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Process-based selection of copula types for flood peak-volume relationships in Northwest Austria: a case study

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Abstract: The case study aims at selecting optimal bivariate copula models of the relationships between flood peaks and flood volumes from a regional perspective with a particular focus on flood generation processes. Besides the traditional approach that deals with the annual maxima of flood events, the current analysis also includes all independent flood events. The target region is located in the northwest of Austria; it consists of 69 small and mid-sized catchments. On the basis of the hourly runoff data from the period 1976–2007, independent flood events were identified and assigned to one of the following three types of flood categories: synoptic floods, flash floods and snowmelt floods. Flood events in the given catchment are considered independent when they originate from different synoptic situations. Nine commonly-used copula types were fitted to the flood peak – flood volume pairs at each site. In this step, two databases were used: i) a process-based selection of all the independent flood events (three data samples at each catchment) and ii) the annual maxima of the flood peaks and the respective flood volumes regardless of the flood processes (one data sample per catchment). The goodness-of-fit of the nine copula types was examined on a regional basis throughout all the catchments. It was concluded that (1) the copula models for the flood processes are discernible locally; (2) the Clayton copula provides an unacceptable performance for all three processes as well as in the case of the annual maxima; (3) the rejection of the other copula types depends on the flood type and the sample size; (4) there are differences in the copulas with the best fits: for synoptic and flash floods, the best performance is associated with the extreme value copulas; for snowmelt floods, the Frank copula fits the best; while in the case of

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the annual maxima, no firm conclusion could be made due to the number of copulas with similarly acceptable overall performances. The general conclusion from this case study is that treating flood processes separately is beneficial; however, the usually available sample size in such real life studies is not sufficient to give generally valid recommendations for engineering design tasks.

Key words: process-type flood classification, regional analysis, flood peaks, flood volumes, suitability of copula models

1. Introduction

The studies that deal with the selection and fitting of flood peak – flood volume models generally rely on a purely statistical analysis of a particular bivariate relationship. The statistical models are usually fitted in an automated fashion that follows a prescribed algorithm suitable for the engineering practice. Exploring the driving mechanisms leading to the occurrence of flood events is usually not at the center of interest. Here, the flood type differentiation is carried out first before attempting a process-based selection of copulas.

One of the possible typology approaches consists of classifying flood events into process types. For example, *Waylen and Woo (1982)* accounted for snowmelt-induced spring floods and rainfall-induced winter floods by separately fitting two distribution functions to the subsamples. Another example is *Merz and Blöschl (2003)*, who classified floods in Austria into long-rain floods, short-rain floods, flash floods, rain-on-snow floods and snowmelt floods, based on an extensive spatio-temporal analysis of the flood events and the related hydrological and meteorological variables. The concept of flood typology was the core philosophy of *Gaál et al. (2012)*, who aimed at a better understanding of the hydrological factors controlling the dependence between flood peaks and volumes. They analyzed the ratio of these quantities (designated the "flood time scale") in a regional context in Austria and compared catchments with contrasting characteristics in order to understand the controls in a holistic way. It was shown that flood time scales were controlled by the climatic factors (storm types and the antecedent soil moisture), geology, and land forms. This concept was further developed in *Gaál et al. (2014)*, where, again, the relationship between flood peaks and volumes was examined in the light of flood processes. They showed that the consistency of the peak-volume relationship, as quantified by Spearman's rank correlation coefficient, is related to the consistency of the climatic driving flood processes or flood types. The current paper represents the third part in a series of studies of the flood peak-volume relationship in Austria (see also *Szolgay et al., 2015, 2016a,b*); here, the dependence of these variables is examined in terms of copulas in a comparative regional perspective.

The copula theory mostly became popular in the actuarial sciences and financial applications in the 1980s and 1990s, and gradually attracted the attention of, among others, the field of hydrological engineering. Copulas have successfully been adopted for modelling hydrological phenomena with multidimensional aspects such as floods (e.g., *Kao and Chang, 2012; Requena et al., 2013*), rainstorms (e.g., *Vandenberghe et al., 2010; Gyasi-Agyei and Melching, 2012*) and droughts (e.g., *Lee et al., 2013; De Michele et al., 2013*). A copula-based bivariate analysis of the relationship between flood peaks and flood volumes has especially been the subject of numerous studies (*Favre et al., 2004; Shiau et al., 2006; Zhang and Singh, 2006; Genest and Favre, 2007; Poulin et al., 2007; Karmakar and Simonovic, 2009; Chowdhary et al., 2011; Bačová-Mitková, 2012; Ben-Aissia et al., 2012; Reddy and Ganguli, 2012; Ganguli and Reddy, 2013; Bačová-Mitková and Halmová, 2014; Sraj et al., 2014*). In some cases, an analysis of a flood peak-volume relationship represents a subpart of a broader study where a third variable is usually included and statistically modelled by trivariate copulas, e.g., the duration of the flood events in *Genest et al. (2007)* and *Zhang and Singh (2007)* or sediment concentration in *Bezak et al. (2014)*.

A few common features of the aforementioned studies are:

- They generally provide detailed guidelines for adapting the copula-fitting technique, which include different parameter estimation methods and goodness-of-fit tests such as graphic methods, formal statistical tests or error statistics (see, e.g., *Karmakar and Simonovic, 2009; Chowdhary et al., 2011*);
- They test the goodness-of-fit of a wide variety of copula families, which are mostly selected arbitrarily, but generally on the basis of the most common usage. In the case of trivariate analyses, one can find a targeted choice of copulas, e.g., the metaelliptical copulas in *Genest et al. (2007)*.

- The results are mostly demonstrated on the basis of a single selected catchment; two or more catchments appear in only a few works. *Zhang and Sing (2006, 2007)* present their theory based on examples of more catchments; however, they do not represent any coherent regional perspectives.
- The record lengths of the catchments studied represent a broad scale. Generally, lengths of 30 to 60 years are analyzed, but there are also considerably longer data series (more than 100 years in *Poulin et al., 2007*, and *Chowdhary et al., 2011*) as well as extremely short records (only 18 years in *Reddy and Ganguli, 2012*).
- The copula-based analyses of flood peak-volume relationships are not too conclusive as to the best-fit copulas. The Gumbel copula often appears among the copulas with the most acceptable fit (*Zhang and Singh, 2006; Genest and Favre, 2007; Poulin et al., 2007; Karmakar and Simonovic, 2009; Ben-Aissia et al., 2012; Sraj et al., 2014*), but such a conclusion is sometimes based on subjective decisions (such as a visual comparison of the observed and simulated data on scatter plots). Moreover, in the case of short records, there is no evidence to reject the null hypothesis that makes a particular copula acceptable; therefore, a number of copulas are suitable for modelling the particular peak-volume relationship. *Poulin et al. (2007)* emphasizes that when engaged in the procedure of selecting the copula model that best fits the observed data, it is also essential to reflect the tail dependence in the extremes. Not considering this feature of joint extreme events may lead to severe underestimations in hydrological design and flood protection.

In a copula-based extreme value analysis of any bi- or multivariate relationship, one needs a higher density of data, especially in the region of the right tails of the marginal distributions. This requirement, however, is rarely met in flood frequency estimation, since the studies are generally based on an analysis of the events that represent the annual maxima (i.e., the annual maxima of flood peaks and the corresponding flood volume), and the discharge records are generally still not of a sufficient length. To overcome the limitations stemming from the shortness of the observations, we propose to analyze a larger sample of floods, namely, all the flood events that can be regarded as independent from a hydro-meteorological point of view.

Since the majority of copula-based studies of a bivariate flood peak-volume relationship generally focus on a single selected catchment and the regional perspective does not appear in these studies (even in cases when more locations are investigated, such as in *Zhang and Sing, 2006, 2007)*, in the current case study we have examined a number of catchments and gone beyond a copula-based analysis of the floods and volumes which were carried out locally, to generalize our findings within the selected region. The previous works of the team of co-authors related to the same topic (*Szolgay et al., 2015, 2016a,b*) concluded that if a certain type of copula is acceptable, it may not be restricted to a particular site, but rather to a broader region. Herein, the following specific questions are raised:

- Does the copula-fitting procedure lead to a different set of parametric copulas that are acceptable for different flood types?
- Are there any differences in the set of acceptable copulas when they are fitted to data samples constructed on different principles (i.e., a process-based selection of all the independent flood events vs. the annual maxima of the flood events regardless of the flood processes)?
- Does the sample size influence the rejection rate of the individual copula models?

2. Study region and data

In this paper, the same pilot region is used as in *Gaál et al. (2016)*, and the database of flood events in the region is identical to that analyzed therein and in *Szolgay et al. (2016b)*, where their detailed descriptions were given. Here, a brief summary of the region and data selection is presented for the sake of completeness.

Flood generation mechanisms across Austria is of wide variety (e.g., *Merz and Blöschl, 2003, 2009; Parajka et al., 2007*), which makes analyses of flood-peak relationships difficult, since the different hydrological, climatological and meteorological, geological, etc., settings (*Gaál et al., 2012*) make it complicated to conduct and summarize regional studies. In order to reduce this complexity, we decided to restrict our analysis herein to a geographically more limited area, namely, the northwest of Austria (Fig. 1).

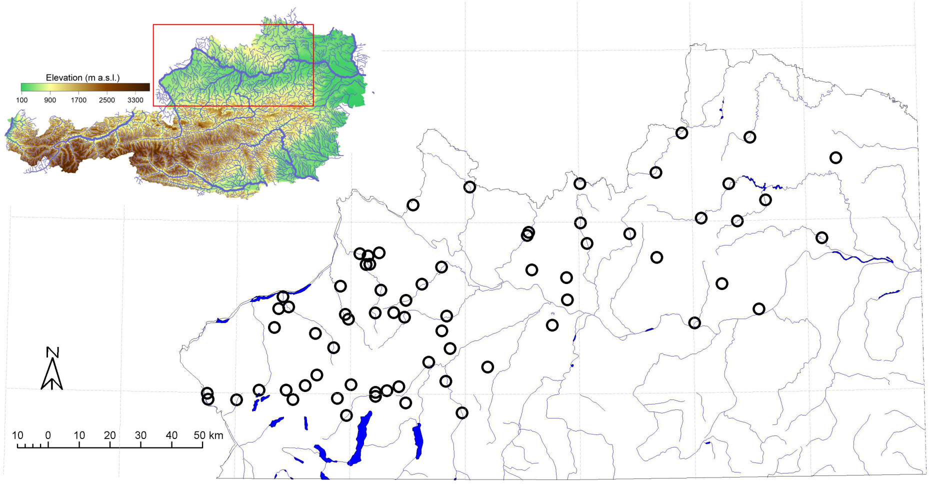

Fig. 1. Geographical location of the 69 catchments selected.

The region covers approximately 1/5–1/6 of the area of the country. It is dominated by lowlands and hilly sites, with elevations ranging from about 400 to 1500 m a.s.l. From a climatological point of view, the northwest region is mostly under the influence of air masses from the Atlantic region, which move from the west or northwest to the east or southeast. Since the orographic enhancement is not significant, the annual rainfall amounts (from about 500 to 1500 mm) are lower than in the Central Alps or at sites located directly along the northern fringe of the Austrian Alps (at some sites exceeding 3000 mm). The mean annual precipitation in the target region shows a slightly decreasing western-to-eastern gradient. Floods may occur both during the summer and winter. Due to the lower elevations of the region, snow/glacier-melt events of long durations do not occur. This fact is beneficial to this study, since the identification of the beginning and/or end of such events is not straightforward (see, e.g., *Merz et al., 2006*). The winter floods are usually induced by rain-on-snow processes when antecedent snowmelt saturates the soils and relatively low rainfall intensities may produce significant floods.

In this paper, runoff data with a time resolution of 1 hour from 69 catchments were used. The runoff records cover the period from 1976 to 2007. The catchment areas range from 10.6 to 444.3 km^2 (median: 74.6 km^2), while the range of their mean elevations is 342 to 888 m a.s.l. (median: 570 m a.s.l.).

The flood events were first identified from the runoff records. Even though there are different approaches for identifying flood events and to separate direct runoff and the base flow (e.g., *Li et al., 2014; Gonzales et al., 2009*), we followed the methodology that was adopted in the series of previous flood studies related to the target area (e.g., *Merz and Blöschl, 2003; Merz et al., 2006; Merz and Blöschl, 2009; Gaál et al., 2012; 2014*). The whole procedure consists of two main steps: first, the base flow and the direct flow were separated by means of the recursive digital filter of *Chapman and Maxwell (1996).* In the second step, the beginning and end of the rainfall-runoff events were identified. This is a rather complex, automated iterative procedure based on a number of criteria that are related to the values (and ratios) of the direct runoff and the base flow at the beginning of the event, the time of the culmination, and the end of the event. More details on the event separation procedure can be found in *Merz et al. (2006).*

3. Methodology

3.1. Identification and classification of flood events

Merz and Blöschl (2003) introduced a flood type classification which served as the basis for the process-based analysis of the events in this study. *Merz and Blöschl (2003)* originally defined five categories of floods (long-rain floods, short-rain floods, flash floods, snowmelt floods and rain-on-snow floods) on the basis of the meteorological situation (the spatial extent, the intensity and type of precipitation, solar radiation, etc.) and the state of the catchment (antecedent rainfall, saturation of soils, snow coverage, snowmelt, etc.) before and during the individual flood events. Later, *Gaál et al. (2014)* reduced the number of flood processes from five to three by merging categories that are acceptably similar to each other from the perspective of the flood generation mechanisms. As a result of this reduction, *Gaál et al. (2014)* used a 3-class differentiation of floods, namely synoptic floods (originally long-rain or short-rain floods), flash floods (no change in the classification), and snowmelt floods (originally rain-on-snow floods or snowmelt floods).

The flood classification of *Merz and Blöschl (2003)* is a very comprehensive and thorough, but extremely time demanding procedure. This is also one of the reasons that in Merz's original flood event database, only the annual flood maxima were treated and assigned to one of the five flood-type categories. This limitation represented the very first serious obstacle to our process-type analysis of flood events. Considering the fact that the discharge data in our study are generally available from the period of 1976–2007, records of a length of 32 years or less do not permit a serious bivariate analysis of flood peaks and flood volumes, particularly when this relatively small number of events is further separated into 3 or 5 classes. Therefore, even though the flood type classification does exist for the annual maxima of flood peaks, a decision was made to significantly increase the number of flood events in the individual flood type categories by using all the independent flood events that were possible to identify.

According to our understanding, two (or more) flood events are independent when they do not originate from the same synoptic situation. Indeed, we did not identify synoptic situations day by day; instead, we adopted a simplified approach. Its basic idea is that in Central Europe, cyclonic situations (fronts, weather types, etc.) do not persist longer than 7 days on average (see, for instance, the long-term statistics of the classification of *Grosswetterlagen* by *Werner and Gerstengarbe, 2010*). In other words, regardless of the precipitation observed, we assume that after a 7-day period, a completely different atmospheric situation occurs. On the basis of this hypothesis, we defined and adopted one of the following two rules of thumb:

1) a flood event is independent when it begins more than 7 days after the end of the previous event, and, at the same time, there is at least a 7-day period between the end of the current event and the beginning of the next one; or

2) a flood event is independent when a dry period with a length of at least 7 days begins during the event. The dry periods are evaluated on the basis of hourly catchment rainfall, while any small values ($\leq$ 0.1 mm) are disregarded, i.e., they are considered as being equal to 0.0 mm.

The types of flood events were identified according to the following approach. First, snowmelt events were classified on the basis of a simple and objective criterion. A rainfall-runoff event was classified as a snowmelt flood when in the highest elevation zone of the catchment (when catchment zones are delineated with a step of 200 m in altitude), a snow cover of a depth of at least 5 cm was observed at the beginning of the event (*Parajka et al., 2007*). Secondly, flash floods were identified on the basis of a set of criteria as follows:

- flash flood events can only occur in the months from May to September,
- flash flood events can only be caused by a rainfall event of a short duration (max. 5 hours),
- flash flood events have to show a characteristic steep hydrograph, and
- flash flood events cannot be related to low air temperatures (i.e., it is expected that they are caused by a rainstorm as a result of thermal convectivity).

All the remaining flood events were then classified as synoptic flood events.

3.2. Copulas

Copulas are mathematical functions that allow for modelling the relationship between two or more interrelated variables by getting rid of the direct influence of their marginal distributions. A copula-based bi- or multivariate frequency analysis can be split into two parts: in the first part, only the marginal distributions are assessed, while in the second part, after a uniform transformation of the marginals, the relationship between the variables can be modelled. This feature of copulas represents a considerable advance in comparison with the conventional approaches of bi- or multivariate frequency analysis, where serious restrictions have to be put both on the marginal distributions as well as on the joint distribution functions of the variables (see, e.g., *Karmakar and Simonovic, 2009*; or *Chowdhary et al., 2011*, for a comprehensive review of the traditional approaches and their drawbacks).

A bivariate copula function can implicitly be formally written as:

$$F_{XY}(x,y) = C(F_X(x), F_Y(y)) \tag{1}$$

(*Nelsen, 2006*), where F_X and F_Y are the respective marginal distribution functions of random variables X and Y which, in this paper, stand for flood peaks and flood event volumes, respectively. The term F_{XY} represents the joint distribution function of random vector (X, Y), and C is a copula, i.e., a function $C : [0,1]^2 \to [0,1]$ satisfying boundary conditions $C(t,0) = C(0,t) = 0$, $C(t,1) = C(1,t) = t$ (uniform margins) for any $t \in [0,1]$, and the so-called 2-increasing property, which is analogous to the non-decreasing property of a cumulative distribution function in a univariate case. As such, a copula can be viewed as a standardized joint distribution function.

Usually, the marginal distribution functions F_X and F_Y are not known. Therefore, they are estimated on the basis of the observations of the random variables (X_i, Y_i), $i = 1, ...n$, using a corresponding empirical distribution function (sometimes referred to as a plotting position formula):

$$F_X(x) = \textstyle\sum_i \mathbf{1}(X_i \leq x)/(n+1) \tag{2}$$

An analogous relationship holds for F_Y. The individual values of F_X and F_Y are called pseudo-observations and are denoted as $U_{j,i}$ $(i = 1, ...n, j = 1, 2)$. The empirical copula C_n is practically a multi-dimensional extension of the univariate plotting position formula:

$$C_n(u_1, u_2) = \textstyle\sum_i \mathbf{1}(U_{1,i} \leq u_1)\mathbf{1}(U_{2,i} \leq u_2)/(n+1) \tag{3}$$

In the paper, we used nine commonly-used one-parametric families from several classes of copulas, i.e.:

- the Archimedean class (the Clayton, Frank, and Joe copulas),
- the extreme-value class (the Gumbel-Hougaard, Galambos, and Hüsler-Reiss copulas),
- the elliptical class (the normal, and the t- [or Student] copula) and finally
- the unclassified Plackett copula.

Archimedean copulas are favored for their ease of handling; extreme-value (hereafter "EV") copulas are particularly appropriate for modelling the dependence between the extremes of random variables; and elliptical copulas are simply the copulas of elliptically contoured distributions. All the copulas used here are symmetrical with respect to the main diagonal, which reflects the exchangeability of the random variables; further, the Archimedean (except for Frank) and EV copulas are non-symmetrical with respect to the secondary diagonal. Some copulas possess a non-zero lower tail or upper tail dependence, i.e., they accumulate more probability mass at points $(0, 0)$ or $(1, 1)$. The Student t-copula is associated with a non-zero tail dependence on both sides. Although it is two-parametric by definition, the second parameter (degrees of freedom) is fixed to value 4 to effectively distinguish it from the normal copula. A brief summary of the copulas used in the current study by classification is in Table 1.

Table 1. A summary of the 9 copula types used in this paper.

Copula type	**Abbre-viation**	**Copula class**	**Lower tail dependence**	**Upper tail dependence**
Clayton	cla	archimedean	yes	no
Frank	fra	archimedean	no	no
Galambos	gal	extreme value	no	yes
Gumbel-Hougaard	gum	extreme value	no	yes
Hüsler-Reiss	hus	extreme value	no	yes
Joe	joe	archimedean	no	yes
normal (Gaussian)	nor	elliptical	no	no
Plackett	pla	unclassified	no	no
t (Student)	tco	elliptical	yes	yes

All of them, except for the Student t-copula, are single-parameter copulas. The parameter θ controls the degree of association between peaks and volumes and can be directly related to the rank correlation coefficient (see, e.g., *Nelsen, 2006*). The parameter θ was estimated from the flood peak-volume data by maximizing the so-called pseudo-likelihood function:

$$L(\theta) = \sum_i \log[c_\theta(U_{1,i}, U_{2,i})] \,, \tag{4}$$

where c_θ denotes the copula density, and $U_{j,i}$ $(i = 1, ...n, j = 1, 2)$ are the pseudo-observations of flood peaks and volumes.

The goodness-of-fit of the copulas to the data was tested by one of the 'blanket' tests (as denoted and designed in *Genest et al., 2009*, to demonstrate their general applicability with no necessity for a strategic choice of any parameters, kernels or weights) with the Cramér-von Mises measure of distance:

$$S_n = \sum_i [C_\theta(U_{1,i}, U_{2,i}) - C_n(U_{1,i}, U_{2,i})]^2 \tag{5}$$

between parametric copula C_θ and empirical copula C_n. The probability distribution of S_n, given that the null hypothesis (H_0: C_θ fits well) holds, is unknown and has been bootstrapped.

Since the test statistic, in principle, represents a distance, an intuitive approach for comparing their similarity using the outcomes of the test was adopted here. Instead of the test statistics of the goodness-of-fit that are size dependent, the p-values obtained through resampling (number of bootstrap samples: 1000) are used. The rationale (though not statistically strictly correct) behind this is that from a practical point of view, the smaller the distance S_n between the empirical values and the model, the better it describes (fits) the data (in the given sense). This does not of course guarantee or mean that the "better" model is the "more" correct one, e.g., for predictions (which anyway cannot be expected, given the rather short data series available). For our purpose of a regional assessment or comparison, we only used it to discriminate among the models in a regional sense. Since the lower values of S_n (a smaller departure from the null hypothesis statement) lead to larger p-values, we used these in turn as a measure of similarity. The suitability of the models in a strict statistical sense was the subject of examination in a similar regional setting in our previous studies (*Szolgay et al., 2015, 2016a,b*).

4. Results

Figure 2 contains the percent ratio of the three different flood types at the individual sites of the target region. It shows the clear prevalence of the synoptic flood types in the region, which at most sites exceeds two thirds of the total number of floods locally (overall, it is about 3/4). The second largest group is that of the snowmelt events, with the overall percent ratio close to 20% (see also Table 2). Locally, the percent ratio of the snowmelt floods varies between 8 and 46% . When considering all the flood events in the region, the smallest number of events (about 7%) belongs to the class of flash floods. Even though there are a small number of catchments where the absolute number of flash floods is low, there are no catchments in the selection with a missing flood type.

The cumulative distribution functions (hereafter "CDF") of the p-values obtained from fitting the 9 selected copula types to the samples of the

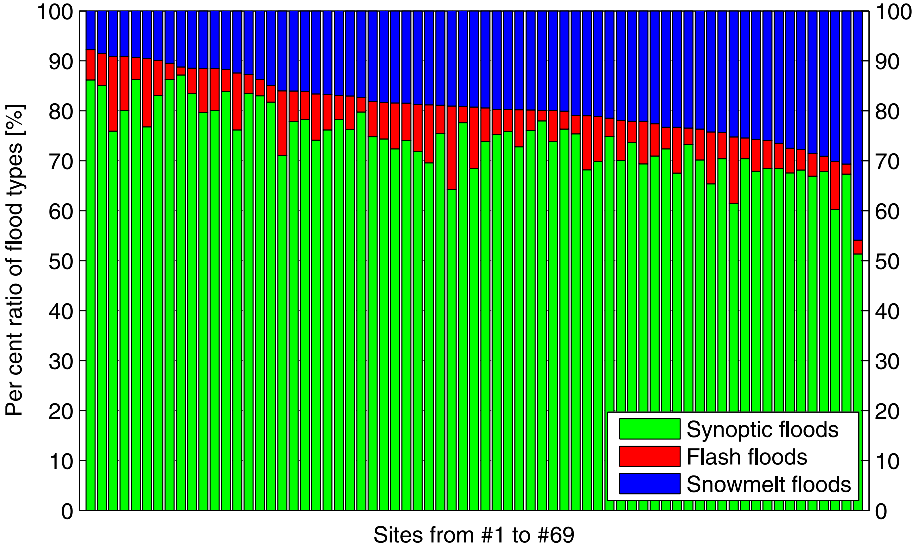

Fig. 2. Percent ratio of the three different flood types at the individual sites of the target region.

Table 2. Flood types locally, expressed in percentages.

	Minimum	Maximum	Median
Synoptic floods	51.4	87.2	74.3
Flash floods	1.6	16.7	5.8
Snowmelt floods	7.8	45.9	19.4

independent flood peak and flood volume pairs locally, and separately for the three individual flood types, are shown in Fig. 3. Note that in the case of flash floods, there are a few cases when the particular copulas could not be fitted (usually due to the small sample size). Therefore, the CDFs in Fig. 3 (middle) were only drawn on the basis of a smaller number of catchments (i.e., where it was possible to assign p-values). Table 3 presents the overall measure of the goodness-of-fit of the 9 selected copula types for different processes, which are expressed by the median p-values through all the catchments.

The best copula types here were selected predominantly on the basis of the whole cumulative distribution of the p-values (Fig. 3). The figures in

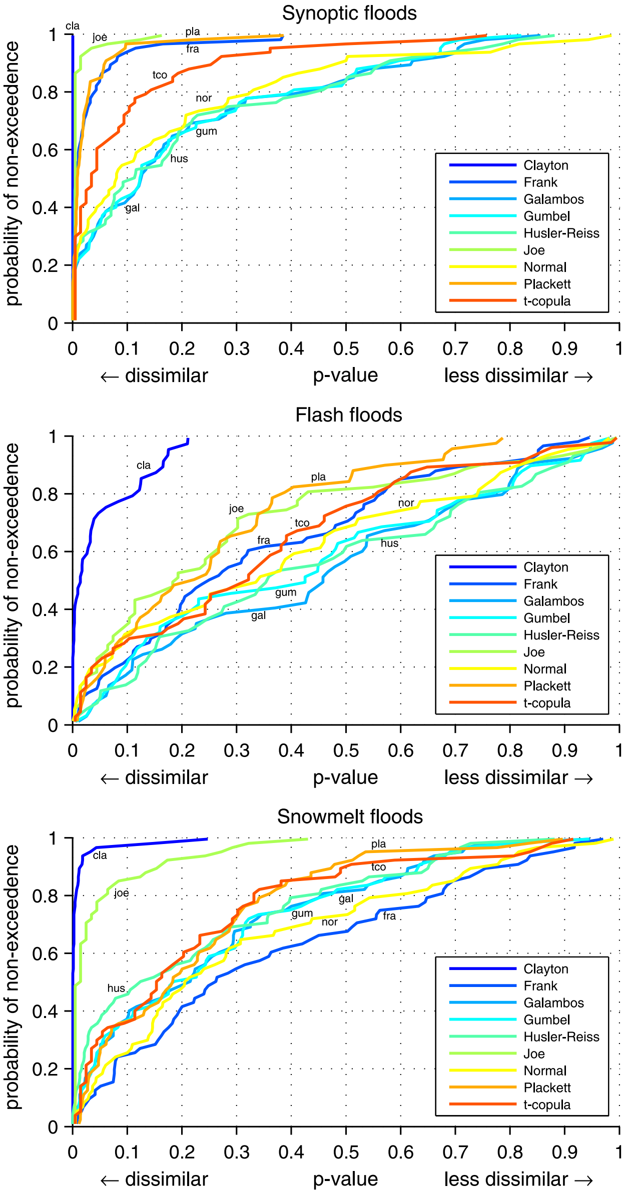

Fig. 3. Cumulative distribution functions of the p-values obtained from fitting the 9 selected copula types to the samples of the Q-V pairs locally and separately for the individual flood types: synoptic floods (top), flash floods (middle) and snowmelt floods (bottom).

Table 3. Overall median p-values of the goodness-of-fit measures between the empirical copulas at individual sites and the 9 parametric copulas. The copula models rejected at the significance level of $\alpha = 0.1$ are indicated in italics, while the copulas with the best performance are in bold.

	cla	**fra**	**gal**	**gum**	**hus**	**joe**	**nor**	**pla**	**tco**
Synoptic floods	*0.000*	*0.005*	**0.125**	0.123	0.113	*0.005*	*0.080*	*0.008*	*0.035*
Flash floods	*0.014*	0.250	**0.458**	0.425	0.356	0.188	0.339	0.222	0.322
Snowmelt floods	*0.000*	**0.262**	0.215	0.185	0.124	*0.015*	0.205	0.183	0.153

Table 3, which are point characteristics, serve as a supplementary tool in the decision making.

The following were selected as the best models:

- **synoptic floods**: the Galambos, Gumbel, and Husler-Reis copulas (i.e., the EV copulas),
- **flash floods**: the Galambos, Gumbel, and Husler-Reis copulas (i.e., the EV copulas),
- **snowmelt floods:** the Frank copula.

The extreme value copulas (which have a non-zero upper tail dependence coefficient, Table 1) show the best performance both for synoptic and flash floods. This can be attributed to the fact that flash floods and synoptic floods are associated with generally higher estimates of the upper tail dependence coefficient compared with the snowmelt floods (not shown here).

Figure 4 shows the CDF of the p-values for the copulas with the most acceptable fit.

The Clayton was depicted as the worst copula type, regardless of the flood process. It shows an unacceptable performance for all the processes. This can be explained by its shape, since it has a non-zero lower tail dependence, while in the current analysis, we are focusing on the extremes in the upper tails of the distributions. The second worst performer in the case of snowmelt floods is clearly the Joe copula. In the case of synoptic floods, the Joe, Frank and Plackett copulas have the most remarkably different positions in the top left corner of the CDF plot (Fig. 3). In the case of flash floods, however, the Clayton is the only copula that was rejected. Due to the small sample sizes of the flash floods, the other copula types cannot be rejected (Table 3).

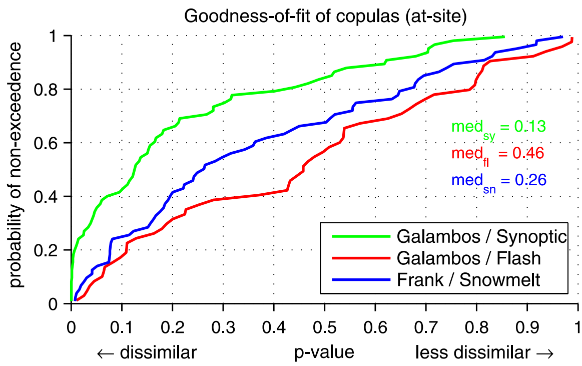

Fig. 4. Cumulative distribution functions of the p-values for the copulas with the most acceptable fit.

In practical design-oriented studies, the annual maxima of floods are usually used both in the univariate and bivariate cases. This inevitably results in less data as compared to our method of selecting all the independent events, such as in the case of synoptic floods. Moreover, at many sites with a rich typology of flood-generating processes, the data sample consists of a mix of events of different types. Consequently, the i.i.d. requirement (i.e., the data sample being independent and identically distributed) of the frequency analysis is rarely met, and it would probably need to be based on a mixed model (however, this is rarely done in practice). The effect of a mixed distribution of data types on the selection of frequency models can be even more pronounced in the bivariate case.

We illustrate this concern in Fig. 5, where all the floods in the database of independent events at a selected catchment are shown. The scatter plots (Fig. 5, right) clearly indicate that there are substantial differences between the empirical copulas. As a consequence, the i.i.d. requirement of the frequency analysis would most probably not be met in this case, and the copula fitting would probably need to be based on a mixed model.

A similar copula-fitting procedure was carried out for the samples of flood events based on the annual maxima of flood peaks (Fig. 6, Table 4). In terms of the least suitable copula types, the Clayton copula is ranked at the first place, which is followed by the Joe copula (nevertheless, the latter one cannot be rejected, as indicated by the median p-value in Table

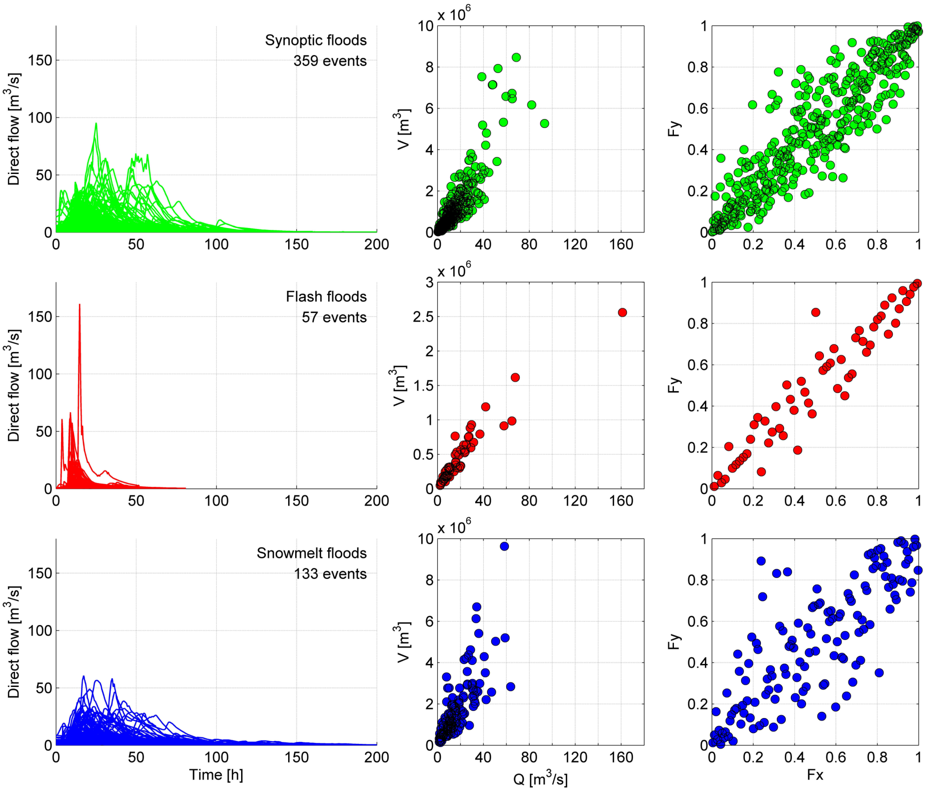

Fig. 5. Aurachkirchen catchment (at the stream Aurach, located in the southwestern parts of the target region), where a substantial difference is indicated between the empirical copulas. Left: flood hydrographs. Middle: relationship between the observed values of the flood peaks and flood volumes. Right: the relationship between the flood peaks and flood volumes, each of which is rescaled to the unit interval (i.e., pseudo-observations). Top (green color): synoptic floods; middle (red color): flash floods; bottom (blue color): snowmelt floods.

4). These results are independent of the fact whether they are formulated on the basis of the whole CDF (Fig. 6) or the point statistics (median, Table 4). Judged on the basis of the point statistics, the normal copula shows the best performance. Nevertheless, when looking at the CDFs in Fig. 6, the picture is not so clear: the normal copula is not the best overall copula type, but is only for the medium and larger p-values.

It is also interesting to take a look at the performance of the Gumbel

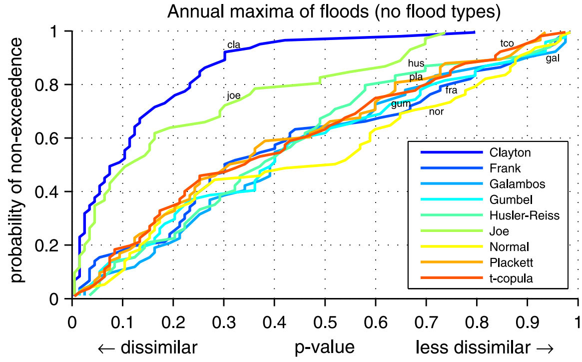

Fig. 6. Cumulative distribution functions of the p-values obtained from fitting the 9 selected copula types to the samples of the annual maxima of the Q-V pairs locally, where the flood types are not considered.

Table 4. Overall median p-values of the goodness-of-fit measures between the empirical copulas of the annual maxima of the floods at individual sites and 9 parametric copulas. The copula models rejected at the significance level of $\alpha = 0.1$ are indicated in italics, while the copulas with the best performances are denoted in bold.

	cla	**fra**	**gal**	**gum**	**hus**	**joe**	**nor**	**pla**	**tco**
Annual maxima of floods	*0.084*	0.302	0.396	0.396	0.376	0.114	**0.520**	0.312	0.342

copula, which, on the basis of a number of studies of the Q-V relationship (see the literature overview in the introduction), is very often selected as the best copula family for modelling this bivariate relationship. Our results indicate that the Gumbel copula does not show an outstanding performance: it is only one of a number of copula types that perform acceptably well regionally (i.e., it cannot generally be rejected on the basis of the median statistics).

5. Discussion and conclusions

In this paper (and in our companion studies, see *Szolgay et al., 2015, 2016a,b* and *Gaál et al., 2016*), we decided to use all the flood events that can be considered independent from a hydro-meteorological point of view. This decision resulted in an enhanced number of events: we had an approximately

10 to 20 times larger database of flood events than we would have been able to prepare on the basis of the principle of annual maxima.

A problem in our data selection was related to the flood processes identified, i.e., the relatively small number of flash flood events in the given region. At some catchments, the number of flash floods do not exceed 10. Generally, it is not reasonable to fit any statistical model to such small sample sizes, even in a univariate case. Nevertheless, we did not exclude catchments with a small number of flash events since they also contribute valuable information on synoptic and snowmelt floods. In *Szolgay et al. (2016a,b)*, we commented extensively on this issue in a slightly different setting with respect to the selection of the correct statistical model based on the usual "accept-reject the hypothesis" test framework. Here, the emphasis was put on an overall regional evaluation of the acceptable models in an intuitive subjective framework. To reflect the small sample problem in this type of analysis, an additional experiment was performed to see the influence of the inclusion of catchments with small samples of flash floods in the analysis. We excluded all the catchments where the number of flash floods did not exceed 20. This action reduced the total number of catchments in the target region from 69 to 38. The CDFs of the p-values of the goodness-of-fit of the copula models are shown in Fig. 7. Practically, no major change is discernible between the two sets of CDFs; they follow each other in a similar way: [cla] $\rightarrow$ [joe or pla] $\rightarrow$ [tco or fra] $\rightarrow$ [nor, hus, gal or gum].

Overall we can conclude that we cannot restrict the choice of model regionally despite the enlarged dataset. On the other hand, the larger sample

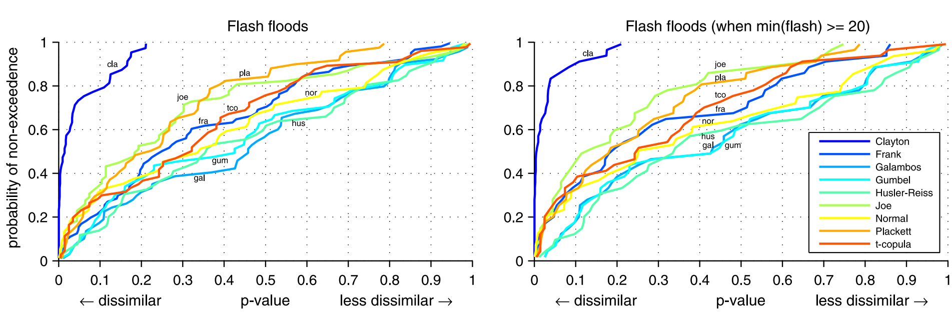

Fig. 7. Cumulative distribution functions of the p-values of the goodness-of-fit of the copula models in all 69 catchments (left) and in 38 catchments where the number of flash floods exceeds 20 (right).

sizes helped in reducing the number of plausible copulas to fit. Small samples, on the contrary, are not helpful in selecting a suitable copula model.

There are no universally and acceptably good copula types in common use for all three flood processes. For synoptic and flash floods, the extreme value copulas seem to perform better than the others, while for snowmelt floods, the Frank copula shows the best fit. In a reverse formulation, the Clayton copula shows an unacceptable performance for all the processes. The rejection of the other copula types depends on the flood type.

The current approach based on the flood processes and all the independent events was also compared with the traditional one where the annual maxima of flood events are selected regardless of the flood processes. The Clayton copula is, again, the least suitable copula, while no firm conclusion can be made on the best-fit copula type. The Gumbel copula, which is generally preferred in the literature, is only one of several copulas with an acceptably good performance.

The worst copula type (with the highest rejection rate) is undoubtedly the Clayton. This result was obtained regardless of the process-based samples and is also valid for the results based on the annual maxima. It is asserted that this is due to the fact that the Clayton copula is the one with a non-zero lower tail dependence and a zero upper tail dependence, which predetermines its use for low flows or phenomena with similar behavior instead.

On the other hand, different behavior can be seen in the case of the best-fit copulas for the annual maxima of flood peaks and respective flood volumes. The extreme value copulas dominate in the case of flash floods and synoptic floods, but in the case of snowmelt floods, they are ranked worse due to the better performance of the elliptical normal copula and the non-elliptical Frank copula. In the case of synoptic floods, the Frank copula belongs to the spectrum of the worse copulas, while for snowmelt floods, it shows the best overall fit. This is certainly not the effect of the sample size since for the smallest sample sizes (flash floods and the samples of the annual maxima), the Frank copula shows an intermediate performance.

Despite the not unambiguous outcomes of the model comparison in the given region, we prefer a regional perspective: the results of a local fitting from a particular catchment may be overridden by more robust overall results from a region. Therefore, we recommend simple engineering guidelines as to how to take the advantage of the regional information:

1) prepare a larger process-based dataset,
2) go to a broader region,
3) fit the copulas locally,
4) look at the results regionally, and
5) make the decision on the best/worst copula model for the given site in the region.

The above-described regional perspective is also related to the sample size issue: if there is a sufficiently large data sample at a given site, a local analysis can yield an acceptable copula model(s) to fit or reject. On the other hand, in the case of a small local data sample, one may come to unclear or incorrect conclusions on the suitability of the copula models, so in order to reduce the risk of a wrong choice, one has to look at the local results from the broader region of the particular site.

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