x_C = 0$, $y_C = 0$, z_C), as in Figure 1.

The value of W_{zz} can be directly computed from the gravity measurements at points S and C (see Eq. (2.4)). For the other four Eötvös components at point S we proceed as follows:

A parametric vector equation for the actual equipotential surface of point S around this point, expressed in the local Cartesian system, is:

$$\bar{s} : \Re^3 \supset U \to \Re^3 : (x, y) \to \bar{s}(x, y) = (x, y, z(x, y)) \,, \tag{2.1}$$

and the tangent vectors of the equipotential surface are:

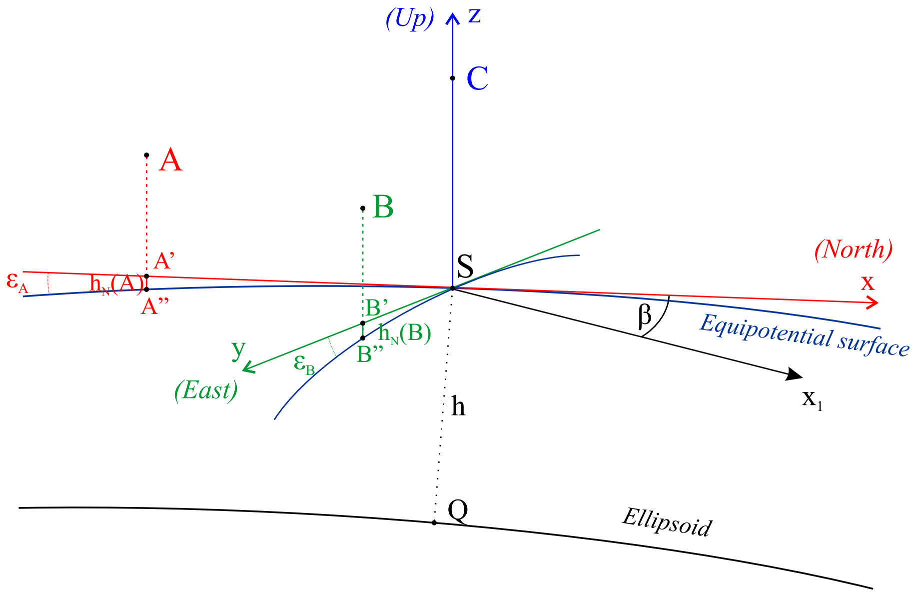

Fig. 1. The geometry of the local gravimetric network, showing the main concepts and the quantities involved in the computations.

$$\frac{\partial \bar{s}}{\partial x} \equiv \bar{s}_x = \left(1, 0, -\frac{W_x}{W_z}\right), \tag{2.2}$$

$$\frac{\partial \bar{s}}{\partial y} \equiv \bar{s}_y = \left(0, 1, -\frac{W_y}{W_z}\right). \tag{2.3}$$

The value of W_{zz} at point S is obtained by:

$$W_{zz}(S) \equiv W_{zz} = -\frac{g_C - g_S}{z_C}. \tag{2.4}$$

Approximate, temporary values of W_{xx} and W_{yy} at point S can be obtained as follows:

Let x_1, y_1 be the axes of the principal directions at point S, namely the directions along which the sectional curvature takes its minimum and maximum values. We set:

$$W^t_{yy}(S) \equiv W^t_{yy} \approx W_{y_1 y_1}(S) \equiv W_{y_1 y_1} := -g_S k^e_2(S). \tag{2.5}$$

The quantity k_2^e is the principal curvature of the normal equipotential surface in the East–West direction at point S, which (*Manoussakis, 2013*) is equal to:

$$k_2^e(S) \equiv k_2^e = -\frac{U_{yy}(S)}{\gamma_S}\,, \tag{2.6}$$

where

$$U_{yy}(S) = -\gamma_Q k_2(Q) + \gamma_Q k_2^2(Q) h_S\,, \tag{2.7}$$

and γ_Q, γ_S, are the values of normal gravity at points Q and S, respectively, and h_S is the geometric height of point S. In (2.7), point Q is the projection of point S on the ellipsoid of revolution along the vertical line to the ellipsoid (*Hofmann-Wellenhof and Moritz, 2006*), k_2 is the principal curvature of the ellipsoid along the East–West direction and U stands for the Eötvös matrix of the normal gravity field. From Poisson's equation we can also estimate W_{xx}^t at point S:

$$W_{xx}^t = 2\omega^2 - W_{zz} - W_{yy}^t\,. \tag{2.8}$$

We now proceed to estimate W_{xz} and W_{yz}.

Let A' be the projection of point A on the tangent plane of the equipotential surface at point S and A'' the intersection of the line AA' with the actual equipotential surface passing through point S (see Figure 1). Using a Taylor series expansion, we get:

$$\begin{aligned}\bar{s}(A'') - \bar{s}(A) = {} & \bar{s}_x(S)x_A + \bar{s}_y(S)y_A + \\ & + \frac{1}{2}\left(\bar{s}_{xx}(S)x_A^2 + 2\bar{s}_{xy}(S)x_A y_A + \bar{s}_{yy}(S)y_A^2\right),\end{aligned} \tag{2.9}$$

where the symbols $\bar{s}_{xx}$, $\bar{s}_{xy}$ and $\bar{s}_{yy}$ stand for the second order partial derivatives of the vector equation of the actual equipotential surface passing through point S.

Multiplying both sides with the unit normal vector and, having in mind that point A is on the xz plane, we end up with:

$$h_N(A) \equiv |z(A'')| = \left|\frac{1}{2}L_{act}x_A^2\right|, \tag{2.10}$$

where L_{act} is an element of the second fundamental form of the actual equipotential surface at point S, namely:

$$L_{act} = \langle \bar{N}, \bar{s}_{xx} \rangle \,, \tag{2.11}$$

where

$$\bar{N} = \frac{1}{|\bar{s}_x \times \bar{s}_y|} (\bar{s}_x \times \bar{s}_y) \,. \tag{2.12}$$

The value of L_{act} is given by

$$L_{act} = -\frac{W^t_{xx}}{g_S} \,. \tag{2.13}$$

The small angle ε_A (see Fig. 1) is computed by:

$$\tan \varepsilon_A \cong \varepsilon_A = \frac{|z(A)|}{|x_A|} = \left| \frac{1}{2} L_{act} x_A \right| . \tag{2.14}$$

Since A' is the projection of point A on the tangent plane of the actual equipotential surface at point S ($z_{A'} = 0$), we obtain:

$$g_{A'} = g_A - W_{zz}(z_{A'} - z_A) = g_A + W_{zz} z_A \,, \tag{2.15}$$

$$W_z(A') = -g_{A'} \cos \varepsilon_A \,. \tag{2.16}$$

The values of W_z are known at points A' and S, so the value of the second order partial derivative W_{xz} at point S is given by:

$$W_{xz} = \frac{W_z(A') - W_z(S)}{x_A} = \frac{g_S - g_{A'} \cos \varepsilon_A}{x_A} \,. \tag{2.17}$$

Repeating the above procedure for a point B on the West–East direction, we have similar relations, which lead to the estimation of the value of the second order partial derivative W_{yz} at point S:

$$h_N(B) \equiv |z(B'')| = \left| \frac{1}{2} N_{act} y_B^2 \right| , \tag{2.18}$$

$$N_{act} = \langle \bar{N}, \bar{s}_{xx} \rangle = -\frac{W^t_{yy}}{g_S} \,, \tag{2.19}$$

$$\tan \varepsilon_B \cong \varepsilon_B = \frac{|z(B'')|}{|y_B|} = \left| \frac{1}{2} N_{act} y_B \right| . \tag{2.20}$$

Since B' is the projection of point B on the tangent plane of the actual equipotential surface at point S, we obtain:

$$g_{B'} = g_B - W_{zz}(z_{B'} - z_B) = g_B + W_{zz} z_B \,, \tag{2.21}$$

$$W_z(B') = -g_{B'} \cos \varepsilon_B \,. \tag{2.22}$$

Therefore, the second order partial derivative W_{yz} at point S is given by:

$$W_{yz} = \frac{W_z(B') - W_z(S)}{y_B} = \frac{g_S - g_{B'} \cos \varepsilon_B}{y_B} \,. \tag{2.23}$$

2.2. Estimation of the value of the Eötvös matrix component W_{xy}

Let β be the angle of the principal direction x_1 with the x axis (see Fig. 1). The transformation between the two coordinate systems is given by:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \cos\beta & -\sin\beta & 0 \\ \sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix}. \tag{2.24}$$

Using Eq. (2.24), we can relate the gradients of the potential between the two systems:

$$W_{x_1} = W_x \frac{\partial x}{\partial x_1} + W_y \frac{\partial y}{\partial x_1} + W_z \frac{\partial z}{\partial x_1} = W_x \cos\beta + W_y \sin\beta \,, \tag{2.25}$$

which leads to:

$$W_{xx}^t \left(-\frac{1}{2} \sin 2\beta \right) + W_{xy}^t \cos 2\beta + W_{yy}^t \left(\frac{1}{2} \sin 2\beta \right) = W_{x_1 y_1} \,. \tag{2.26}$$

By definition, the right-hand side of Eq. (2.26) is equal to zero. Dividing the terms of this equation by $\cos 2\beta$ (tentatively assuming that $\beta \neq \pi/4$) we end up with the following relation:

$$W_{xy}^t = \frac{1}{2}(W_{xx} - W_{yy}) \tan 2\beta \,. \tag{2.27}$$

Therefore, in order to determine the value of W^t_{xy}, the angle β between the principal axis x_1 and the x axis has to be known (see Fig. 1). This means that additional information, for the behaviour of the gravity field along the equipotential surface of point S, is needed. If this information is not given (since the actual value of the angle β cannot be extracted from gravity measurements alone), it is necessary to find an estimate of the angle β using known components of the Eötvös matrix. One estimate of the angle β may be deduced from (*Völgyesi, 1993*):

$$\beta := \arctan \frac{W_{yz}}{W_{xz}} \,. \tag{2.28}$$

From Eqs. (2.28) and (2.27) we can estimate a temporary value of W^t_{xy}.

However, one should examine the limitations of this approach. From Eqs. (2.27) and (2.28), the angle β is equal to zero when either of the following conditions are met:

$$W_{xy} = 0 \quad \text{or} \quad W_{yz} = 0 \,. \tag{2.29}$$

If the first condition holds, for small variations along the y-axis we have that:

$$W_x(B') = W_{xy} y_B = 0 \,, \tag{2.30}$$

and for small variations along the x-axis:

$$W_y(A') = W_{xy} x_A = 0 \,. \tag{2.31}$$

The Eötvös matrix can also be expressed as (*Dermanis, 1993*):

$$E_S = \begin{bmatrix} W_{xx} & W_{xy} & W_{xz} \\ W_{xy} & W_{yy} & W_{yz} \\ W_{xz} & W_{yz} & W_{zz} \end{bmatrix}_S = \begin{bmatrix} -gk_1 & -g\tau_1 & -gk_{NS} \\ -g\tau_1 & -gk_1 & -gk_{EW} \\ -gk_{NS} & -gk_{EW} & 2\omega^2 + gk_1 + gk_2 \end{bmatrix}_S \,, \tag{2.32}$$

where τ_1 is the torsion of the geodesic line which is tangent to the x-axis at point S and k_{NS} and k_{EW} are the North–South and East–West curvature components of the plumbline passing through point S, respectively.

Since W_{xy} is equal to zero, the torsion of the geodesic line is equal to zero at point S. If the geodesic line is locally a plane curve, so that τ_1 is equal to zero for a small area around point S, then (*Weatherburn, 1995*) the geodesic is also a line of curvature and parametric line (x = const. or y = const. along the geodesic). In the case of y = const. the x-axis is tangent to the geodesic, so it is an axis of principal direction with $\beta = 0$. The equipotential surface is locally a surface of revolution and W_{yz} is equal to zero, hence the same result for angle β is found from Eq. (2.28). There is a possibility that the axis of revolution is the y-axis (x = const.). In this case W_{xz} is equal to zero and angle $\beta = \pi/2$, therefore our $(x.y)$ axes are again principal directions.

Another possibility, when W_{xy} is equal to zero, is that the geodesics, which are tangent to the x and y axes at point S, are not plane curves. In this case again the aforementioned axes are principal directions but neither W_{xz} nor W_{yz} is equal to zero. This case is not well described by Eq. (2.28).

If the second condition of Eq. (2.29) holds, for small variations along the y-axis we have that:

$$W_z(B') = W_z(S) + W_{yz} y_B = W_z(S) = -g_S \,. \tag{2.33}$$

This means that either the equipotential surface is locally a surface of revolution – which was already discussed above – or the gravity field is symmetrical but the shape of the equipotential surface around point S is cylindrical. The cylindrical shape of the equipotential surface classifies point S as a parabolic point (*Grossman, 1976*), therefore the sectional curvature at point S along the y-axis is equal to zero. But since point S is a parabolic point, the value of the sectional curvature along the y-axis has an extreme value, hence the y-axis is a principal axis and the same holds for the x-axis, so that the angle β is equal to zero.

The case that point S is singular is excluded, thus we do not study the case when one of the parametric lines has a cusp at point S. Finally, the case when $\beta = \pi/4$ remains to be examined.

From Euler's theorem, we can determine the value of the sectional curvature k_n of the actual equipotential surface at point S as a function of the angle a ($a = 0$ along the x_1-axis), namely:

$$k_n(S) = k_1(S) \sin^2 a + k_2(S) \cos^2 a \,, \tag{2.34}$$

where k_1(S) and k_2(S) are the principal curvatures at point S. Since $\beta = \pi/4$, the angle a is also equal to $\pi/4$ (see Fig. 2) and from Eq. (2.34) we conclude that:

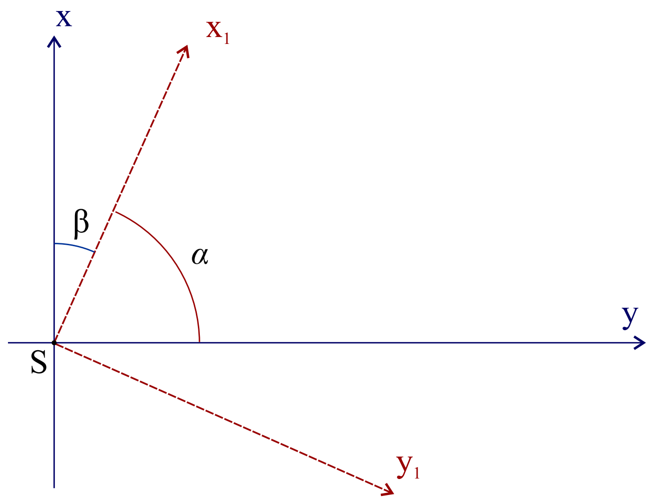

Fig. 2. Orientation of axes and relevant angles on the horizontal plane.

$$k_y(S) = k_x(S) = \frac{1}{2}(k_1(S) + k_2(S)) = \\ = -\frac{1}{2}\left(\frac{W_{x_1x_1} + W_{y_1y_1}}{g}\right)_S = -\frac{1}{2}\left(\frac{W_{xx} + W_{yy}}{g}\right)_S , \tag{2.35}$$

where k_x and k_y are the sectional curvatures along the x-axis and the y-axis, respectively. The last equality of Eq. (2.35) holds because the sectional curvature is equal to the mean curvature of the equipotential surface at point S, which is an invariant quantity. But (*Hofmann-Wellenhoff and Moritz, 2006*):

$$k_x(S) = -\frac{W_{xx}}{g_S} , \\ k_y(S) = -\frac{W_{yy}}{g_S} . \tag{2.36}$$

Hence $W_{xx} = W_{yy}$ and point S is an umbilical point. In this case all directions are principal directions and the gravity field is locally spherically symmetric so that $W_{xz} = W_{yz} = W_{xy} = 0$. This means that if point S is an umbilical point – something which is extremely rare – the angle β is undetermined.

2.3. Refinement of the values of the Eötvös matrix components

From the transformation described in Eq. (2.24) we can express the inverse transformation (from the $(x,\, y)$ to the $(x_1,\, y_1)$ system) and construct relations similar to Eq. (2.26) for the other surface derivatives, so we end up with the following system of equations:

$$W_{x_1x_1}\cos^2\beta + W_{y_1y_1}\sin^2\beta = W_{xx}\,, \tag{2.37}$$

and

$$W_{x_1x_1}\sin^2\beta + W_{y_1y_1}\cos^2\beta = W_{yy}\,, \tag{2.38}$$

because, by definition:

$$W_{x_1y_1} = 0\,. \tag{2.39}$$

Recalling our assumption for the estimate of W_{yy} (see Eq. 2.5)), we introduce the temporary values W^t_{xx} and W^t_{yy} to the left-hand side of Eq. (2.37) and Eq. (2.38), so that:

$$W_{x_1x_1} = W^t_{xx}\,, \quad W_{y_1y_1} = W^t_{yy}\,. \tag{2.40}$$

Hence, the solution is expressed as:

$$W_{xx} = W^t_{xx}\cos^2\beta + W^t_{yy}\sin^2\beta\,, \tag{2.41}$$

and

$$W_{yy} = W^t_{xx}\sin^2\beta + W^t_{yy}\cos^2\beta\,. \tag{2.42}$$

Further refinement of the value of W_{xy} is accomplished by Eq. (2.27), using the new values of the Eötvös matrix components W_{xx} and W_{yy}. If necessary, one can repeat the whole procedure, from Eq. (2.10) onwards, in order to obtain a better estimation of the Eötvös matrix components (as we did in the numerical test that follows).

3. Numerical Test

In order to examine the performance of the proposed method, we made a numerical simulation. We chose 12 arbitrary points on the Earth's physical surface, described in Table 1 and Figure 3, and we computed the simulated Eötvös matrix components and gravity values at those (S) and at nearby points (A, B, C). The simulated values were computed using the EGM2008 zero-tide gravity model (*Pavlis et al, 2012*), at maximum degree and order 720. Then, we applied the proposed method to estimate the Eötvös matrix components from the gravity values.

Table 1. The geodetic coordinates of the sites used in the numerical simulation.

Site	$\boldsymbol{\varphi^o}$ (geodetic latitude [°])	$\boldsymbol{\lambda^o}$ (geodetic longitude [°])	$\boldsymbol{h}$ (geometric height [m])
1	57.20	357.70	131
2	50.40	355.90	94
3	43.30	359.60	247
4	37.30	354.00	56
5	43.45	11.90	310
6	52.20	4.50	43
7	57.00	9.90	42
8	49.90	11.60	460
9	40.60	23.60	346
10	47.80	21.70	155
11	55.20	30.20	157
12	45.00	39.00	30

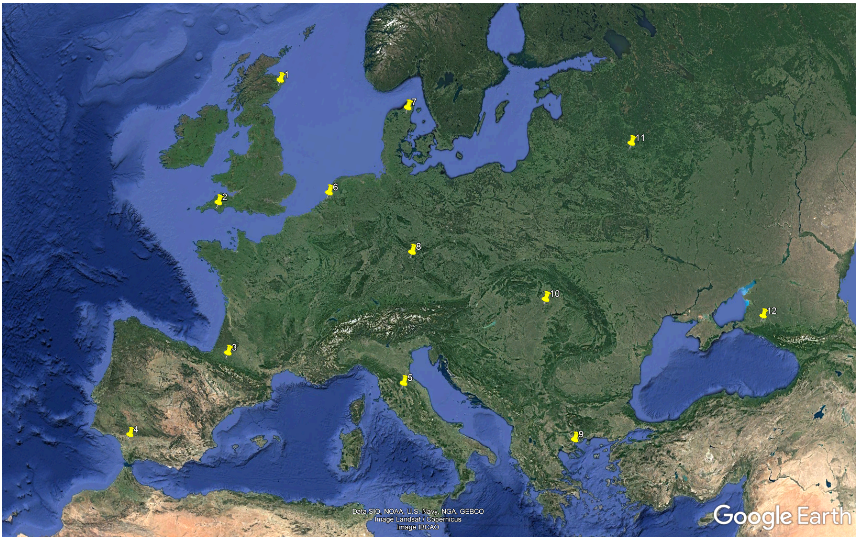

Fig. 3. Map showing the distribution of the test points around Europe.

In each case, the x, y coordinates of the three nearby points (A, B, C) had randomly assigned values in the range $[-5\,\mathrm{m},\ 5\,\mathrm{m}]$, while the z coordinates had random values in the range $[0.5\,\mathrm{m},\ 2\,\mathrm{m}]$. For the computation of the Eötvös matrix components, the model gravity values at all points were accurate to 1 ngal (1 ngal $= 10^{-11}\,\mathrm{m/sec^2}$). Using these values, in Table 2 below we summarize the statistics of the differences found between the simulated (from the EGM2008 model) and the estimated Eötvös matrix components, in Eötvös units, for the disturbing potential T.

Using exactly the same points but values of the gravity measurements rounded-off to the nearest μgal (1 μgal $= 10^{-8}\,\mathrm{m/sec^2}$), which today is a realistic accuracy level, we obtained the statistics for the differences shown in Table 3, again in Eötvös units.

4. Discussion and conclusions

In this work, we outlined a method for the estimation of the components of the Eötvös matrix using local gravity measurements. We chose a point on the Earth's physical surface (point of interest), with known geometric height, and three neighbouring points and obtained gravity measurements at all points. We estimated the value of W_{zz} from the gravity measurements and initial values of the surface derivatives W_{xx} and W_{yy}, using an approximation for the curvature of the actual equipotential surface along the East–West direction. The initial values of W_{xx} and W_{yy} were then used for the estimation of W_{xz} and W_{yz}. They were also used to compute an approximate value of W_{xy}, using an estimate for the angle β of the principal directions. Finally, the initial values of W_{xx} and W_{yy}, along with the angle β, were used to compute refined values of W_{xx} and W_{yy}. The whole procedure was repeated, in order to further refine the values of the estimated Eötvös matrix components.

The proposed method was tested by a numerical simulation, choosing twelve points on the Earth's physical surface, scattered over a wide area in Europe, and simulating the gravity measurements and the Eötvös matrix components using the EGM2008 gravity model (zero tide, degree and order 720). We determined the statistics of the differences found between the computed and the estimated components of the Eötvös matrix for two different accuracy levels of the simulated gravity values. Despite the ap-

Table 2. Statistics of the simulation results using gravity values accurate to 1 ngal.

Point	ΔTxx	ΔTxy	ΔTxz	ΔTyy	ΔTyz	ΔTzz
1	2.217	4.027	−0.0011	−2.217	0.0026	−0.0010
2	−1.626	−0.611	−0.0020	1.626	−0.0009	0.0002
3	0.196	−5.694	0.0008	−0.196	0.0002	−0.0023
4	−0.173	0.828	−0.0005	0.173	0.0007	−0.0013
5	2.295	0.885	0.0003	−2.295	0.0012	−0.0001
6	−0.411	2.299	−0.0006	0.411	0.0016	−0.0019
7	3.102	−3.487	0.0001	−3.102	0.0000	0.0034
8	1.114	−4.542	0.0005	−1.114	−0.0013	−0.0003
9	1.880	−0.837	0.0008	−1.880	−0.0017	−0.0015
10	4.251	2.000	−0.0010	−4.251	0.0009	0.0016
11	−0.580	2.012	0.0013	0.580	−0.0013	0.0022
12	4.058	−5.922	−0.0010	−4.058	0.0006	0.0028
Min. value	**−1.626**	**−5.923**	**−0.0020**	**−4.250**	**−0.0018**	**−0.0023**
Max. value	**4.250**	**4.027**	**0.0013**	**1.626**	**0.0026**	**0.0034**
Mean value	**1.36**	**−0.75**	**−0.0002**	**−1.36**	**0.0002**	**0.0001**
Std. dev.	**1.73**	**3.00**	**0.0010**	**1.73**	**0.0014**	**0.0018**

Table 3. Statistics of the simulation results using gravity values accurate to 1 μgal.

Point	ΔTxx	ΔTxy	ΔTxz	ΔTyy	ΔTyz	ΔTzz
1	3.530	4.602	−1.676	−0.977	1.245	−2.553
2	2.079	0.503	−1.760	2.758	−0.011	4.837
3	−0.813	−7.599	1.674	3.351	0.484	−2.538
4	−1.877	1.400	0.572	−0.956	0.183	2.833
5	3.014	5.192	1.011	−2.275	1.039	−0.739
6	2.614	1.131	0.382	1.063	0.189	−3.677
7	1.936	−3.105	−0.405	−4.541	−0.434	2.605
8	1.513	−4.568	0.118	−1.112	0.340	0.401
9	3.981	−0.068	0.837	−0.121	−0.360	−3.860
10	8.062	2.438	0.219	−4.130	0.521	−3.932
11	0.164	2.014	−1.755	−0.569	0.314	0.405
12	1.905	−6.439	−0.890	−3.704	0.686	1.799
Min. value	**−1.877**	**−7.599**	**−1.760**	**−4.541**	**−0.434**	**−4.837**
Max. value	**8.062**	**5.192**	**1.674**	**3.351**	**1.245**	**2.833**
Mean value	**2.18**	**−0.37**	**−0.14**	**0.93**	**0.35**	**−1.24**
Std. dev.	**2.53**	**3.73**	**1.14**	**2.37**	**0.49**	**2.55**

proximation made for the curvature of the actual equipotential surface, we have shown that the gravimetric estimation of the components of the Eötvös matrix is satisfactory. Using gravity values at the accuracy level of current gravimeters, we estimated W_{xy} and the vertical gradient components of the Eötvös matrix with an accuracy comparable to that obtained by torsion balance instruments. It is to be noted that, in principle, the values of the vertical gradient components W_{xz}, W_{yz} and W_{zz} were very accurate, while the accuracy of the values of W_{xx}, W_{yy} and W_{xy} may be further improved, through a better estimation of the curvature.

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