Evaluation of recent Earth's global gravity field models with terrestrial gravity data

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Abstract: In the context of the rapid development of environmental research technologies and techniques to solve scientific and practical problems in different fields of knowledge including geosciences, the study of Earth's gravity field models is still important today. The results of gravity anomaly modelling calculated by the current geopotential models data were compared with the independent terrestrial gravity data for the two territories located in West Siberia and Kazakhstan. Statistical characteristics of comparison results for the models under study were obtained. The results of investigations show that about 70% of the differences between the gravity anomaly values calculated by recent global geopotential models and those observed at the points in flat areas are within ± 10 mGal, in mountainous areas are within ± 20 mGal.

Key words: Earth's gravity field, differences, gravity anomalies, global geopotential model, terrestrial gravity data

1. Introduction

Due to the technological progress and advanced techniques in studying Earth's gravity field the accuracy investigations of recent global gravity field models for various territories are of current interest. Satellite gravimetric missions based on the satellite-to-satellite observations and gradiometry make it possible to obtain the large-scale features of Earth's gravity field described by spherical harmonics. The benefit of satellites is that they can provide global and sufficient coverage in a reasonable time. It allows us to study the Earth's gravity field at scales down to hundreds and thousands of kilometres. The main disadvantage of using satellites for gravity field exploration is low sensitivity to high-degree geopotential spherical harmonic coefficients due to a strong signal attenuation with altitude (*Heiskanen and Moritz, 1967*).

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To study the global-scale gravity field based only on the terrestrial gravity data is not possible due to the state-of-the-art and number of observations. The above mentioned techniques for studying the Earth's gravity field face some problems, which may be overcome by means of low-orbit satellite systems. Among the current international space research projects focused on the investigation of Earth's gravity field over the past decade, GOCE (Global Ocean Circulation Experiment)-data provides the most detailed information (*Bruinsma et al., 2010; Drinkwater et al., 2003; Karpik et al., 2015; Kanushin et al., 2014; Šprlák et al., 2015; Voigt and Denke, 2015*).

The main purpose of this work is to evaluate the accuracy of the recent global geopotential models based on the GOCE data for two territories located in West Siberia and Kazakhstan.

2. Models

For the purposes of our investigations we have chosen two satellite-based gravity field models *go_cons_gcf_2_tim_r5, jyy_goce04s* and the mixed model *eigen–6c3stat* which were created in 2014 using the results of gravity measurements from the GOCE mission (*Brockmann et al., 2014; Förste et al., 2013*).

General information on these models under study created by the results of gravity measurements from the GOCE mission is given in Table 1 (*ICGEM – International Center for Global Gravity Field Models*).

Table 1. General information on models based on the GOCE data.

No.	Model	Published (year)	Degree N	Input
1	jyy_goce04s	2014	230	S(Goce)
2	go_cons_gcf_2_tim_r5	2014	280	S(Goce)
3	eigen–6c3stat	2014	1949	S(Goce, Grace, Lageos), G

The most convenient form for representation of the Earth's global gravity field is a spherical harmonic expansion of the geopotential in geocentric coordinates (*Torge and Müller, 2012; Vanicek and Krakiwsky, 1982*), namely the geocentric radius vector r, latitude φ, and longitude λ as:

$$V = \frac{fM}{r}\left[1 + \sum_{n=2}^{\infty}\left(\frac{a_e}{r}\right)^n \sum_{m=0}^{n}\left(\bar{C}_{nm}\cos m\lambda + \bar{S}_{nm}\sin m\lambda\right)\bar{P}_{nm}\left(\sin\varphi\right)\right], \quad (1)$$

where fM is the geocentric gravitational constant; a_e is the Earth's equatorial radius; $\bar{C}_{nm}$ and $\bar{S}_{nm}$ are the geopotential's fully normalized spherical harmonic coefficients of degree n and order m; $\bar{P}_{nm}$ are associated Legendre functions. While the origin of coordinates coincides with the center of the Earth's mass, the formula (1) does not include the coefficients $n = 1$.

Free-air gravity anomalies of degree N are represented in terms of the spherical harmonic expansion (*Hofmann Wellenhof and Moritz, 2007*):

$$\Delta g = \gamma \sum_{n=2}^{N}(n-1)\sum_{m=0}^{n}\left(\Delta\bar{C}_{nm}\cos m\lambda + \bar{S}_{nm}\sin m\lambda\right)\bar{P}_{nm}\left(\sin\varphi\right), \quad (2)$$

where:$\Delta\bar{C}_{nm} = \bar{C}_{nm} - \bar{C}^0_{nm}$; $\bar{C}^0_{nm}$ are the spherical harmonic coefficients of the normal gravity potential; $\bar{P}_{nm}\left(\sin\varphi\right)$ are the fully normalized associated Legendre functions.

3. Analysis

Comparative accuracy evaluation of the Earth's global gravity field model under study was carried out by the following formula:

$$\delta g(P) = \Delta g_T(P) - \Delta g_S(P)\,, \quad (3)$$

where $\Delta g_T(P)$ are free-air gravity anomalies observed at the terrestrial gravity points; $\Delta g_S(P)$ are free-air gravity anomalies computed from the model.

Quantitative accuracy evaluation of the Earth's global gravity field modelling was based on the following common criteria for accuracy:

– Average value

$$E(\Delta) = \frac{1}{k}\sum_{i=1}^{k}\delta g(P_i)\,, \quad (4)$$

– Standard deviation

$$D(\Delta) = \left[\frac{1}{k}\sum_{i=1}^{k}\left(\delta g(P_i) - E(\Delta)\right)^2\right]^{1/2}. \quad (5)$$

Calculations were carried out with a special software package developed by the authors at the Siberian State University of Geosystems and Technologies (*Goldobin et al., 2015*).

To evaluate the accuracy of the geopotential model under study, the model-based data were compared with the independent observed free-air gravity anomalies at the terrestrial gravity points for the two territories located in West Siberia (Territory 1) and Kazakhstan (Territory 2).

Twenty-seven gravity stations with observed free-air gravity anomalies were chosen for Territory 1. The free-air gravity anomalies were determined with the accuracy not less than 1 mGal. The relief of this territory is predominantly flat. Variations in gravity anomalies are negligible and about 65 mGal (*Kanushin et al., 2014*).

Sixty-one gravity stations with observed free-air gravity anomalies were chosen for Territory 2 (*Kanushin et al., 2014*). The free-air gravity anomalies were determined with an accuracy of no less than 1 mGal. The territory under study is characterized by the compound anomalous gravity field with the range of variation about 176 mGal. At these stations the gravity anomalies were calculated for every model under study. Then the differences between observed and calculated gravity anomaly values were determined.

4. Results

Statistical distribution parameters of the differences between observed and calculated gravity anomaly values in mGal for Territory 1 are given in Table 2. The histograms of differences between observed and calculated gravity anomaly values for Territory 1 are shown in Figure 1.

It is evident from Figure 1 and Figure 2 that the distributions of differences calculated by the formulas and those based on the terrestrial gravity data correspond to the normal distribution.

The maps of gravity anomaly differences between the observed values and the values calculated by the models *jyy_goce04s, go_cons_gcf_2_tim_r5, eigen-6c3stat* for Territory 1 are presented in Figure 2.

Statistical distribution parameters of the differences between observed and calculated gravity anomaly values in mGal for Territory 2 are given in Table 3.

Table 2. Statistical distribution parameters of the differences for the Territory 1.

	jyy_goce04s [mGal]	go_cons_gcf_2_tim_r5 [mGal]	eigen-6c3stat [mGal]
Degree	230	280	1949
Minimum	−20.049	−20.344	−15.515
Maximum	19.751	18.407	14.923
Average	0.228	0.687	1.065
Standard deviation	10.040	9.677	7.509
Asymmetry	−0.116	−0.317	−0.348
Kurtosis	−0.448	0.005	−0.232

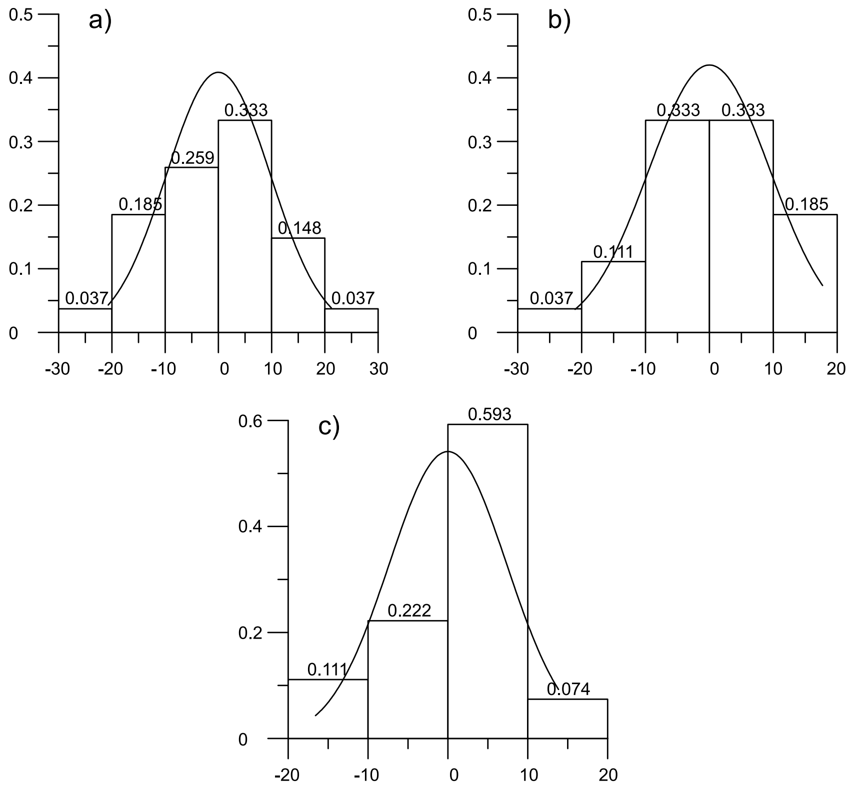

Fig. 1. Histograms of differences between observed and calculated gravity anomaly values (in mGal) for Territory 2: a) *jyy_goce04s*, b) *go_cons_gcf_2_tim_r5*, c) *eigen-6c3stat* (with the percentage on the vertical axis and the difference in mGal on the horizontal axis).

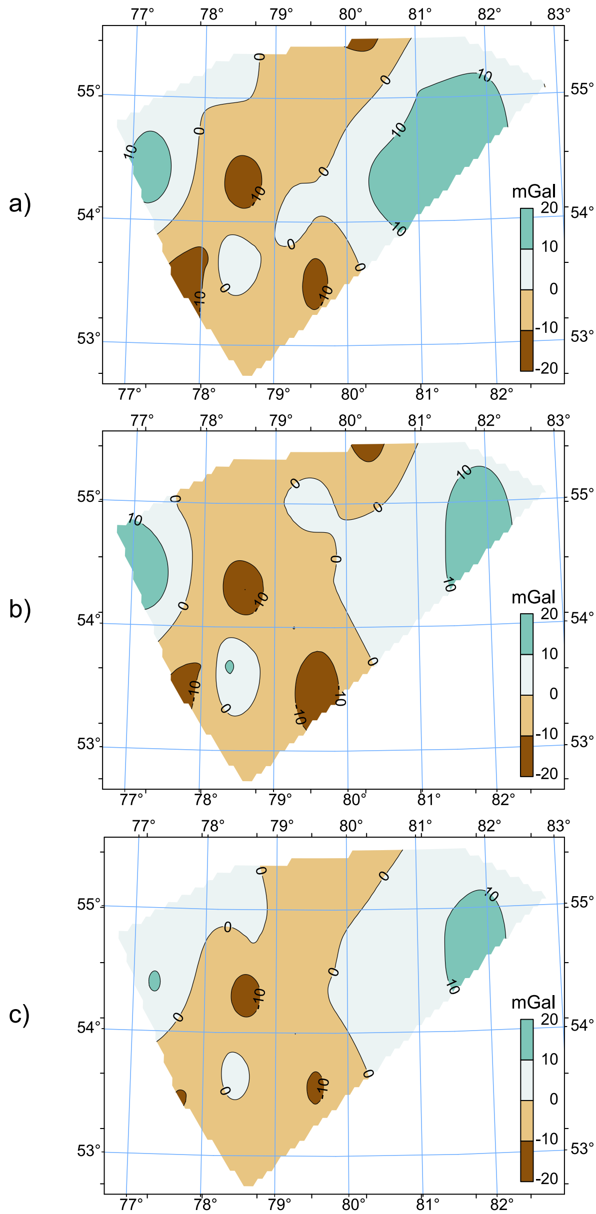

Fig. 2. Maps of gravity anomaly differences between the observed values and the values calculated (in mGal) for Territory 1: a) *jyy_goce04s*, b) *go_cons_gcf_2_tim_r5*, c) *eigen-6c3stat*.

Table 3.Statistical distribution parameters of the differences for the Territory 2.

	jyy_goce04s [mGal]	go_cons_gcf_2_tim_r5 [mGal]	eigen-6c3stat [mGal]
Degree	230	280	194
Minimum	−69.425	−62.266	−35.934
Maximum	80.220	63.655	47.917
Average	−0.662	−41.462	−42.100
Standard deviation	25.454	21.287	14.435
Asymmetry	0.738	0.379	0.900
Kurtosis	1.943	1.544	2.476

The histograms of differences between observed and calculated gravity anomaly values for the Territory are shown in Figure 3.

The maps of gravity anomaly differences between the observed and the values calculated by the models *jyy_goce04s, go_cons_gcf_2_tim_r5, eigen-6c3stat* for Territory 2 are presented in Figure 4.

The maps of gravity anomaly differences in Figures 2 and 4 show that the spatial distribution of their minimum and maximum correlates with the gravity anomaly maxima and minima in the territories under study. High gradient zones of gravity field anomalies have a greater error than zones with lower anomalous field gradient.

5. Conclusion

The results of the evaluation of the selected GOCE-based Earth's gravity field models have shown the following:

- About 70% of differences between the gravity anomaly values calculated by the recent global geopotential models and those observed at the points in flat areas are within ±10 mGal, in mountainous areas they are within ±20 mGal.
- Standard deviation is within a range of 10 mGal to 7.5 mGal.

According to Table 2 representing the investigations of the territory with the anomalous gravity field of 65 mGal it is evident that:

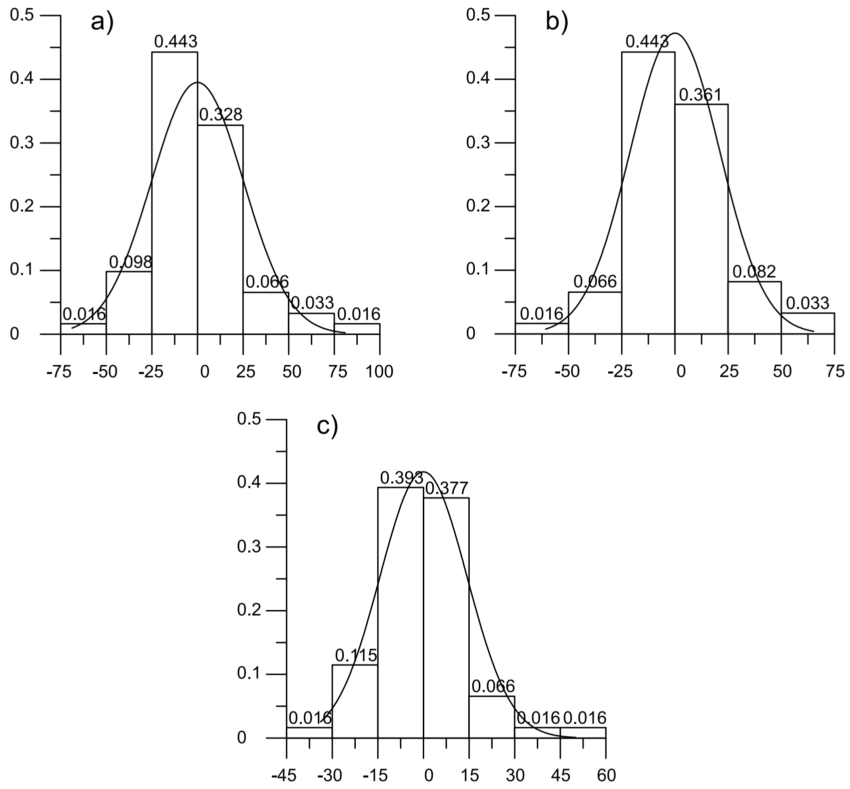

Fig. 3. Histograms of differences between observed and calculated gravity anomaly values (in mGal) for Territory 1 by the models: a) *jyy_goce04s*, b) *go_cons_gcf_2_tim_r5* c) *eigen-6c3stat* (with the percentage on the vertical axis and the difference in mGal on the horizontal axis).

– The least standard deviation was obtained for the combined gravity field model (using terrestrial gravity data) *eigen-6c3stat* 7.5 mGal at the average value of $E = 1.06$ mGal.

– A symmetrical distribution of differences close to the normal distribution has been observed for the models under study in the given territory.

From Table 3 representing the investigations of the territory with the compound anomalous gravity field of 176 mGal it is apparent that the least standard deviation for the combined gravity field model *eigen-6c3stat* is 14.4 mGal at the average value $E = -2.1$ mGal.

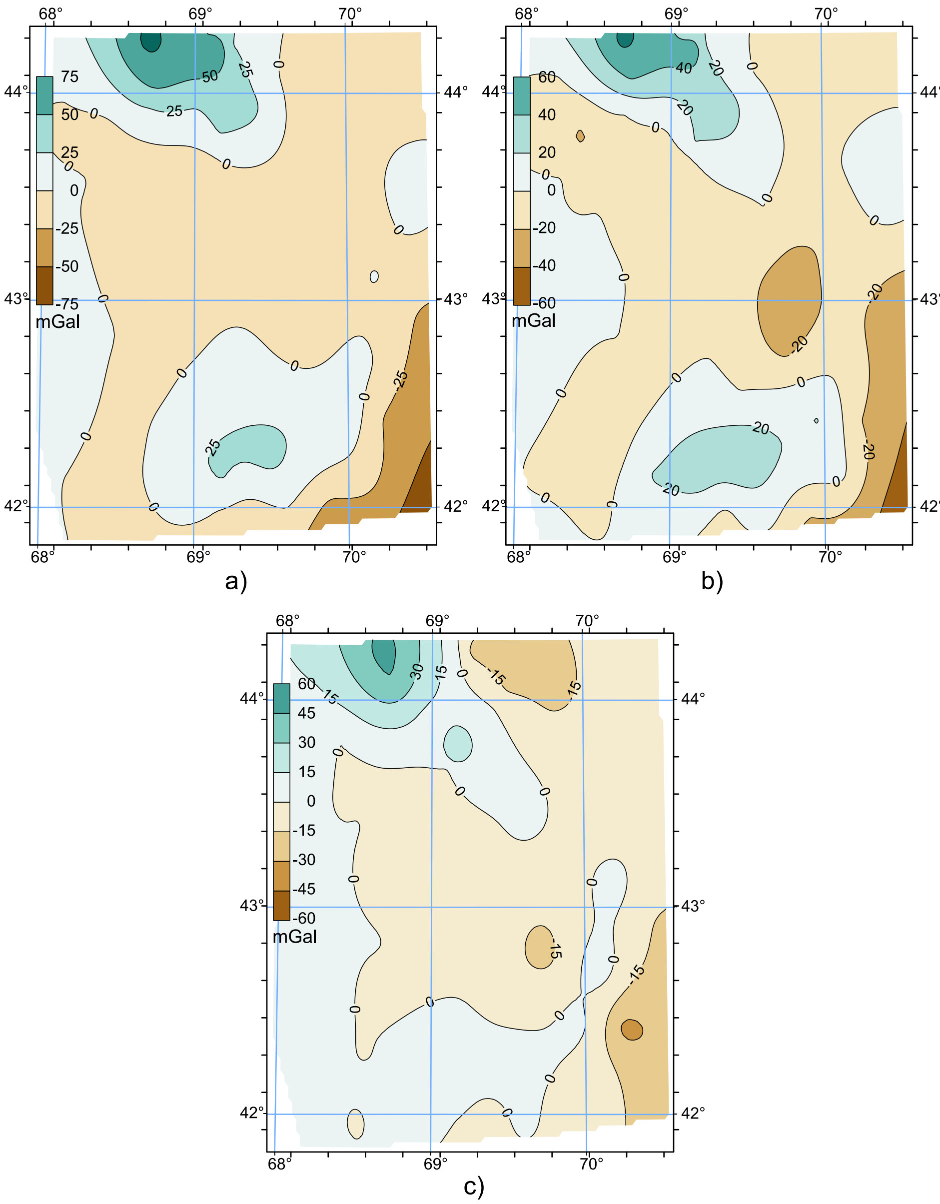

Fig. 4. Maps of gravity anomaly differences between the observed values and the values calculated (in mGal) for Territory 2 by the models: a) *jyy_goce04s*, b) *go_cons_gcf_2_tim_r5*, c) *eigen-6c3stat*.

The accuracy of the models under study approach the levels claimed during their creation but actually do not reach them (*Förste et al., 2013; Šprlák et al., 2015; Voigt and Denke, 2015*). Recent global models of the gravity field give new opportunities and significantly expand the scope of tasks for studying the Earth's gravity field and figure.

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