The iterative complex demodulation applied on short and long Schumann resonance measured sequences

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Abstract: The precise determination of instantaneous frequency of Schumann resonance (SR) modes, with the possibility of application to relatively short signal sequences, seems to be important for detailed analysis of SR modal frequency variations. Contrary to commonly used method of obtaining modal frequencies by the Lorentz function fitting of DFT spectra, we employ the complex demodulation (CD) method in iterated form. Results of iterated CD method applied on short and long measured sequences are compared. Results for SR signals as well as the comparison with Lorentz function fitting are presented. Decrease of frequencies of all first four SR modes from the solar cycle maximum to solar cycle minimum has been found using also the CD method.

Key words: Schumann resonances, complex demodulation, instantaneous frequency

1. Introduction

The well-known phenomenon of Schumann resonances (SR) has been a subject of monitoring at many observatories around the world. For extracting of quantitative information buried in the measured and recorded raw data, it is necessary to apply a suitable method of spectral (frequency) analysis.

The determination of instantaneous (short-term) modal frequencies from sample sequences of non-stationary signal consisting of several damped harmonic components (modes), plus wideband noise and hum, is not a simple task from both theoretical and experimental (computational) point of view. Moreover, in the case of Schumann resonances (SR) signals, which exhibit the frequency and amplitude variations also in short-time (minutes and

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shorter) scales, the choice of proper signal analysis method has no unique solution.

In order to determine the SR modal frequency variation it is essential to determine the central frequency of individual spectral peaks. The Lorentz Function Fitting (LFF) method, see numerous literature, e.g. *Madden and Thompson (1964), Williams et al. (2006)* or *Mushtak and Williams (2008)*, solves this problem directly in spectral domain as approximation of the SR spectrum by the sum of Lorentz functions. On the contrary, the Complex Demodulation (CD) method operates directly with time series and so it is completely different from the LFF method.

The CD method, as a very powerful tool for frequency analysis of signals, was described in numerous literature, see *Childers (1972), Lee and Park (1994), Draganova and Popivanov (1994).* There were many successfull applications of the CD method for analyzing the geophysical signals, e.g. *Hao et al. (1992), Myers and Orr (1995), Gasquet and Wootton (1997).* The first use of CD for SR signal can be found in *Sátori (1996), Sátori et al. (1996), Verö et al. (2000).* An iterative modification of CD method was explained in detail and directly applied to frequency analysis of SR signals (the electric field component) in *Ondrášková and Ševčík (2013).*

An important question is how to apply the CD method. There are two possibilities, either CD is applied on the longer, in our case the whole (full-length) 327.68 s long SR records or it is applied on short sub-blocks. Tests and the first results of the latter possibility were presented in *Ondrášková and Ševčík (2013).* As the faint SR electric field is sometimes disturbed by local meteorological conditions, the output of measurements or their parts are then strongly saturated. Such outputs or their parts can give values of the searched central peak frequency which cannot represent the reality. The advantage of CD method applied on the short sub-blocks lies in the fact that such values of the frequency resulting from saturated (or otherwise corrupted by extraordinary noise and hum) sub-blocks could be eliminated simply by discarding these sub-blocks.

In this paper, the results of iterative use of complex demodulation method for analyzing the real Schumann resonance signals are presented. The long-term measurements have been processed by an iterative variant of complex demodulation of our measured 327.68 s sequences, as well as on sub-blocks. Both results are compared with those obtained previously by the

LFF method. The differences are discussed.

2. Data

The SR data used in this study were obtained at the Astronomical and Geophysical Observatory of Comenius University (AGO), Modra, Slovakia. Measurements of vertical electric field component of Schumann resonances were performed in period October 2001 – July 2009. The experimental set-up is described in *Kostecký et al., (2000)* and results of processing the long-term measurements using the LFF are in *Ondrášková et al. (2007)* and in *Ondrášková et al. (2011)*.

3. Short and long data blocks

At AGO observatory the experimental data are stored in the form of 327.68 second long data blocks (every block consists of 65,536 data samples, taken at 200 Hz sampling frequency). At the receiving antenna site, situations appear rather frequently, that the noise (caused by local meteorological conditions – wind) or technogenic hum influence only very short part of a complete data block. The best remedy seems to be splitting the block into sub-blocks, process the sub-blocks separately and discard the results from contaminated sub-blocks.

Thus, there are principally two possibilities how to apply the CD method. First, it is possible to process every data blocks as a whole and as a result, we obtain a single value of peak frequency (for every SR mode). Note that the beginning and the end of data block is clamped by 5.0 s wide Hann window. Second possibility consists of splitting every data block in 16 sub-blocks (4096 samples, which corresponds to 20.48 seconds). The peak frequency is obtained as an arithmetic mean of 16 (or less, if some have to be discarded) values from "good" sub-blocks. In this case, the Hann window has to be shortened to 0.5 seconds.

The essential part of the CD method is a low-pass filter. The low-pass filter unit-impulse response was computed and stored in a length of 5000 samples (with sampling frequency of 200 Hz it corresponds to 25 seconds).

When the CD method is applied on short sub-blocks only 1000 samples are used in convolution calculations. If the CD method is applied on the whole 327.68 s long data blocks all 5000 samples of the filter can be used to receive precise results. Unfortunately, calculations are very time consuming. We tested the influence of the length of the low-pass filter unit impulse response on the final result on about 2000 full-length data blocks. It has been found that the same value of resulting modal frequency is obtained when 2000 samples of filter response is used.

What concerns the rate of convergence of the iterative CD method it has been found that it depends on the time length of processed signal. A frequency difference under 0.001 Hz between adjacent iterations is used as a stop criterion in our iterative CD method. Sometimes the signal is contaminated by wideband noise and/or technogenic hum. Then the CD procedure showed no convergence and the results of successive iterations monotonically decreased to unrealistic values below 1 Hz. For this reason, another criterion has to be set – a number of iterations. Test has showed that uncontaminated blocks reached the first criterion (difference < 0.001 Hz) before 10 iterations in case of short data sequences (sub-blocks) and before 30 iterations in case of whole blocks, see also *Ondrášková and Ševčík (2013)*.

Sometimes the complex demodulation procedure converged, but the resultant frequency values were lying significantly outside the reasonable physical limits – say, over the 8.5 Hz or under the 7.2 Hz for the first SR mode, (or outside the $13.0 \div 15.0$ Hz for second one, etc.). Because of their rareness, we have decided simply to discard them.

The first results of application of the iterative CD method on sub-blocks can be found in *Ondrášková and Ševčík (2013)* (Figs. 9–11 therein). Results of new calculations when the CD method has been applied on whole blocks are merged with former results and are presented here in Fig. 1. Green (light grey) points represent results of CD procedure runs applied to the sub-block and limited to 1 iteration. Red points (dark grey) show the same procedure runs but maximal number of iterations was set 5. Black points are results of the same CD procedure runs but maximal number of iterations was 10. Blue points (dark points on light-gray background) in the middle represent results of CD procedure runs applied on whole data blocks. Shown are data from 4 complete days, i.e. 240 "blue" points per day (1 data block every 6 minutes), and $240 \times 16 = 3840$ green, red and black points per day, as every

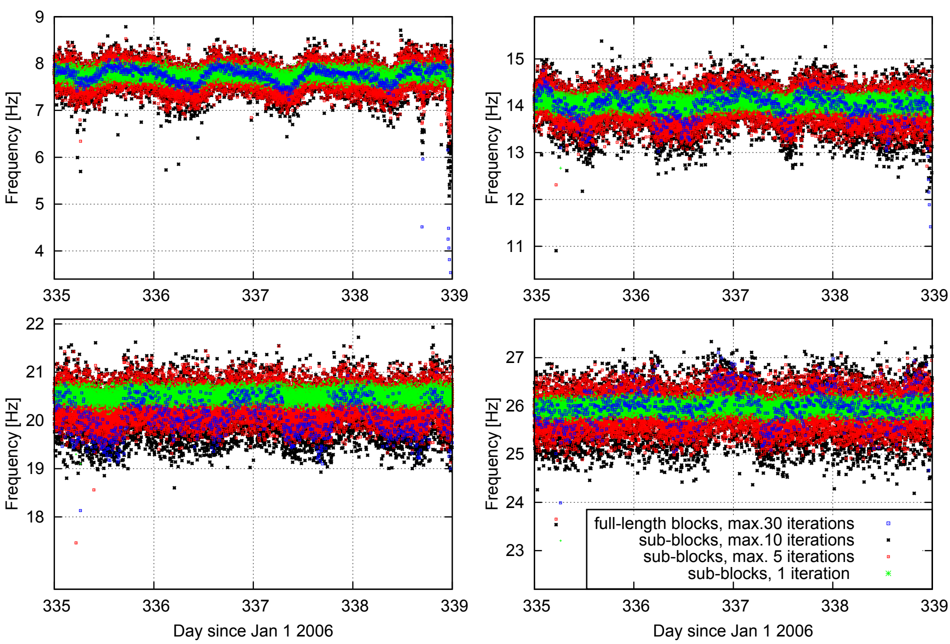

Fig. 1. The results of complex demodulation of the four Schumann modes from December 1–4, 2006. The CD method is applied on sub-blocks, i.e. all of 240 data blocks per day are divided into 16 sub-blocks and every sub-block was processed independently. Thus, there are 3840 points of sub-block for every day. Points of various colors and types differentiate between demodulation procedure runs, which were limited to either 1, 5 or 10 iterations. Moreover, results of complex demodulation applied on whole data blocks are shown as blue points (240 points per day). Note that disturbed conditions at the end of the fourth day affected results of CD of both sub-blocks as well as whole blocks.

block is divided into 16 sub-blocks. It can be seen that the final frequency of a sub-block after one iteration step does not differ from estimated frequency (initial guess) by more than 0.3 Hz. Figure 1 also shows that only small fraction of procedure runs terminated after more than 5 iterations (black points) in which the final frequency reached values below 7 Hz or over 8.5 Hz (for the 1st mode). Furthermore, Fig. 1 shows that individual modal frequencies obtained by the CD method of whole data blocks are less scattered than individual values of modal frequencies of sub-blocks. At the end of the fourth day, disturbed conditions caused that red, black as well as blue points converged to values outside Schumann resonance interval, see

Fig. 1 for the 1st and 2nd modes.

Another comparison of the two variants of the CD method can be found in Fig. 2. Black points represent arithmetic mean of 16 frequencies obtained by the CD method applied on all 16 sub-blocks where maximal number of iterations was set to 10. Blue points again represent results of the CD method applied on whole data blocks where maximal number of iterations was set to 30. The same whole (complete) data blocks were processed by the Lorentzian fitting method, i.e. the modal frequencies were determined by computing DFT spectrum which was then fitted by the least-square method by the sum of 5 Lorentz functions (*Rosenberg, 2004*) – violet points. Naturally, modal frequencies obtained as means of 16 sub-blocks vary from a

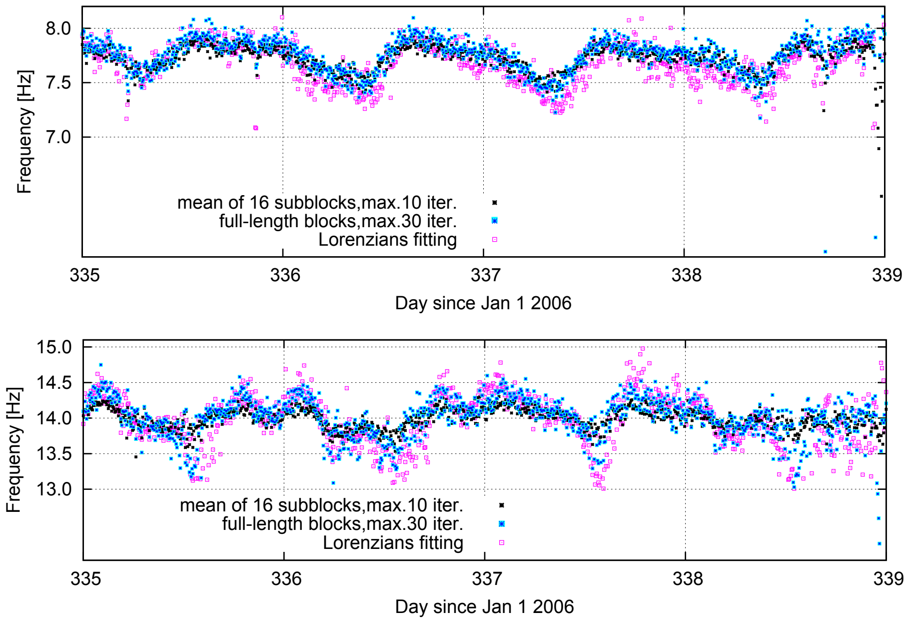

Fig. 2. Modal frequencies for days Dec. 1 to Dec. 4, 2006, obtained by 3 different methods. Black points – the iterative CD method applied on sub-blocks and subsequently averaged (240 point per day); blue points – results obtained by iterative CD method applied on whole blocks, 240 per day; violet points – results of Lorentzian function fitting method applied on the same whole data blocks, 240 per day. Upper plot is for the 1^{st} SR mode, lower plot for the 2^{nd} SR mode. Note that all methods give unreasonable frequencies below 7.2 Hz at the end of 4^{th} day.

mean value of 7.8 Hz with smallest amplitude. The plots show that, as a rule, "Lorentzian" values are most scattered of all methods. A great disturbance of unknown origin at the end of the 4^{th} day caused significant deviations of modal frequency values from usual values, which can be seen in Lorentzian values as well as in values of both variants of the CD method used in this study.

4. Daily variation of SR

One of the goals of SR analysis is determination of the daily variation and the daily frequency range (DFR) for a given month of a year. The DFR is the difference between maximal and minimal value on the monthly mean daily frequency curve. The importance of DFR lies in the fact that this quantity has direct relation to the geometrical (angular) global thunderstorm foci area (*Nickolaenko and Hayakawa, 2002; Ondrášková et al., 2011*).

As an example, monthly mean daily variation obtained using the iterative CD method applied on sub-blocks from our observatory data for January 2007 were showed in *Ondrášková and Ševčík (2013).* It was showed that daily variation obtained by CD method when maximum number of iterations was 5 is practically the same as in the case when maximum number of iterations was 10. That is why the latter variant alone with maximal number of iterations of 10 is used in the daily variation determinations presented here.

Monthly mean values of the 1st SR mode frequencies with a step of 30 minutes for year 2008 are presented in Fig. 3. Then the Bezier interpolation curve shows a smoothed monthly mean daily variation.

Results of four different methods of computation are depicted for comparison. As expected, amplitude of the daily variation determined from results after 1 iteration step of CD (strictly speaking this is not the iterative CD method) is very small in all depicted months. This is true also for year 2002 in Fig. 4, as well as for the 2^{nd} SR mode (not depicted).

The greatest DFR gives the LFF method. The CD applied on the whole data blocks (of the same length as LFF) gives daily variation of very close pattern and only slightly smaller DFR. It is interesting that in winter months the curves of daily variation obtained by these two latter methods are also

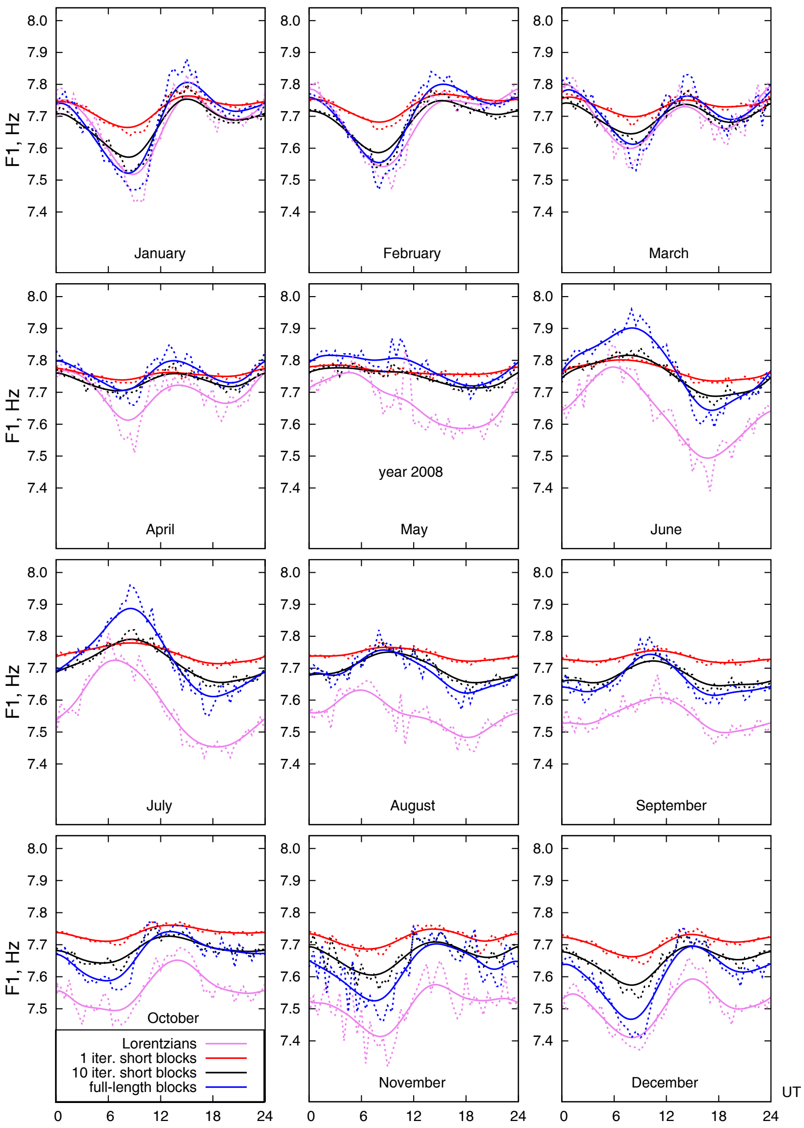

Fig. 3. Mean daily variation of the 1st SR mode in 2008 computed by 4 different methods: by CD method with 1 step (1 iteration) – red dotted lines, by CD method with maximum 10 iterations – black dotted lines, both applied on short sequences (sub-blocks) about 20 seconds long, by CD method applied on full-length blocks 327.68 seconds long – blue dotted lines, as well as by the Lorentz function fitting method – violet dotted lines. Smooth full lines are the Bezier interpolations.

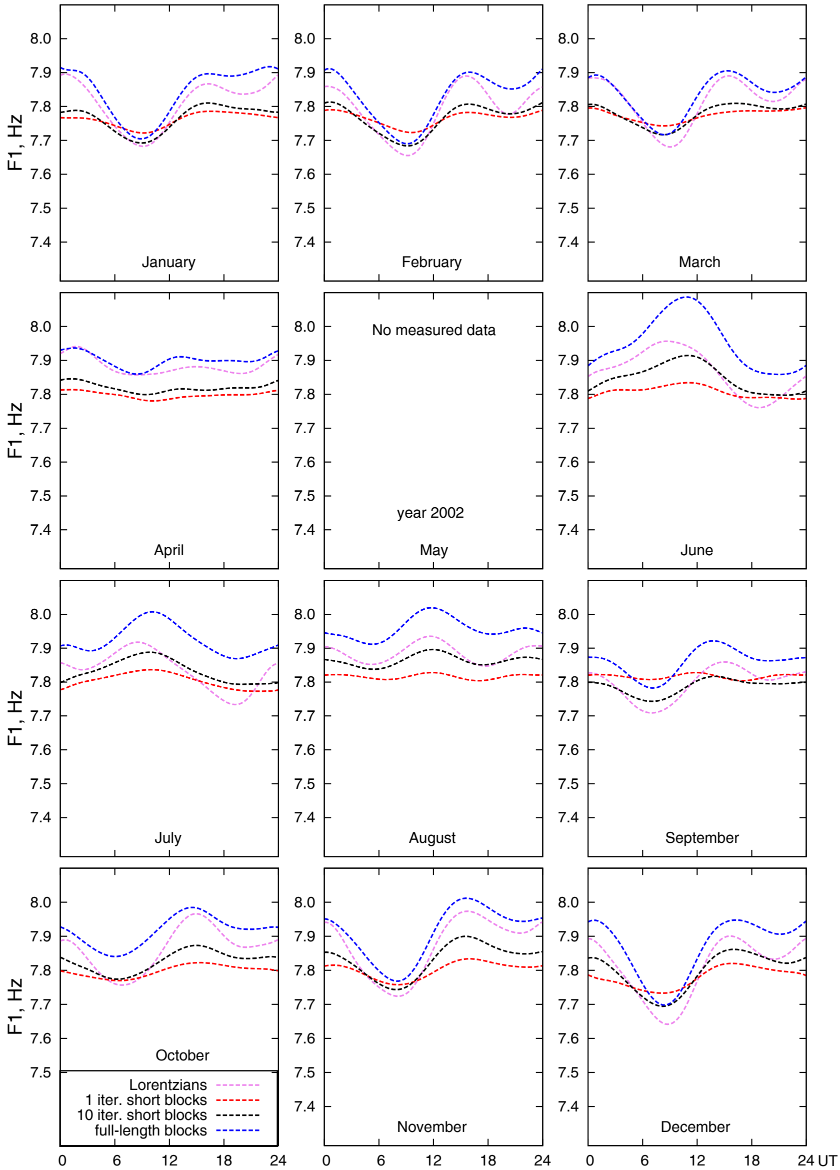

Fig. 4. Similar to Fig. 3 but for year 2002. Here only the Bezier interpolations are plotted, using dashed lines.

absolutely very close, while in other months they are afar off up to 0.15 Hz, e.g. in August 2008. This difference in both frequency level and in the DFR between the LFF and the CD methods has been found also by *Sátori (2009)*.

Daily variation curves for both years 2002 and 2008 are depicted together in Fig. 5. For a better clarity, results of 1 iteration step of CD are not depicted in these graphs. A clear decrease from 2002, a year of solar cycle maximum, to 2008 is seen for all 3 methods depicted.

Figures 6, 7 and 8 depict monthly mean daily variation of the 2$^{\text{nd}}$, 3$^{\text{rd}}$ and 4$^{\text{th}}$ SR modes. A similar conclusion for these higher modes can be made as for the 1$^{\text{st}}$ mode. The pattern of daily variation is similar for all methods, the DFR determined using the CD method is significantly smaller than that obtained by the LFF method in case of the 2$^{\text{nd}}$, 3rd and 4$^{\text{th}}$ modes.

5. Conclusions

The iterative variant of complex demodulation method was applied to whole data blocks recorded at the Astronomical and Geophysical Observatory of Comenius University in Modra, Slovakia. This CD method was then applied to shorter signal sequences obtained by splitting the whole blocks into 16 sub-blocks and the final modal frequencies computed as arithmetic means. Results of both variants were compared mutually and with the Lorentz function fitting method.

The above mentioned methods were used to reveal frequency variations in Schumann resonance signals. It was found that

– monthly mean daily variation in mode frequency determined by these methods shows up principally the same pattern;

– both the mean frequency level and daily frequency range are different when different spectral techniques or their variants are used;

– scattering of individual values and the amplitude of daily frequency variation and thus also the DFR is greatest using Lorentz function fitting method;

– modal frequencies determined by iterative complex demodulation of the whole data blocks are closer to frequencies determined by Lorentz function fitting;

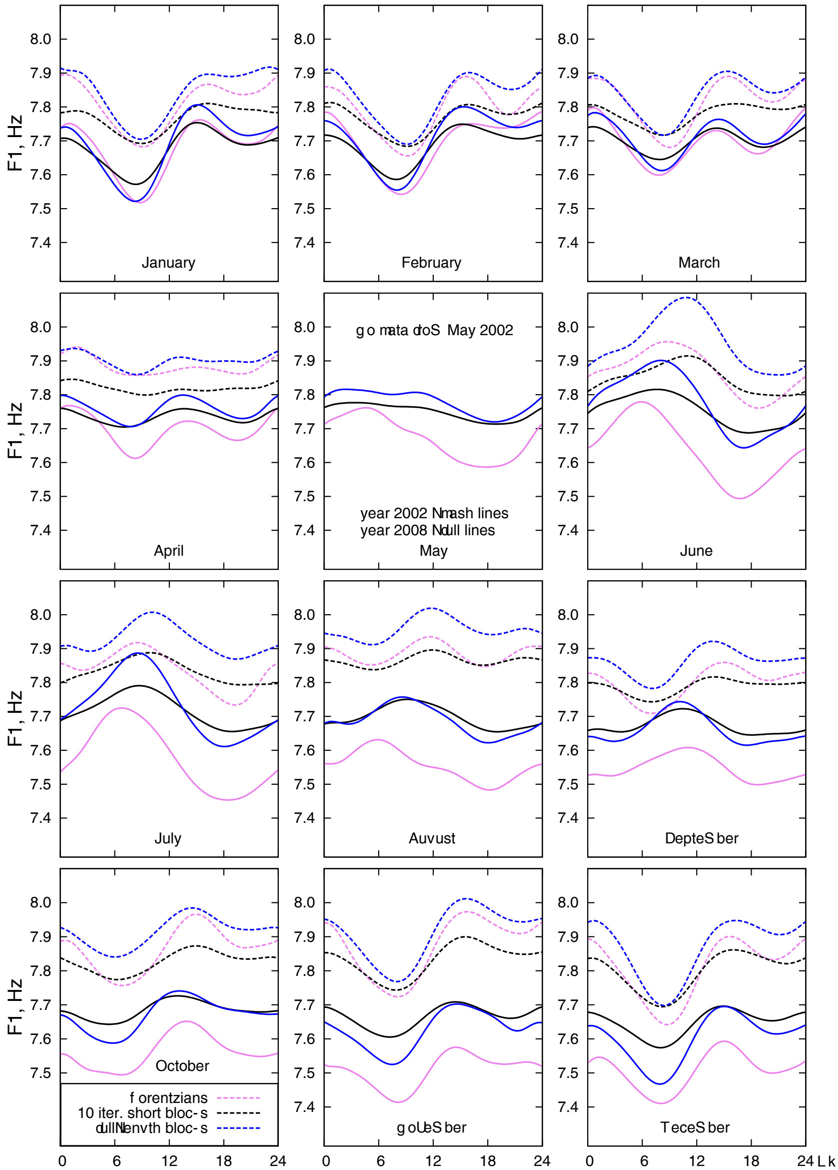

Fig. 5. Mean daily variation of the 1st SR mode for individual months. 3 different colors represent 3 different methods. The full lines are for 2008 and the dashed lines are for 2002. The decrease of modal frequency from year 2002 (solar maximum) to year 2008 (solar minimum) is evident using all discussed methods.

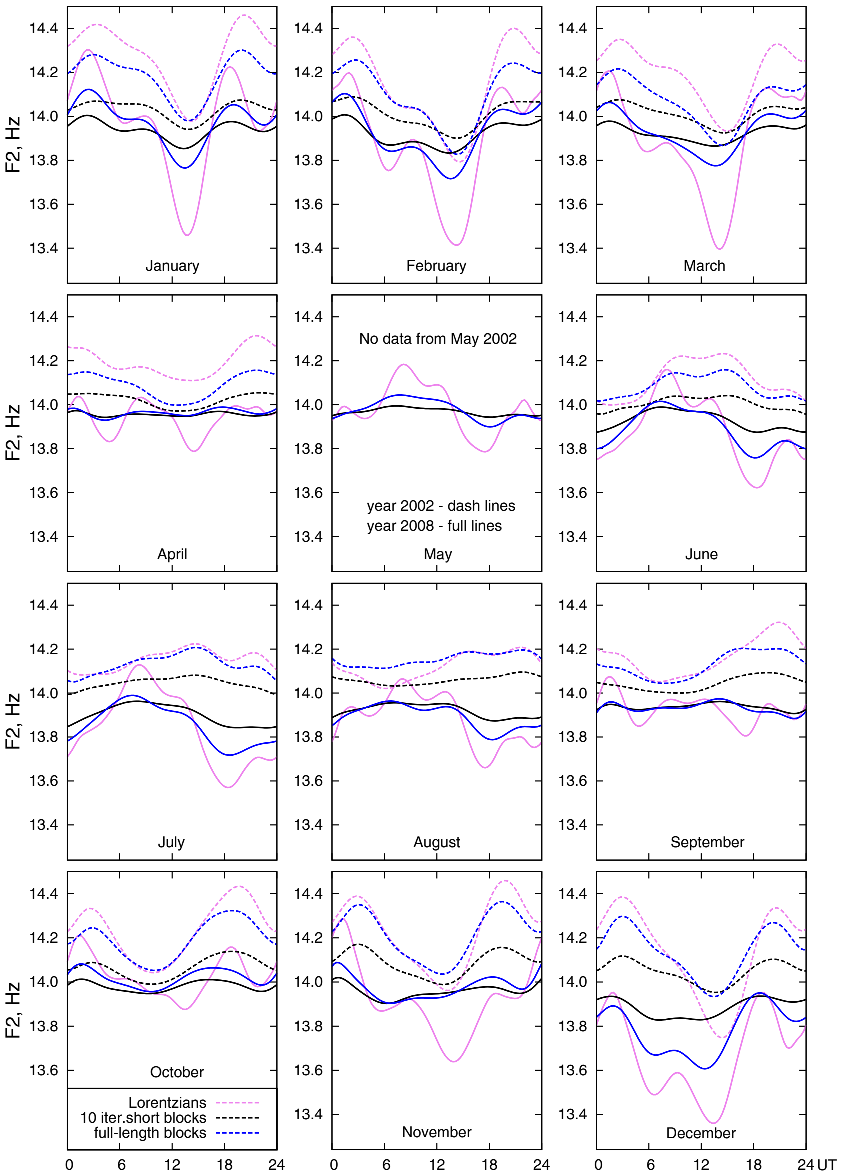

Fig. 6. The same as Fig. 5 but for the 2nd SR mode.

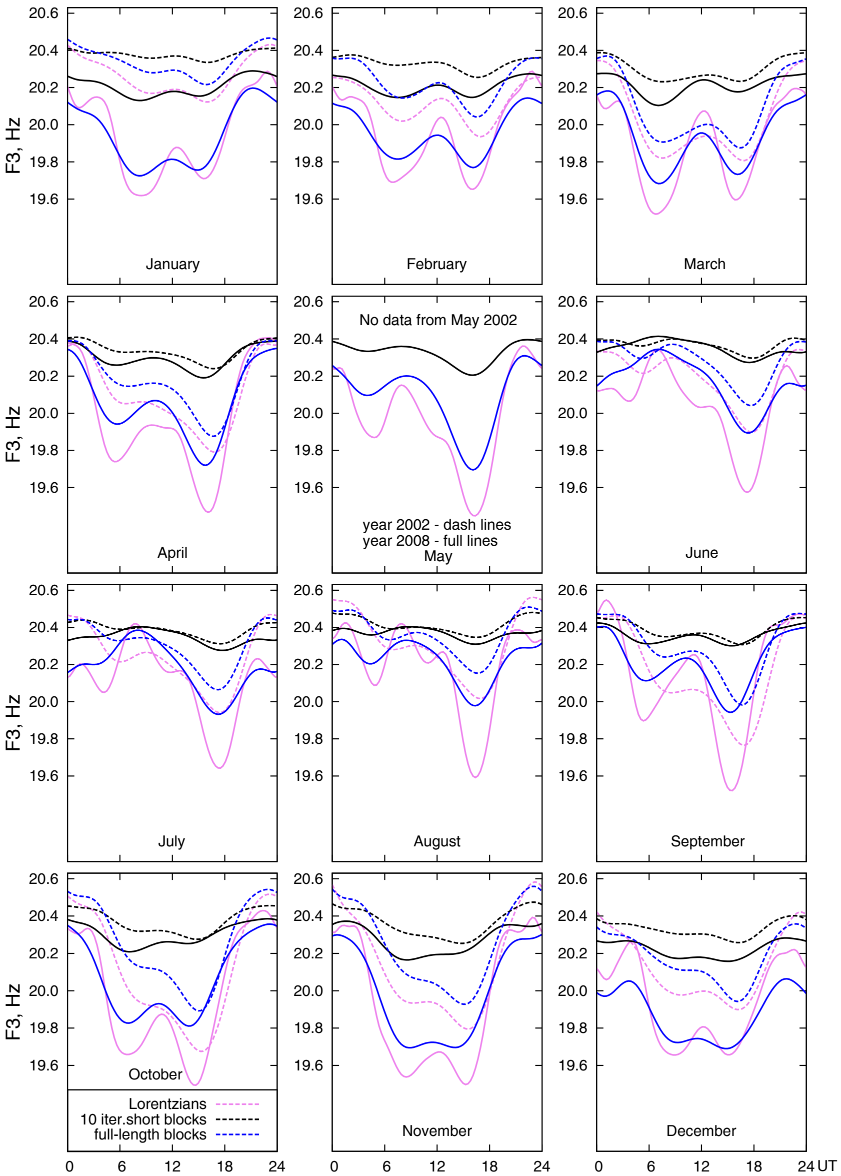

Fig. 7. The same as Fig. 5 but for the 3rd SR mode.

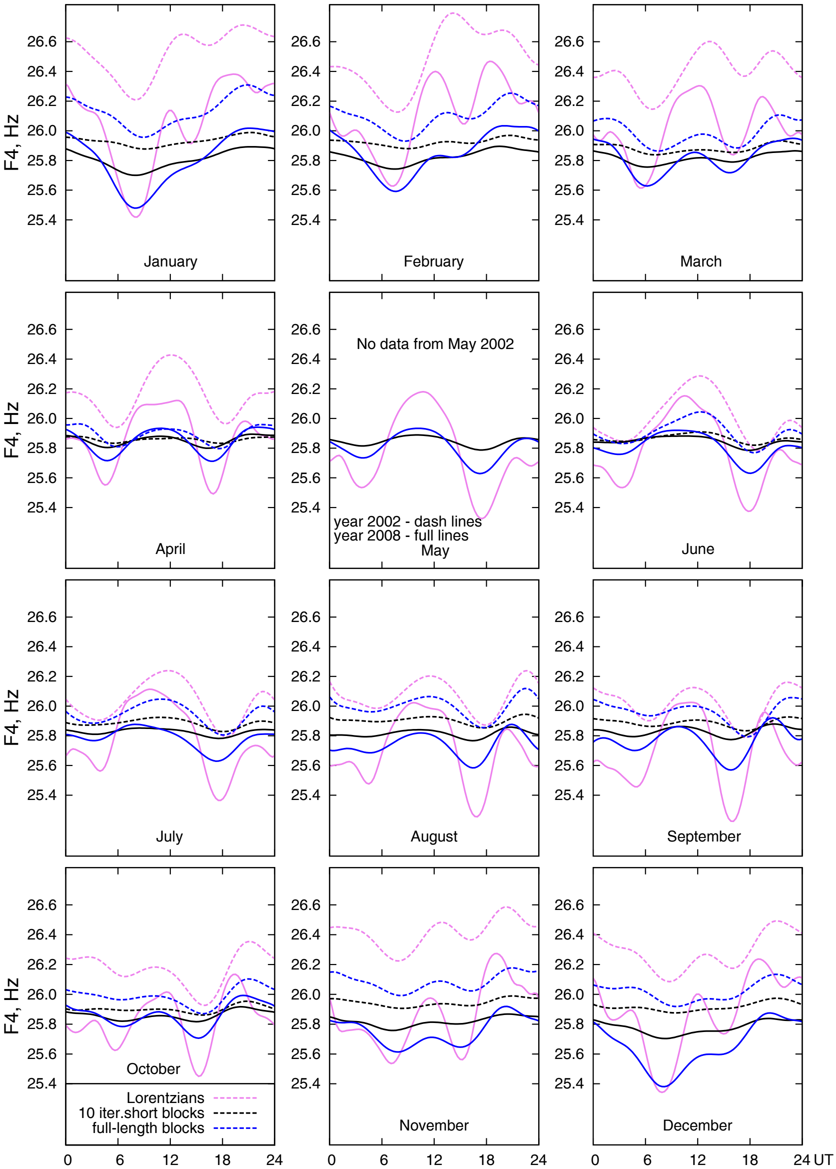

Fig. 8. The same as Fig. 5 but for the 4$^{\text{th}}$ SR mode.

– no matter what spectral technique, calculated modal frequencies show clear decrease from 2002 (solar cycle maximum) to 2008 (solar cycle minimum). Solar cycle variation in modal frequency is again greatest when Lorentz fitting is used *Ondrášková et al., (2011)*.

Computational feasibility of the CD and LFF methods applied to data of equal length is comparable. Modal frequency computation by CD method using short data sequences is fasted likely due to lower number of necessary iteration steps.

Comparison of both methods as to the accuracy of modal SR frequency determination is obviously possible solely when an adequate artificial signal is processed, which is a mixture of different waves with varying amplitudes and phases, noise and with a dominant Schumann mode of known frequency. This comparison has not been done in this work and will be a subject of future research.

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