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Inner zone terrain correction calculation using interpolated heights

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Abstract: The discrepancy between real heights of gravity points and the elevation model has a significant impact on the terrain corrections calculation especially within the inner zone. The concept of interpolated heights of calculation points used instead of measured ones within the specified inner zone can considerably decrease the resulting errors. The choice of appropriate radius of the inner zone for use of interpolated heights is analysed on synthetic topography model as well as real data. The tests with synthetic models showed the appropriate radius of this zone is proportional to the deformation wavelength of the model. Simple statistical analysis of a particular elevation model can give an estimate of the appropriate radius for the calculation using interpolated heights. A concept with interpolated heights in the zone 0–250 m is used in actual practice in Slovakia. The analysis of regional gravity data from the Tatry Mountains test area indicates the searched radius should be about 100 m. Detailed gravity measurements from different areas showed the searched radius does not play so important role but the use of interpolated heights instead of measured ones is still relevant. The more reasonable method instead of using interpolated heights is also presented when calculating the topographic effect.

Key words: terrain correction, digital terrain model, Bouguer anomaly, synthetic topography model

1. Introduction

The standard recommended way to evaluate innermost zone terrain correction (up to approximately 100 m) is to use in-situ geodetic measurements (e.g. *LaFehr et al., 1988; Steinhauser et al., 1990; Lyman et al., 1997; Hinze et al., 2005; Schiavone et al., 2009* and others). However, this approach demands additional costs and it is time consuming. In addition, as we have realized during our test measurements in the High Tatra Mountains (*Zahorec, 2014*), it is very difficult to realize geodetic measurements, even those using remote (laser) methods, in high mountains areas. There

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was a problem to obtain quasi-regularly spaced data using laser range-finder farther than about 30 m around the measurement points, because the obstructed view and inaccessible terrain as well as the frequent occurrence of fog (e.g. *Lyman et al., 1997*) also pointed to the problem with dense fog). Therefore the use of available digital terrain models (DTM) for terrain correction calculations is still topical, even in the inner zone.

The gravimetric database of Slovak Republic contains actually more than 300 000 terrestrial gravity measurements. The in-situ geodetic measurements around the measured points were not performed during the acquisition of these data, which was realized in Slovakia (then in Czechoslovakia) mainly during the 1970's and 1980's. Thus also the inner zone terrain corrections were estimated on the basis of available topographic maps and later using various elevation models. *Grand et al. (2001)* pointed out that the DTM used for the inner zone terrain correction calculation was insufficient and they were talking rather about estimation of these corrections than about their calculation. We have estimated the error to be as much as 15 mGal (for the density 2670 kg/m^3) from the inner zone 0–250 m within the Tatry Mountains (*Zahorec et al., 2010*). Today we have possibility to recalculate terrain corrections for the whole database using actual DTMs. These models are much more accurate than previous ones, but there is still a problem with discrepancy between real (measured) heights of calculating points and model heights, which leads to large calculation errors. It is because we estimate the terrain correction within the nearest zone in points which are several tens of metres "hung" in the air, or "dipped" under the topography surface when we use measured heights. One simple concept to decrease resulting error is to use interpolated heights of calculating points instead of measured ones. In this way, we "put" the calculating points on

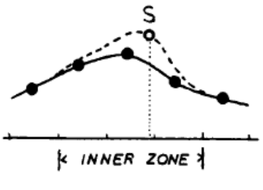

Figure 49 Spline interpolation of elevations in an inner zone and possible modification to give the "correct" elevation at computation point s.

Fig. 1. Solving a problem of discrepancy between calculation point and elevation model by *Forsberg (1984)*.

the topography surface which is the usual situation with measured gravity points. When we accept this approach, there is a question to which distance around the calculation point we should apply it or, in other words, which radius of the innermost zone we should use. I have studied this problem on synthetic topography models as well as the real data. The aim of this paper was to confirm and improve the concept of interpolated heights.

2. Problem "interpolated vs. measured heights of calculation points"

The standard method of terrain correction calculation used in Slovakia during the last decades is as follows (*Grand et al., 2004*): the calculation area around the point is divided to four circular zones, inner zone T1 (0–250 m), intermediate zone T2 (250–5240 m), outer zones T31 (5.24–28.8 km) and T32 (28.8–166.7 km). Different DTM grids are used within particular zones, the most detailed one is used for the inner zone T1 calculation (today 10 × 10 m or better, if available). In addition, the interpolated heights of calculation points instead of measured ones are often used within the inner zone T1 to avoid possible errors resulting from the above mentioned discrepancies between real (measured) and model heights. This approach was established more or less intuitively during the recalculation of the Slovak gravimetrical database (*Grand et al., 2001*) and therefore it is necessary to study this problem in more detail. I have used our new software Toposk (*Marušiak et al., 2013*) for current calculations. This software enables us to calculate a topographic effect (and consequently the terrain correction) at arbitrary calculation point position, so also above or below the topographic surface.

The problem of discrepancy between measured point and available elevation model was recognized several decades ago. For example *Krohn (1976)* solved the problem using the system of multiquadric equations defining a smooth surface which also passes through the station, *Forsberg (1984)* used a bicubic spline interpolation (Fig. 1). *Cella (2015)* has recently used a set of triangular prisms defined by the calculation point itself and nearest elevation grid nodes. However this is not an adequate way in the case of errors within the elevation model and consequent large differences between point elevation and model (it could lead to unrealistic "topographic" forms represented by the nearest prisms). In contrast with those methods the concept of interpolated heights does not change (adapt) the topography model shape in the vicinity of the point, the calculating point is just "laid" on the surface.

So the question is: In which zone (subzone) around the calculation point should we use this approach? We can suppose the measured heights are accurate (the measurement error is not considered) and therefore the discrepancy is due to errors in DTM. Thus we should consider what errors (deformations) a particular DTM contains. If we expect only local deformations, the calculation zone using interpolated height should be equally local, because it is obvious the farther (undeformed) topography had to be considered in regard to the correct (measured) height of the calculating point. We can suppose local DTM deformations due to "smoothing" of valleys and tops, as shown, for example, by *Cogbill (1990).* On the other hand we could also expect larger deformations over forest areas as an example. It is obvious the calculation zone with interpolated height should be somehow equivalent to the extent of DTM deformation around the calculation points. In the future this problem will diminish as the high detailed elevation data (e.g. LIDAR) will be available for the whole territory of Slovakia.

3. Tests with synthetic topography models

The outlined idea can be proved by a simple test with synthetic topography models. I have generated a simple sinusoidal topography model with detailed grid cell size equal to 1 m (Fig. 2, left). This model was subsequently deformed by a periodical deformation with amplitude of 50 m and a wavelength equal to 200 m (Fig. 2, right). Since the real topography is usually more rugged than the available DTM, we can regard the second model as the real topography while the first model can represent a smoothed DTM. The terrain corrections calculated for the "real" model are regarded as an ethalon. Then we can compare the values calculated for the "DTM" model both for interpolated heights and "measured" heights while the "measured" ones represent the heights obtained from the "real" model.

Since the width of zones with systematic positive or negative DTM deformations is equal to 100 m (which corresponds to the half-wavelength of

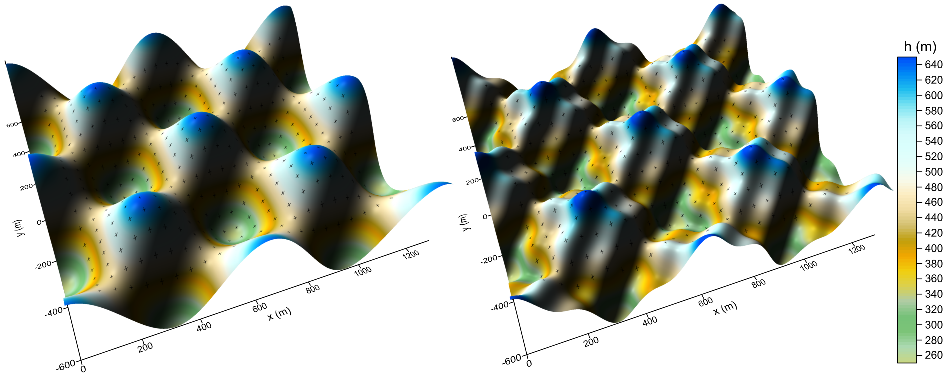

Fig. 2. Synthetic topography models representing DTM (left) and "real" topography (right). Small marks indicate calculating points.

loaded deformation, see bottom graph in Fig. 4), we can suppose that the appropriate dimension of the zone for use of interpolated heights is equal to several tens of metres. Therefore it is useful to compare calculated terrain correction values within the successive ten-metre wide intervals (Fig. 3 and Table 1). As we can see, the evidently larger calculation errors were obtained for "measured" heights within the innermost subzones up to 20 m. On the other hand, interpolated heights exhibit gradually worse results within subzones beyond 30 m. This test thus showed that the appropriate radius of the zone for use of interpolated heights should be equal approximately to

Table 1. Statistical results of calculated terrain corrections errors shown in Fig. 3.

Calculation error	Calculation height	Subzone				
		0–10 m	10–20 m	20–30 m	30–40 m	40–50 m
Minimum (mGal)	measured	–0.349	–0.326	–0.286	–0.238	–0.191
	interpolated	–0.349	–0.326	–0.286	–0.282	–0.329
Maximum (mGal)	measured	0.981	0.682	0.414	0.256	0.194
	interpolated	0.351	0.336	0.305	0.256	0.194
Mean (mGal)	measured	0.397	0.124	0.026	–0.007	–0.019
	interpolated	–0.059	–0.058	–0.056	–0.054	–0.051
SD (mGal)	measured	0.293	0.220	0.153	0.115	0.086
	interpolated	0.158	0.149	0.134	0.116	0.096

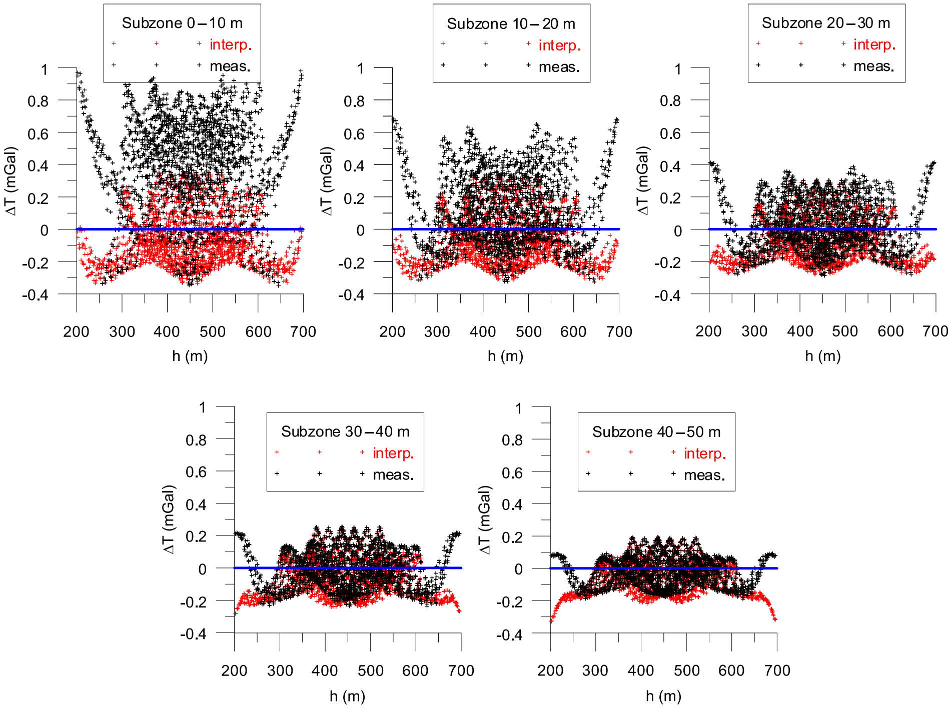

Fig. 3. Errors of terrain corrections calculation (for the density 2670 kg/m^3) for "measured" (black) and interpolated (red) heights of calculating points, respectively, within ten-metres wide subzone intervals. Blue lines indicate the zero error.

one quarter of the deformation half-wavelength (in this case about 25 m). We have to keep in mind the proposed method of analysis is only statistical, not analytical in any way. On the other hand, the test clearly showed the concept of interpolated heights is legitimate within the specified innermost zone around the calculation point.

It is also instructive to see the behaviour of terrain correction errors along a profile in Fig. 4. It displays the calculation errors obtained by both approaches for the zone 0–50 m this time. As one can see, the approach with measured heights (black marks) produces the errors of higher amplitudes, particularly in a positive sense, which is in agreement with real data experience (see below). In addition they show rugged behaviour (abrupt changes) along the profile, which is an undesirable property for gravity data inter-

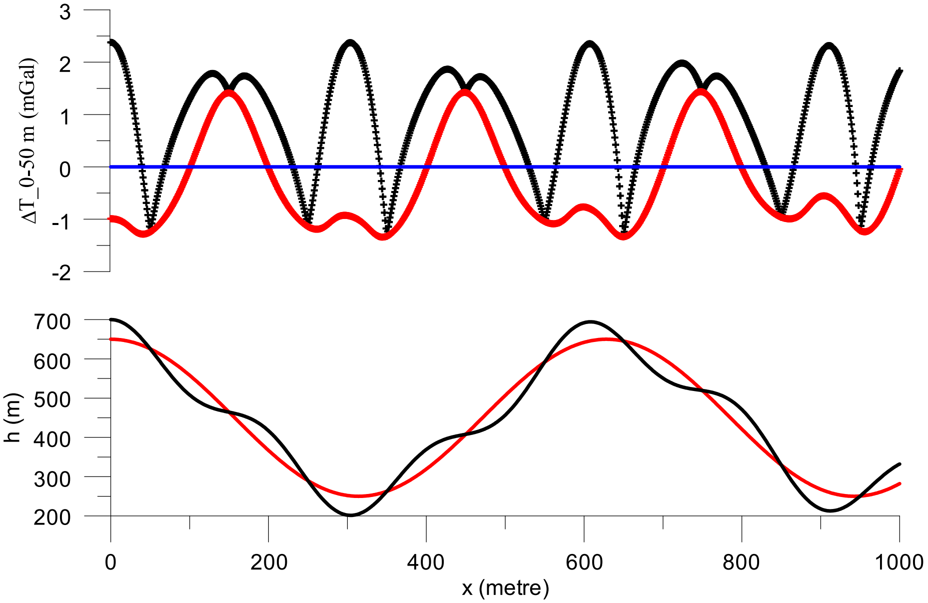

Fig. 4. Errors of terrain corrections calculation within the zone 0–50 m (upper graph, the density 2670 kg/m^3) for "measured" (black) and interpolated (red) heights of calculation points, respectively. Blue line indicates the zero error. Bottom graph shows the elevation profile, black curve represents "real" topography while the red one represents "smoothed DTM".

pretation. The periodicity of errors coincides naturally with deformation wavelength.

4. Real gravity data study

It is not easy to compare modelled results with real data because we are missing appropriate data for analysis. In reality we do not have something like ideal or ethalon DTM. As an approximation I have used a comparison of terrain corrections calculated with two different kinds of DTM in the test area of the Tatry Mountains (point positions are displayed in Fig. 8). As an approximate "ethalon" I have used terrain correction values calculated using actual detailed DTM DETAIL with the resolution of 10×10 m (*Zahorec et al., 2010; Mojzeš and Papčo, 2004*) while a tested model was an older

DTM ATLAS used during the above mentioned recalculation of the Slovak gravimetrical database (*Grand et al., 2001*). Statistical comparison of these models based on the 152 extremely situated measurement points is shown in Fig. 5. We can see the height errors of DTM DETAIL are significantly smaller, therefore I regarded this model as an "ethalon" for the purpose of this test.

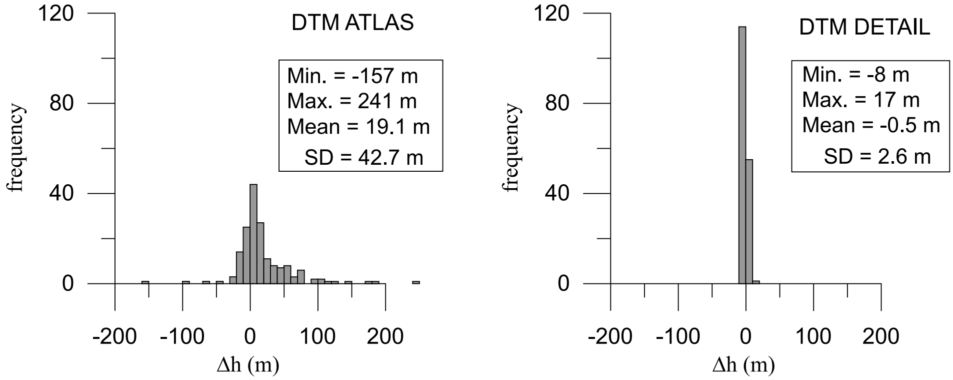

Fig. 5. Statistical comparison of two different DTMs from the Tatry Mountains based on the 152 in-situ measured gravity points covering the elevations from 919 up to 2 631 m (*Zahorec et al., 2010*).

The errors of terrain corrections T1 calculation based on the aboved mentioned models within fifty-metres wide intervals are displayed in Fig. 6. As we can see, evidently larger calculation errors were obtained for measured heights (black dots) within the zones up to 100 m. Hence on the basis of this approximating test it seems to be better to use the inner zone radius of 100 m for the calculation with interpolated heights instead of 250 m used in practice. On the other hand, since the errors resulting from the zone 100–250 m are smaller than errors from the zone 0–100 m, it is evident the use of interpolated heights for the terrain correction calculation within total zone T1 (0–250 m) is still more correct than the use of measured heights.

I have analysed the test area using a simple algorithm which finds a radius of systematic (positive or negative) height deformation of the given DTM around each calculation point. The algorithm compares the height deformations (the difference between DTM ATLAS and DETAIL in this case) at the calculation point itself and surrounding elevation grid points

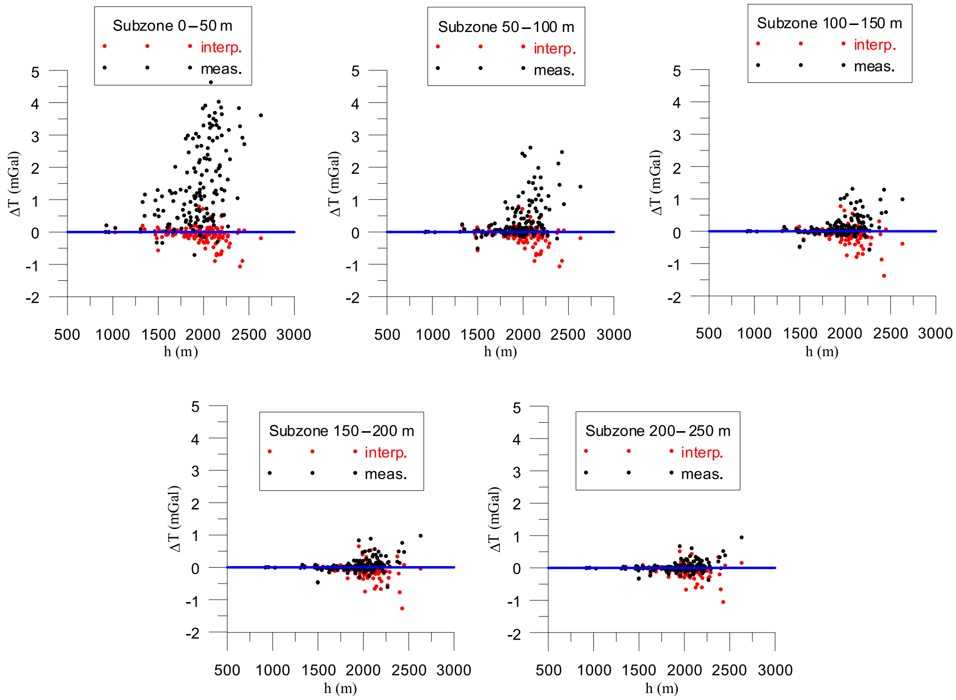

Fig. 6. Errors of terrain corrections calculation (differences between values calculated with DTM ATLAS against the ones calculated with DTM DETAIL; the density 2 670 kg/m^3) using measured (black) and interpolated (red) heights of calculation points, respectively, within fifty-metres wide subzone intervals.

and calculates a maximum distance over which the deformations are systematic (it means with the same sign). The histogram in Fig. 7 shows the results of this analysis. The searched radii lie in the range of several metres up to more then 400 m. The mean value equals approximately 80 m which roughly corresponds to calculated terrain correction errors shown in Fig. 6 (the largest errors are within the subzones up to 100 m). On the other hand, the statistical dataset of 152 points is too small to produce relevant results, because, for example, a median value of this dataset is only about 50 m, which differs from the mentioned mean value.

When we apply this simple analysis to the synthetic model situation described in previous section, we get a mean radius equal to about 22 m, which corresponds to the results shown in Fig. 3. It seems that such simple

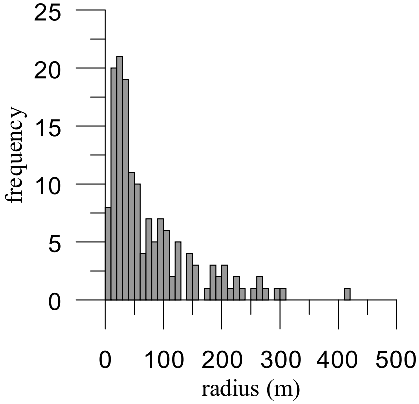

Fig. 7. Statistical results of optimal searched radius for calculation with interpolated heights in the Tatry Mountains area (152 gravity points). Mean radius equals approximately 80 m.

statistical DTM analysis can give us an estimation of appropriate radius for the calculation with interpolated heights. However it is obvious that such analysis results will depend strongly on the used DTM as well as the selected calculation area. The Tatry Mountains are the most extreme mountainous area in Slovakia, therefore it is not a representative area. On the other hand, the problem of discrepancy between the real terrain and DTM is inherent just to the mountainous areas. Therefore we could consider this analysis valid for the whole territory of Slovakia providing that we are looking for a uniform approach.

5. Examination of terrain corrections using Bouguer anomaly data

As it was mentioned before we do not have available “ethalon” terrain corrections to clearly decide which approach is the best one. However, besides the statistical tests shown in a previous section there is also another way to evaluate the correctness of the calculation approach. Since the test area of the Tatry Mountains is in geological sense (and therefore also in its rock density distribution) more or less homogeneous, composed mainly of crystalline rocks (Fig. 8), we should not expect an intensive anomaly behaviour within the Bouguer anomaly map or profiles.

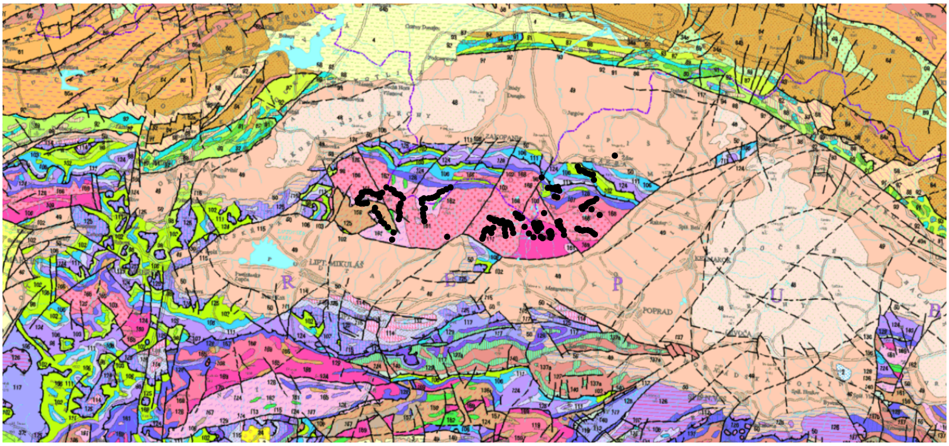

Fig. 8. Geological map of the Tatry Mountains and surrounding area (*Lexa et al., 2000*), with test gravity points (black dots).

The following graphs constructed from selected gravity point groups (daily pseudo-profiles, distance between measured points equals to several hundreds of metres) in Fig. 9 and Fig. 10 confirm this assumption. The complete Bouguer anomaly (CBA) curves calculated with terrain corrections derived from the DTM ATLAS using three different approaches, namely for measured heights, for interpolated heights within the zone 0–250 m and within the zone 0–100 m, are compared with CBA calculated using DTM DETAIL. The terrain corrections for DTM DETAIL were also calculated using mentioned three approaches, but there are very small differences among them due to small DTM errors (maximum error of about ±3 m). We can see the CBA calculated using DTM DETAIL exhibit expected smooth behaviour. On the contrary the CBA calculated using the DTM ATLAS exhibit false anomalies while the approach with measured heights gives much worse results than approaches with interpolated heights within the given inner zone. This is also in agreement with the synthetic test results displayed in Fig. 4. The elevation errors of the DTM ATLAS (differences between real heights and the model) achieve several tens of metres, maximum 129 m.

In addition we can also consider which inner zone radius for use of interpolated heights is better, 250 m or 100 m. I have chosen purposely two examples presented in Fig. 9 and 10, the former indicates the radius equal

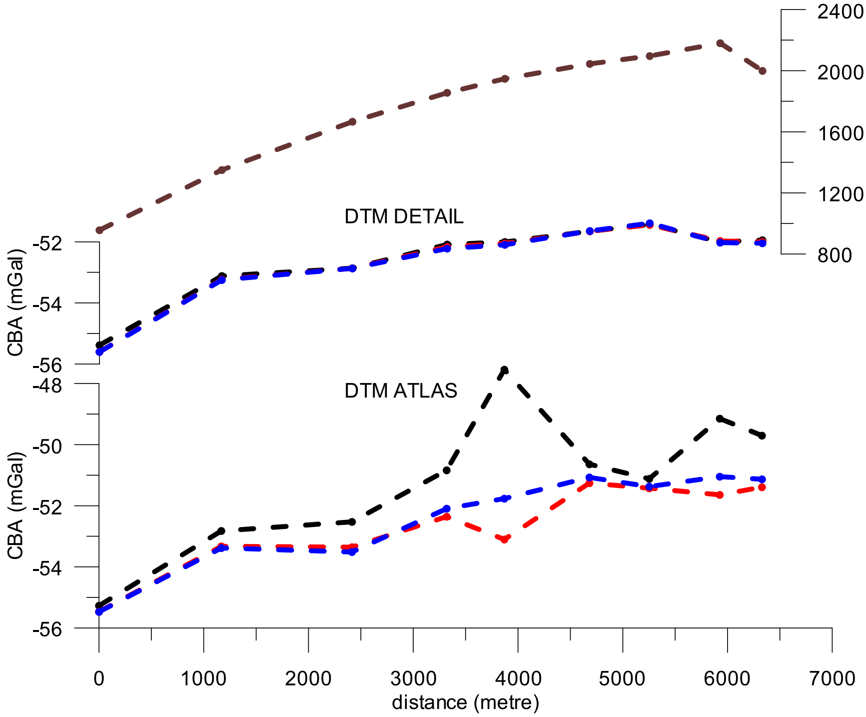

Fig. 9. Complete Bouguer anomaly (correction density 2670 kg/m^3) calculated using three different approaches of terrain correction calculation from two different DTMs. Black lines represent the approach with measured heights of calculating points, red lines represent approach with interpolated heights within the zone 0–250 m and blue ones represent approach with interpolated heights within the zone 0–100 m. Upper brown line shows gravity points elevations.

to 100 m should be preferable (it confirms the results displayed in Fig. 6) but the latter does not. This implies we can consider the searched radius only as a general approximation. However the majority of points (not shown here) indicate the radius of 100 m is generally more appropriate than the radius of 250 m, which was used during the recalculation of gravimetrical database.

The presented graphs show the problem of the heights on the regional scale is inherent particularly to the previously used DTMs, such as the DTM ATLAS. The actual models are much more accurate, but there is still the

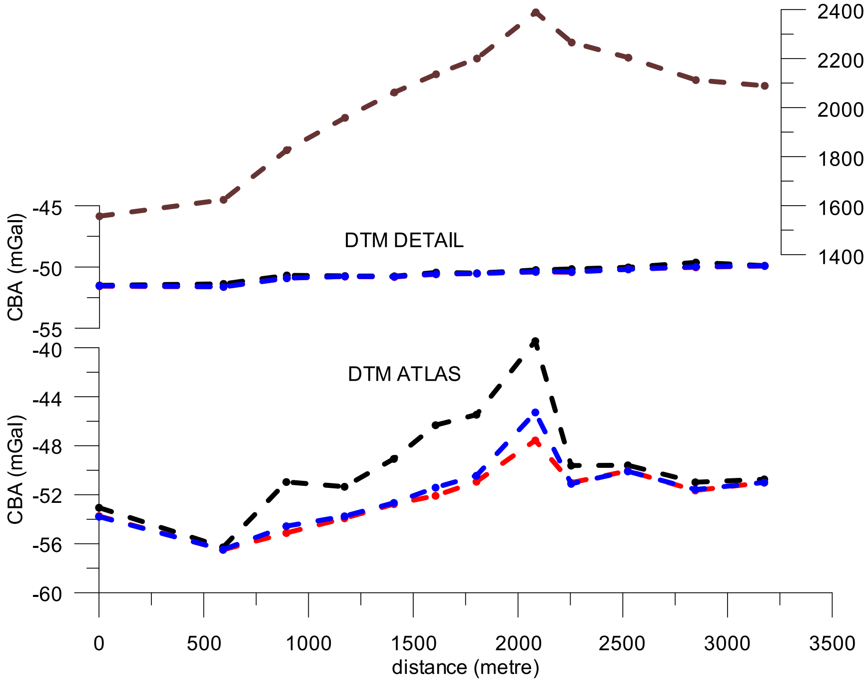

Fig. 10. Complete Bouguer anomalies along a different profile. For explanations see Fig. 9.

similar problem in the local surveys. Fig. 11 shows an example of detailed gravity measurements (distance between measured points equals to 10 m) along a profile crossing the Malé Karpaty Mountains in Slovakia. The terrain corrections were calculated using the actual model DMR-3 (*TOPÚ, 2012*) with resolution 10 × 10 m. This model is of similar quality to the DTM DETAIL used in previous tests. The height errors on profile points do not exceed ±10 m. As we can see, the approach with measured heights produces again the high-frequency false anomalies while both approaches with interpolated heights lead to a relatively smooth CBA curve. In addition we see there is only very little difference between results for the radius of 250 m and 100 m, respectively, for interpolated heights. This means the search for the correct radius does not play so important role in this case but the use of interpolated heights instead of measured ones is still relevant.

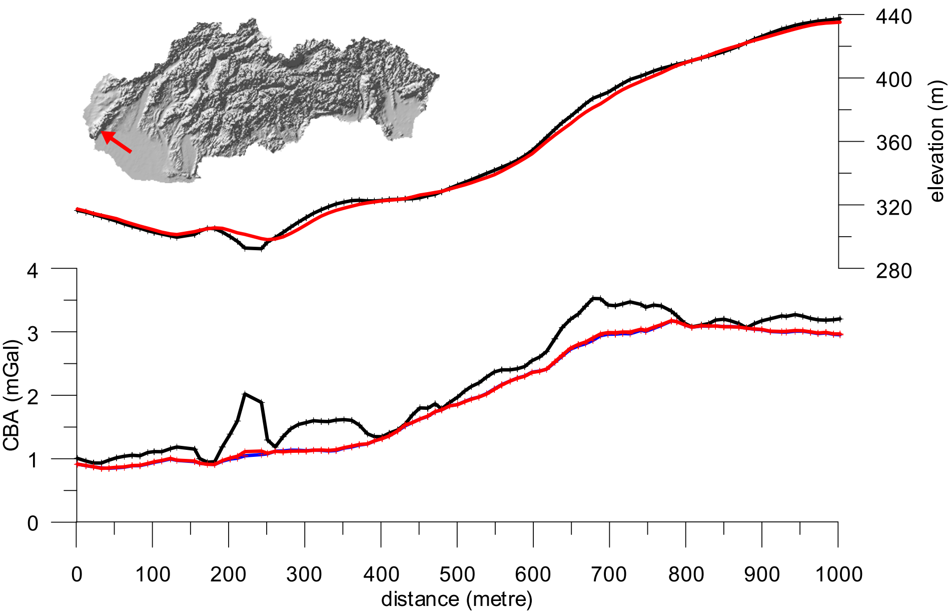

Fig. 11. Complete Bouguer anomaly (correction density 2 670 kg/m^3) on detailed gravity profile (bottom graph) compared with elevations (upper graph). Terrain corrections were calculated using three different approaches: with measured heights of calculation points (black line), with interpolated heights within the zone 0–250 m (red line) and with interpolated heights within the zone 0–100 m (blue line; this line is almost identical with the red one). Black line in the upper graph shows measured heights and red one shows interpolated heights. Relief map of Slovakia with the location of the profile shown at upper left corner.

6. Remarks on the calculation with interpolated heights

Using an interpolated height during the terrain correction calculation we suppose implicitly that the terrain correction is a relative quantity, which depends only on relative terrain undulations around the point and does not depend on the absolute elevation. However when we calculate topographic effect itself, we should also keep the true (measured) height of the calculation point, because the topographic effect is a height-dependent quantity. In such a case we have to "move" the DTM instead of the point. This is possible either by "shifting" the DTM during the calculation (just the zone for which we use the concept of interpolated heights) or by simple

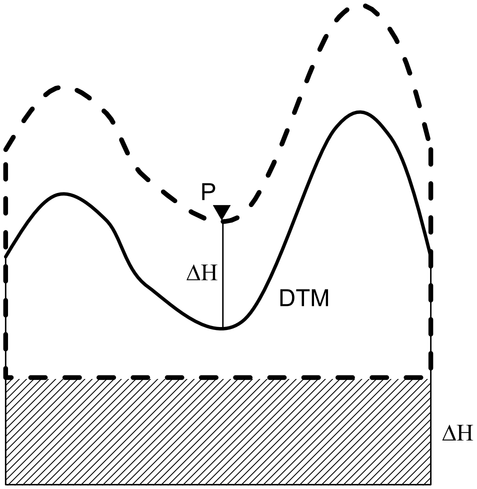

Fig. 12. Topographic effect correction by the effect of vertical cylinder (shaded). ΔH represents the discrepancy between real position of calculation point P and DTM.

follow-up correction. Such correction is equal to the gravitational effect of vertical cylinder with radius equal to the zone in question (e.g. 250 m, 100 m...) and height equal to the difference between measured and interpolated height (Fig. 12). This gravitational effect is calculated for the point P lying above the vertical cylinder in the height equal to the measured or interpolated height, depending on whether the height difference ΔH is negative or positive. Consequently the correction is also negative or positive.

In the case of precise calculation of topographic effect or terrain correction (studies focused on vertical gradient of gravity estimation, microgravity surveys) it is necessary to keep the actual position of the gravity meter sensor in regard of topography. In such cases the calculation point is not "lying" just on the DTM, but for example, about 0.25 m above it, which is the usual sensor height of Scintrex CG-5/3 gravity meters above the ground.

7. Conclusions

I have proved that the concept of using interpolated heights of calculation points instead of measured ones within the specified inner zone is a legitimate approach to terrain correction calculation. The test with synthetic topography models showed the appropriate radius of the zone for use of interpolated heights is proportional to deformation wavelength of the model. Statistical analysis of previously used lower quality DTM on real data from the Tatry Mountains test area indicates the searched radius should be about 100 m rather than the 250 m used in practice in Slovakia nowadays. On the other hand it is not possible to say in general which radius is optimal, it depends on the particular DTM as well as the calculation area. One could say, that if more accurate (detailed) DTM is available, the smaller radius should be used. Qualitative examination of complete Bouguer anomaly profile data showed that the use of measured heights leads to the origination of high-frequency false anomalies. The errors are smaller but still relevant in the case of local (microgravity) surveys while the size of the searched radius does not play so important a role in this case. Therefore I think the use of interpolated height of calculation point within the inner zone (with the radius of 100 m or similar) is the best practice in terrain correction calculation using present-day DTMs.

The use of interpolated heights is not completely legitimate when calculating the topographic effect itself seeing that this effect is a height-dependent quantity. In such a case the calculation with interpolated height can be subsequently corrected by a simple method using the effect of a vertical cylinder.

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