Analysis of precision and accuracy of Precipitable Water Vapour derived from GPS observations

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Abstract: A Precipitable Water Vapour (PWV), one of the parameters defining an instant state of troposphere, can be derived from the combination of GPS observations and ground meteorological measurements. The precision of the PWV values depends on many variables used in the computation. Their influence is discussed and analysed. The accuracy of the GPS based PWV values is estimated by comparison to the radiosonde based PWV values within the period 2004–2007. Due to different behaviour of the PWV with respect to time of the year, the analysis is performed in each season separately.

Key words: GPS, Precipitable Water Vapour (PWV), radiosonde measurements, precision, error propagation law, accuracy

1. Introduction

Precipitable Water Vapour (PWV), one of the parameters defining the instant state of the troposphere, can be modelled using the combination of GPS observations and ground meteorological measurements (*Igondová and Hefty, 2008*). PWV values are produced once per hour what is several-fold more than other methods can provide.

Accuracy as the degree of closeness of a measured or calculated quantity to its actual (true) value and precision also called reproducibility or repeatability, the degree to which further measurements or calculations show the same or similar results (*Taylor, 1999*) are discussed. Limiting factors of the GPS based PWV precision are analysed using the error propagation law. By comparing to the radiosonde based PWV values, we can estimate the accuracy and the stability of PWV derived using GPS observations.

2. Determining Precipitable Water Vapour from GPS observations and surface meteorological measurements

Zenith Total Delay - ZTD caused by the troposphere can be divided into the hydrostatic (also called dry) component *ZHD* (Zenith Hydrostatic Delay) and the wet component ZWD (Zenith Wet Delay). The large part of the delay (typically about 90 percent, which corresponds to delay about 2.3 m in zenith direction) is due to the hydrostatic component of the air (mainly nitrogen and oxygen). *ZHD* [m] can be easily derived from the surface pressure p [hPa], latitude φ and ellipsoidal height H_{el} [km] using the Saastamoinen model (*Saastamoinen, 1972*).

$$ZHD = \frac{0.002277 \cdot p}{1 - 0.00266 \cdot \cos(2\varphi) - 0.00028 \cdot H_{el}} \tag{1}$$

Having ZTD [m] as a result from processing of the network of permanent GNSS stations Zenith Wet Delay [m] is computed as

$$ZWD = ZTD - ZHD. \tag{2}$$

Precipitable Water Vapour [m] can be computed using the dimensionless proportional constant κ'

$$PWV = \kappa' \cdot ZWD, \tag{3}$$

$$\kappa' = \frac{10^8}{\rho \cdot R_v \cdot (c_3/T_m + c'_2 - m \cdot c_1)}, \tag{4}$$

where $c_1 = (77.604 \pm 0.014)\,\mathrm{K} \cdot \mathrm{hPa}^{-1}$, $c'_2 = (17 \pm 10)\,\mathrm{K} \cdot \mathrm{hPa}^{-1}$, $c_3 = (3.776 \pm 0.004) \cdot 10^5\,\mathrm{K}^2 \cdot \mathrm{hPa}^{-1}$, $m = 0.62198$ is ratio of molar weight of water vapour to molar weight of dry air, $\rho = 998\ \mathrm{kg} \cdot \mathrm{m}^{-3}$ is density of water and $R_v = R/M_w$ is the specific gas constant for water vapour, $R = 8.314472 \cdot 10^3\ \mathrm{J} \cdot \mathrm{K}^{-1} \cdot \mathrm{mol}^{-1}$ is gas constant and $M_w = 18.0153\ \mathrm{kg} \cdot \mathrm{mol}^{-1}$ is molecular weight of water vapour. Weighted “mean temperature” of the atmosphere T_m [K] is within latitude range of 27° to 65° and a height range of 0 to 1.6 km given by linear regression (*Bevis et al., 1992*)

$$T_m = 70.2 + 0.72 \cdot T_s, \tag{5}$$

where T_s [K] is the surface temperature. The value T_m is determined with the relative error less than 2%, which corresponds to approximately 5 K.

The numerator in the original formula for computing κ' (*Bevis et al., 1992*) was multiplied by 10^2 because of the conversion of hPa unit to SI units and it leads to currently used numerator 10^8 in formula (**??**).

Considering formulae (**??**) – (**??**) we can form the complete formula for determining Precipitable Water Vapour [m]

$$PWV = \frac{10^8}{\rho \cdot R_v \left(\frac{c_3}{T_m} + c_2' - m \cdot c_1\right)} \cdot \left[ZTD - \frac{0.002277 \cdot p}{1 - 0.00266 \cdot \cos(2\varphi) - 0.00028 \cdot H_{el}}\right]. \tag{6}$$

3. Precision of GPS based *PWV* values

To determine the standard deviation, the characteristic of the precision of any computed value, we can apply the error propagation law

$$\sigma_\Theta^2 = \left(\frac{\partial f(x_1, \ldots x_n)}{\partial x_1}\sigma_{x_1}\right)^2 + \ldots + \left(\frac{\partial f(x_1, \ldots x_n)}{\partial x_n}\sigma_{x_n}\right)^2, \tag{7}$$

where $\Theta = f(x_1, \ldots x_n)$ defines the estimated parameter Θ as a function of the measured data $\mathbf{x} = (x_1, x_2, \ldots x_n)^{\mathrm{T}}$.

The estimated parameter in our analysed case is the PWV. We have to consider that only a part of input data of (**??**) are obtained from the observations. Values c_1, c_2', c_3 are laboratory determined, mean temperature of the atmosphere is computed using the general formula derived from the regression analysis and the value of ZTD results from the processing of a GNSS network. The original GNSS observations are two phase and two code observations for at least 4 up to 15 satellites every 30 seconds, forming original observed data for each station. Therefore we use the final computed values with their precision (standard deviation - σ) in our analysis although this approach is not exactly correct and bring some uncertainty into our results.

There are values we consider constant and known exactly with the zero σ

– the density of water ρ, the ellipsoidal height H_{el}, the latitude φ, the specific gas constant for water vapour R_v and m. Applying the error propagation law (**??**) the *precision of the* PWV is given by

$$\sigma_{PWV}^2 = \left(a_3 \cdot \frac{c_3}{T_m^2} \cdot \sigma_{T_m}\right)^2 + \left(-\frac{a_3}{T_m} \cdot \sigma_{c_3}\right)^2 + (-a_3 \cdot \sigma_{c_2})^2 + + (a_3 \cdot m \cdot \sigma_{c_1})^2 + \left(\frac{a_1}{\rho} \cdot \sigma_{ZTD}\right)^2 + \left(-\frac{a_1}{\rho} \cdot a_2 \cdot \sigma_p\right)^2, \quad (8)$$

where a_i, $i = 1 \ldots 3$ are defined as

$$a_1 = \frac{10^8}{R_v \left(\frac{c_3}{T_m} + c_2' - m \cdot c_1\right)^2}, \quad (9)$$

$$a_2 = \frac{0.002277}{1 - 0.00266 \cdot \cos(2\varphi) - 0.00028 \cdot H_{el}}, \quad (10)$$

$$a_3 = \frac{a_1 \cdot (ZTD - a_2 \cdot p)}{\rho}. \quad (11)$$

The reference values $\varphi = 45°$, $H_{el} = 0.6$ km, $T_m = 270$ K, $\sigma_{T_m} = 5$ K, $p = 950$ hPa, $\sigma_p = 0.2$ hPa, $ZTD = 2.22$ m and $\sigma_{ZTD} = 0.002$ m were used for the following analysis. Values R_v, ρ, m, c_1, σ_{c_1}, c_2', $\sigma_{c_2'}$, c_3 and σ_{c_3} are generally accepted constants.

Influence of the individual standard deviations on the final σ_{PWV} is summarised in Table 1.

A processing of permanent GPS stations results in ZTD values determined with the precision generally from 1 to 3 mm and the values higher than 10 mm are usually excluded as the outliers. On-site ground measurement of the air pressure at GPS stations is realized with precision better or equal to 0.5 hPa (*website EUREF*). It means that for the mid-latitude stations the uncertainty of ZTD and pressure values do not contribute to the final uncertainty of PWV significantly. On the other hand, the uncertainty of T_m has a great impact on the final uncertainty of PWV and it is the most limiting factor of the final PWV precision.

From Table 2 we can see that the component corresponding to T_m has the highest influence on final σ_{PWV}^2 value. Components corresponding to

Table 1. The influence of individual standard deviations on the final σ_{PWV}. The PWV for the reference values is 9.105 mm and $\sigma_{PWV} = 0.186$ mm

i	RMS_i	RMS_{PWV}
ZTD	0.5 – 10 mm	0.185598 – 0.185602 mm
p	0.1 – 1 hPa 2 – 10 hPa	0.185598 mm 0.185599 – 0.185678 mm
T_m	0.5 – 5 K	0.0705 – 0.1856 mm

Table 2. The values of $\left(\frac{\partial f(x_1,\dots x_n)}{\partial x_i}\sigma_{x_i}\right)^2$ if the ZTD varies from 2.18 to 2.35 m and T_m from 260 to 280 K corresponding to PWV values from 0 to 35 mm and σ_{PWV} from 0.05 to 0.7 mm

x_i	$\left(\frac{\partial f(x_1,\dots x_n)}{\partial x_i}\sigma_{x_i}\right)^2$
T_m	$10^{-1} - 10^{-3}$
c_1	$10^{-8} - 10^{-10}$
c_2'	$10^{-2} - 10^{-4}$
c_3	$10^{-3} - 10^{-6}$
ZTD	10^{-10}
p	10^{-9}

ZTD and p have insignificant influence irrespective of absolute PWV value.

With respect to the results in Table 1 and Table 2 we can simplify the form of (**??**) to

$$\sigma_{PWV}^2 = \left(a_3 \cdot \frac{c_3}{T_m^2} \cdot \sigma_{T_m}\right)^2 + \left(-\frac{a_3}{T_m} \cdot \sigma_{c_3}\right)^2 + (-a_3 \cdot \sigma_{c_2})^2 + (a_3 \cdot m \cdot \sigma_{c_1})^2$$
$$= a_3^2 \cdot \left[\left(\frac{c_3}{T_m^2} \cdot \sigma_{T_m}\right)^2 + \left(\frac{\sigma_{c_3}}{T_m}\right)^2 + (\sigma_{c_2})^2 + (m \cdot \sigma_{c_1})^2\right], \quad (12)$$

with no loss of the final precision.

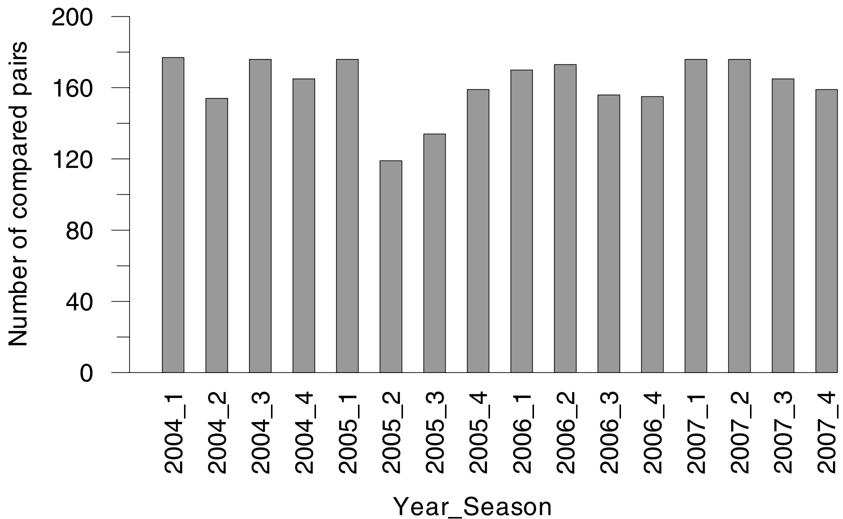

Fig. 1. The number of compared *PWV* pairs within each season of years 2004–2007.

4. Accuracy of GPS based *PWV* values

The accuracy of the *PWV* values derived from GPS observations and surface meteorological measurements is estimated by comparing them to the radiosonde derived *PWV* values. More than 2500 couples of *PWV* within the 4 year period (2004–2007) are included (see Fig. 1). Due to different behaviour of the *PWV* with respect to time of the year, the analysis has been performed in each season separately. The astronomical seasons have been considered (spring: Day of Year DOY 080 to 171, summer: DOY 172 to 265, autumn: DOY 266 to 355, winter: DOY 356 to 079). The comparison has been done using the data from the station GANP (Gánovce near Poprad, Slovakia) where the GNSS observations and the radiosonde measurements are taken at the same place.

The radiosonde based Precipitable Water Vapour $PWV_{radiosonde}$ has been computed by integrating the radiosonde profile using the measured pressure, dry-bulb temperature and dew-point temperature of the radiosonde. Data are available at 0^{h} and 12^{h} UTC (Universal Time Coordinated) which corresponds to 1^{h} and 13^{h} CET (Central European Time) and 2^{h} and 14^{h} CEST (Central European Summer Time), respectively.

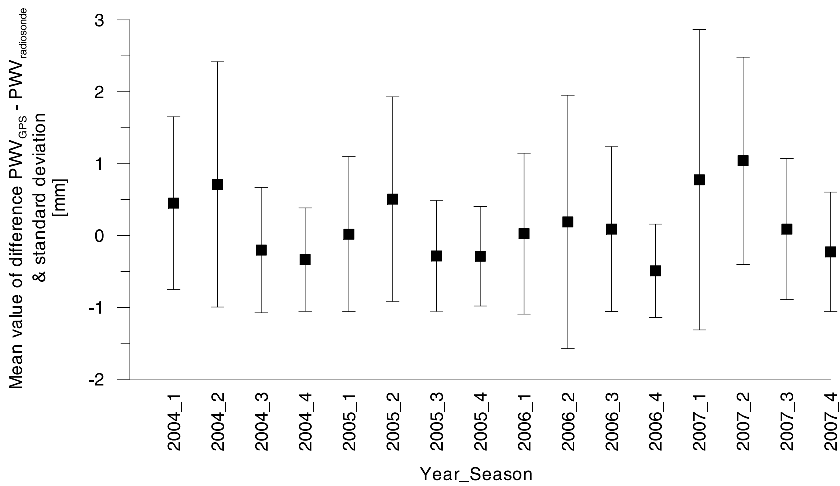

Fig. 2. Mean values of differences between the PWV_{GPS} and the $PWV_{radiosonde}$ with the standard deviation bars within each season of period 2004–2007.

The ZTD has been determined from processing of about 50 permanent GPS stations located in Central Europe using the software Bernese v.5.0 (*Hefty and Igondová, 2008*). The ZHD has been computed from the observed surface pressure using the Saastamoinen model (**??**). Then the GPS based Precipitable Water Vapour PWV_{GPS} has been computed from the formula (**??**) in one hour interval.

The mean value of the difference $\Delta = PWV_{GPS} - PWV_{radiosonde}$

$$\bar{\Delta} = \frac{\sum_{n} \Delta}{n} \tag{13}$$

and the corresponding standard deviation of the difference

$$\sigma_{\bar{\Delta}} = \sqrt{\frac{\sum_{n} \left(\Delta - \bar{\Delta}\right)^2}{n-1}} \tag{14}$$

vary according to season and time of the day as shown in Figs. 2 and 3. During the summer period the differences reach the positive and significantly higher value in comparison to other seasons. The negative values of differences occurred periodically during the winter period. Different behaviour

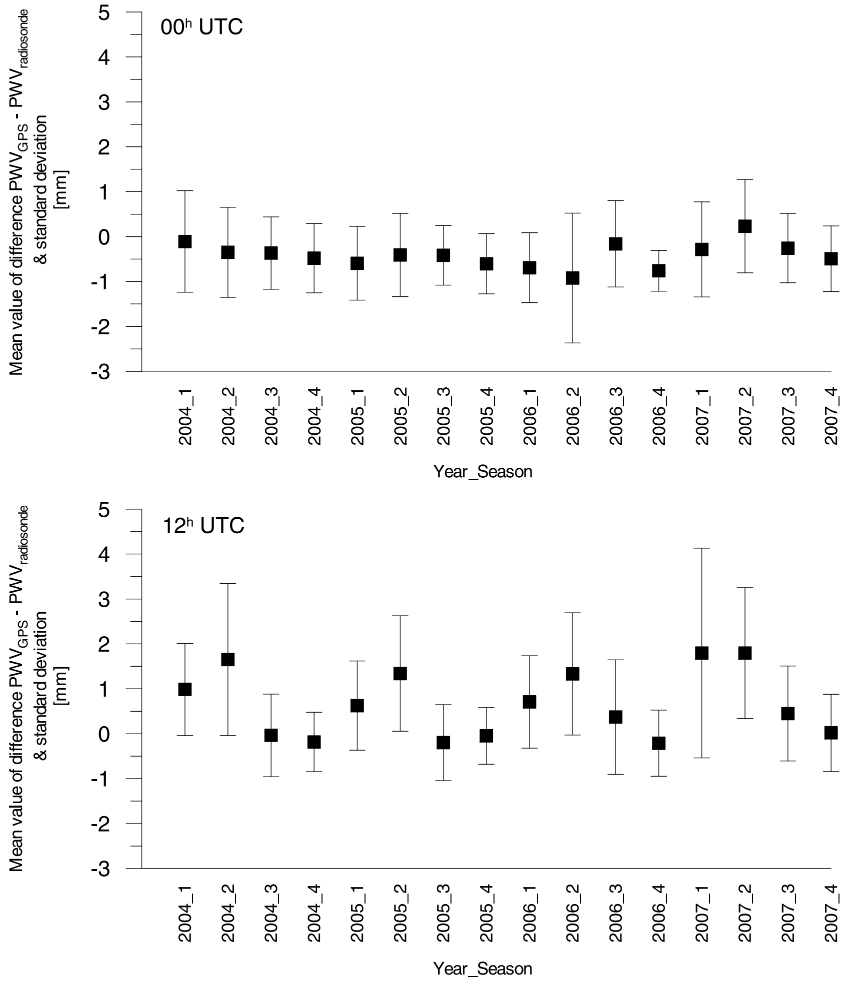

Fig. 3. Mean values of differences between the PWV_{GPS} and the $PWV_{radiosonde}$ with the standard deviation bars at 0^h and 12^h UTC within each season of period 2004–2007.

is also clearly visible when midnight and noon values are investigated separately. Except for one case (summer of 2007) all differences determined at 0^h UTC are negative. The differences at 12^h UTC vary mostly according to season and are almost all positive.

The higher values of differences and standard deviations are generally

Table 3. Average differences between PWV_{GPS} and $PWV_{radiosonde}$ and corresponding standard deviations

Years 2004 - 2007		all	spring	summer	autumn	winter
All data	$\bar{\Delta}$ [mm]	0.135	0.321	0.620	-0.071	-0.335
	$\sigma_{\bar{\Delta}}$ [mm]	1.280	1.374	1.593	0.946	0.724
00^h UTC	$\bar{\Delta}$ [mm]	-0.415	-0.415	-0.366	-0.290	-0.582
	$\sigma_{\bar{\Delta}}$ [mm]	0.935	0.950	1.126	0.814	0.658
12^h UTC	$\bar{\Delta}$ [mm]	0.560	1.042	1.544	0.142	-0.102
	$\sigma_{\bar{\Delta}}$ [mm]	1.369	1.356	1.457	1.020	0.722

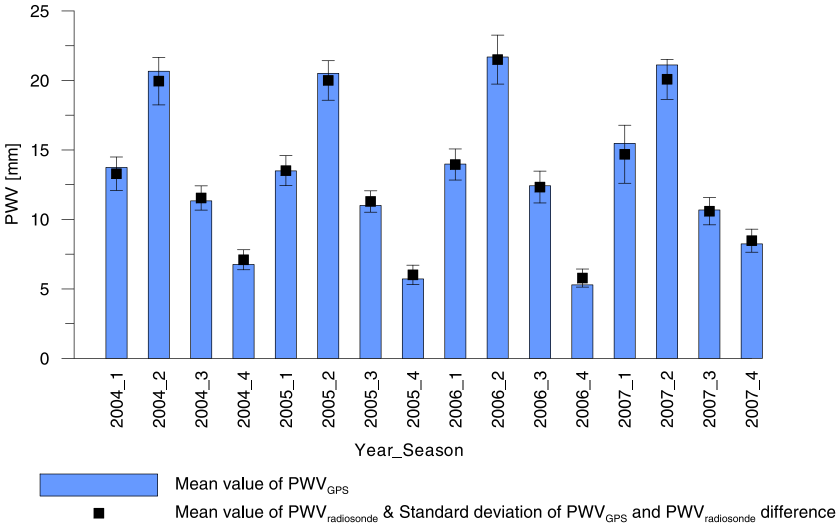

Fig. 4. Mean values of PWV_{GPS} compared to $PWV_{radiosonde}$ and standard deviations of their differences within each season of the period 2004–2007.

connected with the higher values of PWV (see Fig. 4). It is caused by the higher difference in the summer period, mainly at 12^h UTC observations (see Fig. 3).

The average differences between the PWV_{GPS} and $PWV_{radiosonde}$ and corresponding standard deviations for all period and also for midnight and noon measurements separately are summarized in Table 3.

Precision of PWV_{GPS} is given by (**??**) and it varies within 1 mm. We investigate whether the value $PWV_{radiosonde}$ belongs to interval $\langle PWV_{GPS} -$

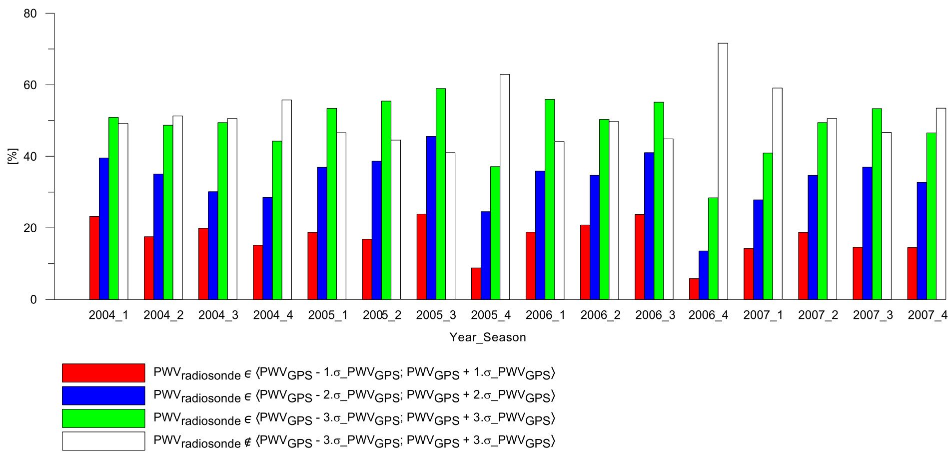

Fig. 5. Percentage of $PWV_{radiosonde}$ data belonging to interval $\langle PWV_{GPS} - i \cdot \sigma_{PWV_{GPS}}; PWV_{GPS} + \ i \cdot \sigma_{PWV_{GPS}} \rangle$, $i = 1, \dots 3$ within each season of the period 2004–2007.

$- t \cdot \sigma_{PWV_{GPS}} \ \ PWV_{GPS} + t \cdot \sigma_{PWV_{GPS}} \rangle$, $t = 1, \dots 3$ or not (see Fig. 5). Only about 15% of $PWV_{radiosonde}$ values belong to interval $PWV_{GPS} \pm 1 \cdot \sigma_{PWV_{GPS}}$ and 50%, on average, belong to interval $PWV_{GPS} \pm 3 \cdot \sigma_{PWV_{GPS}}$. Magnitude of the PWV_{GPS} and $PWV_{radiosonde}$ difference and $\sigma_{PWV_{GPS}}$ varies according to season what is clearly visible if we investigate the number of $PWV_{radiosonde}$ data belonging to interval $\langle PWV_{GPS} - t\,\mathrm{mm};\ PWV_{GPS} + + \ t\,\mathrm{mm} \rangle$, $t = 1, \dots 3$ (see Fig. 6).

Figure 6 shows that 40 to 85% of the $PWV_{radiosonde}$ data belong to the interval of $PWV_{GPS} \pm 1$mm. Percentage of data belonging to intervals is the lowest during winter period and the highest during summer period. Therefore higher number of data belongs to the winter interval. The interval $PWV_{GPS} \pm 3$mm covers 90 to 100% of data regardless of the season.

5. Conclusion

The formula (**??**) for computing a precision of GPS based PWV values was derived applying an error propagation law. Analysis of the influence

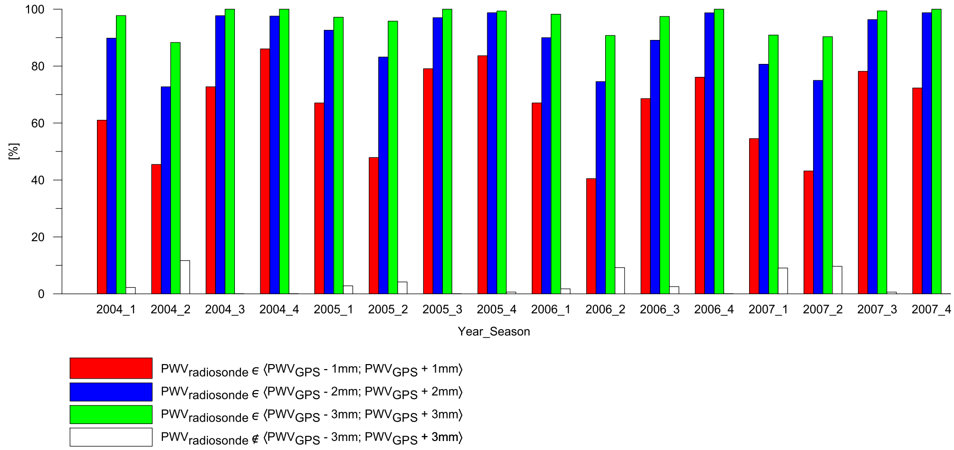

Fig. 6. Percentage of the $PWV_{radiosonde}$ data belonging to interval $\langle PWV_{GPS} - -i\,\mathrm{mm};\ PWV_{GPS} +\ i\,\mathrm{mm}\rangle$, $i = 1, \ldots 3$ within each season of the period 2004–2007.

of individual standard deviation on the final σ_{PWV} shows that the limiting factor of precision is the mean temperature of the atmosphere. Assuming the results from the analysis, a simplified formula (**??**) for computing a precision of GPS based PWV values was determined.

The analysis of simultaneous results from the radiosonde and GPS data led to the accuracy assessment. A high variation in the results was observed according to season and time of day. The mean difference of 0.14 ± 1.28 mm was determined using the 4 years data set with more than 2500 couples of the PWV_{GPS} and $PWV_{radiosonde}$ data.

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